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Design and Hardware Implementation of a Novel 3D Controllable Multi Wing Chaotic System with Constant Lyapunov Exponent Behavior

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Abstract

This article proposes a novel three-dimensional quadratic smooth autonomous system. Each equation in the system contains a nonlinear term, and the equations are highly symmetric. Similar to other chaotic systems, the newly proposed system has rich dynamic behavior and can generate one-,two-,three- and four-scroll attractors by changing a single parameter under certain parameter conditions. And there is more than one unique parameter that can achieve conversion between the four attractors like this. In addition, the system also has a constant Lyapunov exponent and complex transient transition behavior. The basic characteristics and complex dynamic phenomena of the system were studied using methods such as phase diagrams, bifurcation diagrams, and Lyapunov exponent diagrams. Finally, the system was simulated by Multisim software and an actual circuit was constructed to verify the results. The experimental results showed that the software simulation and final actual results had a high degree of agreement with the numerical model on the Matlab platform in the previous stage.

Keywords

Autonomous chaotic system; Controllable multi wing attractor; Constant Lyapunov exponent; Transient transition.

1. INTRODUCTION

In recent decades, chaos has been well-known as a very interesting nonlinear phenomena. Since the discovery of the Lorenz system in 1963,[1] this system has been regarded as the first chaotic model by later generations.[2] on this basis,various well-known Lorenz like system models have been developed , such as the Rössler system, Chen system, Lü system, Liu system, etc., which have been reported one after another.[3] With the deepening of research on chaos, people have found that it can be used in a wide range of applications such as biology, secure communication, image encryption, nonlinear circuits, cellular neural networks, ecology, etc..[4] Distributed in all aspects of our lives, which has attracted more widespread attention.

Most attractors of chaotic systems with complex topological structures can be classified into two categories: multi wing attractors and multi vortex attractors. From the existing proposed systems, multi vortex attractors generally appear in Chua circuits or their generalized systems, while multi wing attractors generally originate from the Lorenz system family.[[5] There are now relatively systematic methods for generating multi vortex attractors. By using sine functions, step functions, switching functions, and saturation functions to increase the number of equilibrium points to achieve the goal of increasing the number of vortices.

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However, how to generate multi wing chaotic attractors from smooth systems is still a research field that needs to be tackled. Due to the complex dynamic characteristics of multi wing chaotic systems compared to general chaotic systems, they have better application prospects in fields such as image encryption, electronic engineering, and secure communication. For example, the multi wing system proposed by W et al. in 2017, it was applied to secure communication implementation based on active-passive drive-response synchronization method.[8]The multi wing system proposed by H et al. in 2020 is suitable for many engineering applications based on chaos. It has also been applied in chaotic image encryption.[9] There are many other applications like this.

According to reports, Qi[10]proposed a new three-dimensional quadratic smooth autonomous system that can generate four wing chaotic attractors with complex topological structures over a wide range of parameters. In 2009, Dadras proposed a three-dimensional system that could achieve dual, three, and four wing variable attractors by simply changing a single parameter. The following year, the person improved on the original system and proposed a four-dimensional autonomous system that could only change a single parameter to achieve one to four wing variable attractors.[12] In recent years, the study of multi wing attractors has remained a hot topic[13]with various systems emerging one after another. Some have implemented multi wing attractors by introducing memory circuits, obtained multi wing systems through fractional order ordinary differential methods,[19] and achieved their goals by introducing state feedback control.[20]However, from both theoretical and practical perspectives, people prefer to obtain a low dimensional and algebraically simple chaotic system. The new three-dimensional autonomous system proposed in this paper, compared to Yu et al. using sawtooth wave functions instead of absolute value functions to construct chaotic systems, or, like Tahir et al.. By applying state feedback controllers to the system. Moreover, the article did not adopt the method of introducing memory circuits such as memristors by Yuan et al. to construct chaotic systems. The equations only contain product nonlinear terms, making the circuit structure easier to design and implement with rich nonlinear phenomena, such as constant Lyapunov exponent behavior.

The Lyapunov index is an important quantitative indicator for measuring dynamic characteristics,[21] which represents the numerical characteristic of the average exponential divergence rate of adjacent trajectories in phase space. Lyapunov exponent diagram is an important tool for observing and analyzing chaotic phenomena. In this article, an interesting exponential spectrum phenomenon arises, which is the constant Lyapunov exponent behavior.When the parameters or initial conditions change within a certain range, the value of the Lyapunov exponent remains unchanged.[22]The occurrence of this phenomenon means that the system is not affected by any parameters within this range and can be used in areas such as secure communication that require high signal stability.

Transient chaos refers to the behavior of a nonlinear system that exhibits a chaotic state within a finite time, and then gradually transforms into a periodic or chaotic attractor over time. It is a common phenomenon in many nonlinear dynamic systems. In the system of this article, instantaneous chaotic phenomena such as the transition from a four wing chaotic state to a periodic state can be observed.

A new chaotic system was constructed on the basis of Ref.[10]. It is a three-dimensional smooth autonomous system with five equilibrium points, each equation containing a quadratic term, and its equation structure is quite symmetrical, which is an interesting phenomenon. Compared to Ref.[10] this system exhibits complex dynamic behaviors, including chaos, period doubling bifurcation, transient transitions, and other phenomena. And this article only uses three-dimensional equations to implement the phenomenon that can only be achieved by using four-dimensional equations, that is, by changing a single parameter to achieve the change of one

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to four wing chaotic attractors, is also discovered in this paper. In addition, another special parameter d can also achieve one to four wing chaotic attractors, which is rare in the current research systems. In addition, the system also has a constant Lyapunov exponent and complex transient transition behavior, proving that the system has complex topological structures and dynamic properties. The stability of the equilibrium point was analyzed, and the basic properties of the system were analyzed using phase diagrams, bifurcation diagrams, Lyapunov diagrams, etc. Finally, the authenticity of the theory was verified through simulation software and actual circuit construction.

The remaining content of this article is as follows. The second section introduces a new three-dimensional chaotic system. The third section analyzes the system proposed in the second section, analyzes its dynamic behavior, and conducts numerical simulations. The fourth section obtains the circuit simulation results of the chaotic system on the Multisim platform and implements the hardware circuit. The fifth section summarizes this article.

2. NEW SYSTEM MODEL

The new three-dimensional quadratic autonomous chaotic system equation can be expressed in the following form:

$$\begin{aligned} \dot{x} &= y + ax - yz + d \\ \dot{y} &= xz - by + z \\ \dot{z} &= xy - cz + x \end{aligned} \tag{1}$$

Where x ,y and z are state variables, a, b, c, and d are system constant parameters.

2.1. Equilibrium point and stability analysis

In chaotic systems, the equilibrium point usually refers to a stable state in the system, and it plays a very important role in controlling and displaying chaotic attractors, especially in displaying multi wings. Some scholars generally believe that the equilibrium point is closely related to the stability and attractors of the system.[23]Below, we can easily solve the equilibrium point of system(1) by simultaneously solving the following algebraic equations(2).

$$\begin{aligned} y + ax - yz + d &= 0 \\ xz - by + z &= 0 \\ xy - cz + x &= 0 \end{aligned} \tag{2}$$

There are five equilibrium points in the system, which can be written as follows:

$$E_1(x_1, y_1, z_1) \quad E_2(x_2, y_2, z_2) \quad E_3(x_3, y_3, z_3) \quad E_4(x_4, y_4, z_4) \quad E_5(x_5, y_5, z_5) \tag{3}$$

When parameters a=7, b=25, c=10.5, and d=0.5, the system equilibrium point can be represented as

$$\begin{gathered} E_1(-17.6475, -9.3904, +14.1019) \\ E_2(-15.7574, +7.7611, -13.1478) \\ E_3(-0.0714, -0.003, -0.0068) \\ E_4(+16.7190, -8.7795, -12.3872) \\ E_5(+14.8287, +8.4491, +13.3446) \end{gathered} \tag{4}$$

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The stability at the equilibrium point is discussed here. By linearizing the system (1) at the point, the Jacobian matrix is obtained as follows

$$J=\begin{bmatrix} a & 1-z & -y \\ z & -b & x+1 \\ y+1 & x & -c \end{bmatrix}$$

$$=\begin{bmatrix} 7 & -13.1019 & 9.3904 \\ 14.1019 & -25 & -16.6475 \\ -8.3904 & -17.6475 & -10.5 \end{bmatrix} \tag{5}$$

Therefore, the obtained linearized eigenvalues are

$$|\lambda I-J|=0 \Rightarrow \lambda_1=3.5144+14.8708i \ ,\lambda_2=3.5144-14.8708i \ ,\lambda_3=-35.5289$$

λ_1 and λ_2 are a pair of complex conjugate eigenvalues with positive real parts. Usually, we refer to fixed points with the same real part and complex conjugate eigenvalues as focal nodes. When the real part is positive, the focus node is unstable, and when the real part is negative, the focus node is stable. On the contrary, when the real part sign is the opposite real eigenvalue, this fixed point is called the saddle focus and is always unstable. Obviously, E_1 is the unstable focus node of system(1).

Similarly, the eigenvalues and stability of the Jacobian matrix at other equilibrium points can be calculated in the same way, as shown in Table1:

Table 1. Eigenvalues and stability of Jacobian matrices at all equilibrium points in system(1)

λ	E1	E2	E3	E4	E5
λ_1	3.5144+14.8708i	2.5270+14.0950i	7.0015	3.0559+13.7668i	2.7114+14.1583i
λ_2	3.5144-14.8708i	2.5270-14.0950i	-10.5083	3.0559-13.7668i	2.7114-14.1583i
λ_3	-35.5289	-33.5540	-24.9932	-34.6117	-33.9228
stability	instability	instability	instability	instability	instability

2.2. Dissipation

To ensure that system(1) is dissipative, the conditions should be met

$$\nabla V=\frac{\partial \dot{x}}{\partial x}+\frac{\partial \dot{y}}{\partial y}+\frac{\partial \dot{z}}{\partial z}=a-b-c<0 \tag{6}$$

Obviously, the parameters set by system(1) satisfy the condition of dissipation. For example, when a=7, b=25, c=10.5, and d=0.5, there is

$$\frac{dV}{dt}=(a-b-c)V=PV \Rightarrow V=V_0e^{Pt} \tag{7}$$

$$\frac{dV}{dt}=(7-25-10.5)V=-28.5V \Rightarrow V=V_0e^{-28.5t} \tag{8}$$

This means that when $t\to\infty$, the volume units of the system trajectory contract exponentially to zero. Thus, all system trajectories ultimately converge to a specific set of zero

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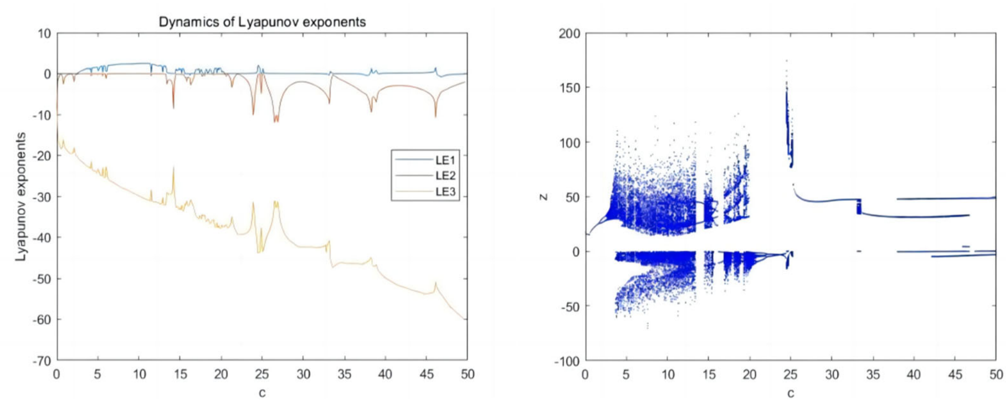
volume, and all trajectories ultimately move around the attractor, confirming the existence of the attractor.[24]

2.3. Bifurcation graph and Lyapunov exponential graph

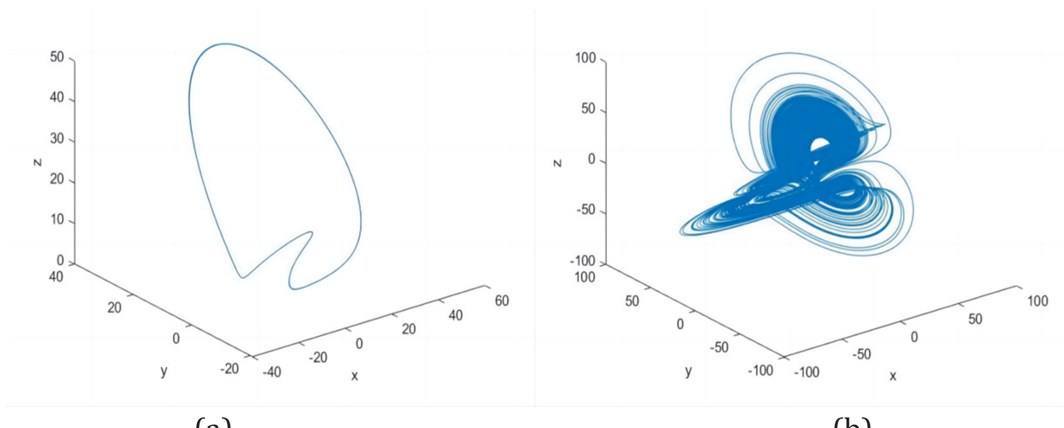
Bifurcation diagrams and Lyapunov diagrams are important means of studying chaos. As shown in Fig.1(a), it is an exponential graph of the variation of system (1) with parameter *c*. From the graph, we can see that the maximum Lyapunov exponent of the system is positive for a long range, which means that the system is in a chaotic state within this range.

Fig.1(b) shows the bifurcation diagram of parameter c as a function of state variable *z*. As is well known, bifurcation diagrams can be used to locate the transition between periodic and chaotic motion. From the graph, we can also see that the system has undergone a period doubling bifurcation to chaos, then transformed into periodic behavior, and then returned to the process of inverse period doubling bifurcation after a brief interval chaos. According to the bifurcation diagram, when we set the parameters a=7, b=25. c=14, and d=0.5, we can observe the periodic behavior shown in Fig.2(a). When we set parameter c to 15 while keeping other parameters unchanged, we can observe chaotic behavior as shown in Fig.2(b). The Lyapunov exponent spectrum in Fig.1(a) matches well with the bifurcation diagram in Fig.1(b).



(a) (b)

Figure 1. (a)Lyapunov exponent plot of system (1) as a function of parameter c .(b) Bifurcation diagram of parameter c with state variable z.



(a) (b)

Figure 2. (a)Periodic behavior at a=7, b=25, c=14, d=0.5.(b) Four wing chaotic attractor at a=7, b=25, c=15, d=0.5.

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3. DYNAMIC BEHAVIOR AND NUMERICAL SIMULATION OF SYSTEMS

This chapter analyzes the dynamic behavior of the system through phase diagrams, bifurcation diagrams, Lyapunov diagrams, and time series diagrams, and uses the built-in Runge-kutta method in Matlab software for numerical simulation. It is interesting that we can observe various complex chaotic behaviors in the system, such as controllable multi wing attractors that can be generated by changing single parameter while keeping other parameters constant, constant Lyapunov exponent behavior that can be used for secure communication, and complex instantaneous chaotic state transitions.

3.1. Controllable multi wing attractor with one to four wings

The system can achieve the transformation of single wing, double wing, three wing, and four wing with only changing c single parameter, and there is more than one such parameter, which is an interesting nonlinear phenomenon. Using Matlab for simulation, when we set the parameters a=7, b=25, d=0.5, and the initial values x (0)=1, y (0)=1, and z (0)=1 to change the value of parameter *c*, we can observe different numbers of chaotic attractors in the image at specific parameters.

[1]When c=3.4, a single wing chaotic attractor appears. Figures 3 (a) - (d) show the observation results on different projection planes. As the differential equation of the system is known, the exact Lyapunov solution of the system equation can be obtained by discretizing the differential equation and using Matlab's built-in Runge-Kutta method. By code calculation, the Lyapunov exponents of the system under this parameter condition can be directly obtained, which are LE1=1.0386, LE2=0, and LE3=-22.1505. According to equation (1),the Lyapunov (LD) dimension of the system is

$$D_L = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 2 + \frac{L_1+L_2}{L_3} = 2 + \frac{1.0386+0}{|-22.1505|} \approx 2.0468 \tag{9}$$

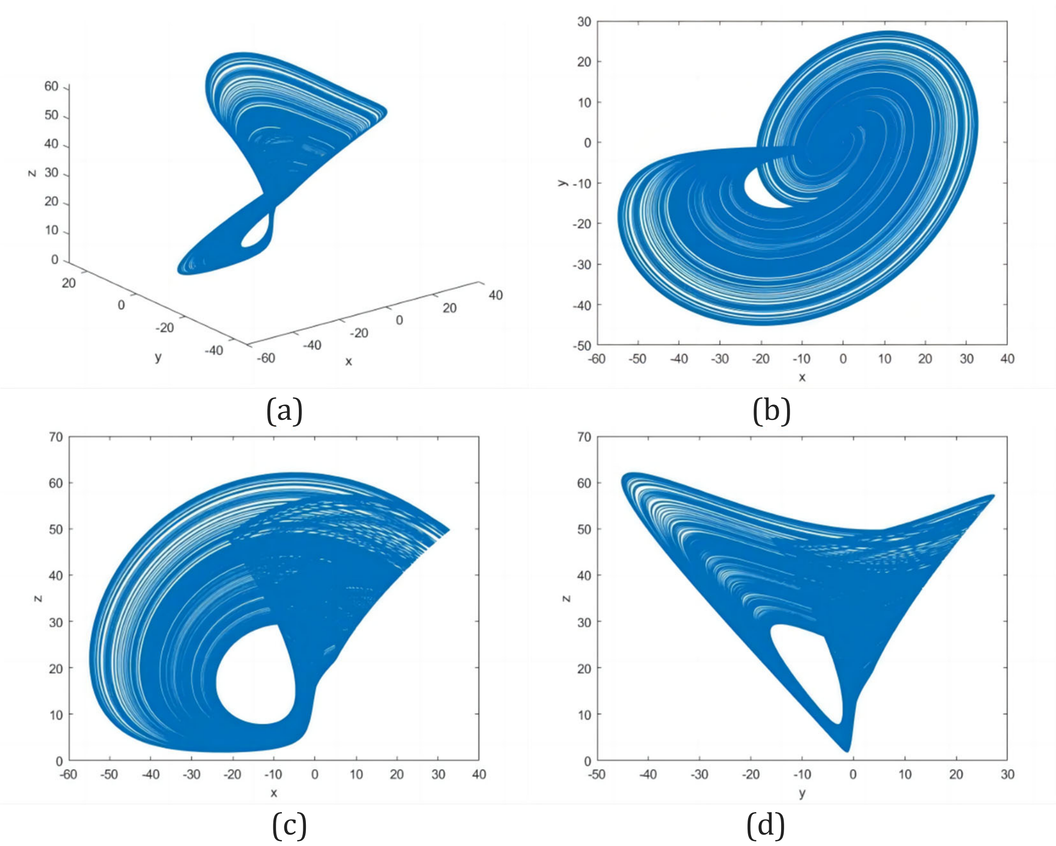

Figure 3. a = 7, b = 25, c = 3.4, d = 0.5 Chaotic attractor of the system.

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From the results obtained in the equation, we can easily see that the Lyapunov exponent of the system is not an integer but a fraction, which means that the system is dissipative. Due to the positive Lyapunov exponent and the presence of singular attractors, the system is clearly in a chaotic state.

[2] When c=20.2, a two wing chaotic attractor appears. Figures 4 (a) - (d) show the observation results on different projection planes. Under this parameter condition, the Lyapunov exponent of the system is LE1=0.5196, LE2=0, and LE3=-37.4139.

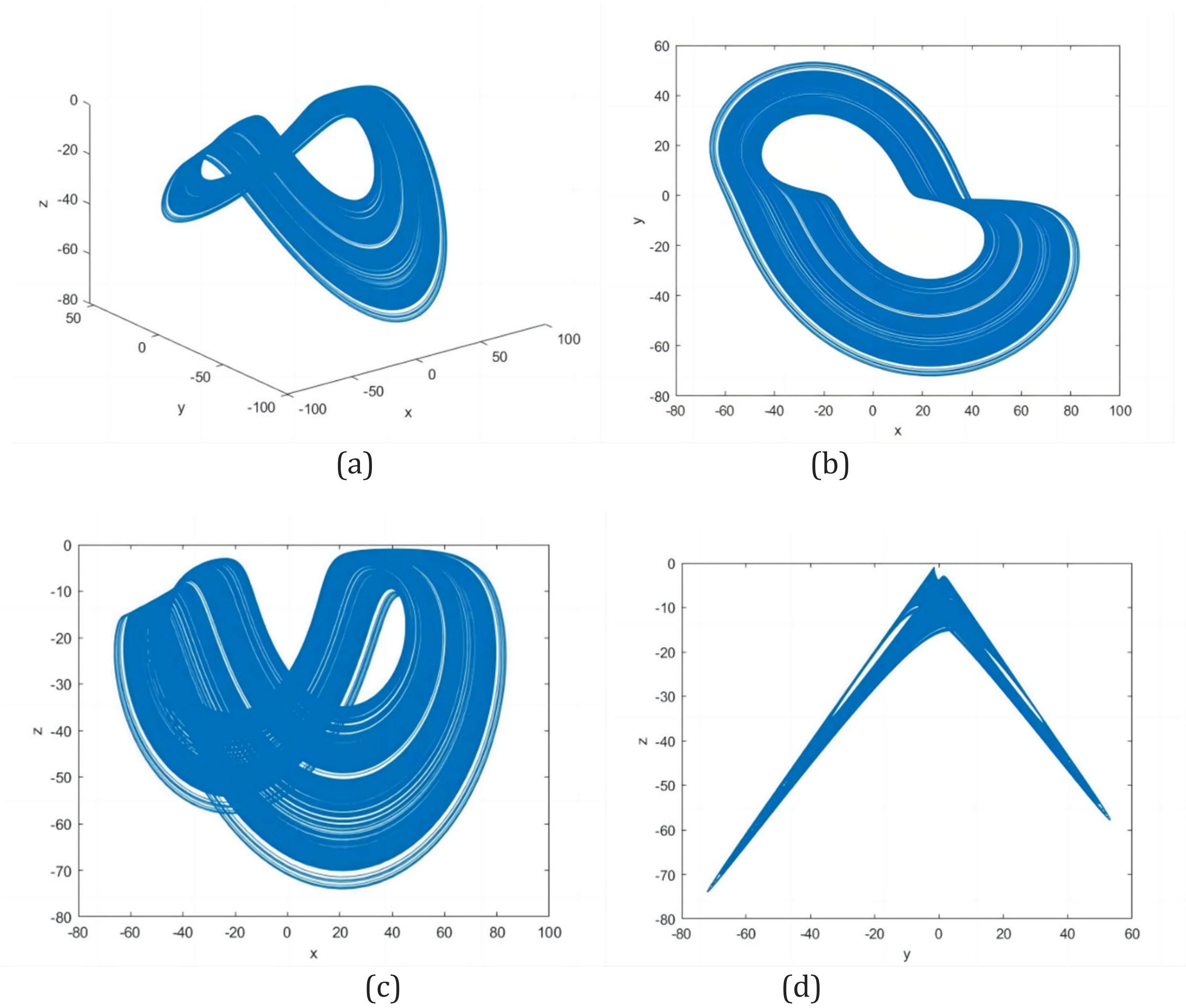

Figure 4. a = 7, b = 25, c = 20.2, d = 0.5 Chaotic attractor of the system.

[3]When c=3.7, a three wing chaotic attractor appears. Figures 5 (a) - (d) show the observation results on different projection planes. Under this parameter condition, the Lyapunov exponent of the system is LE1=1.1595, LE2=0, and LE3=-22.6309.

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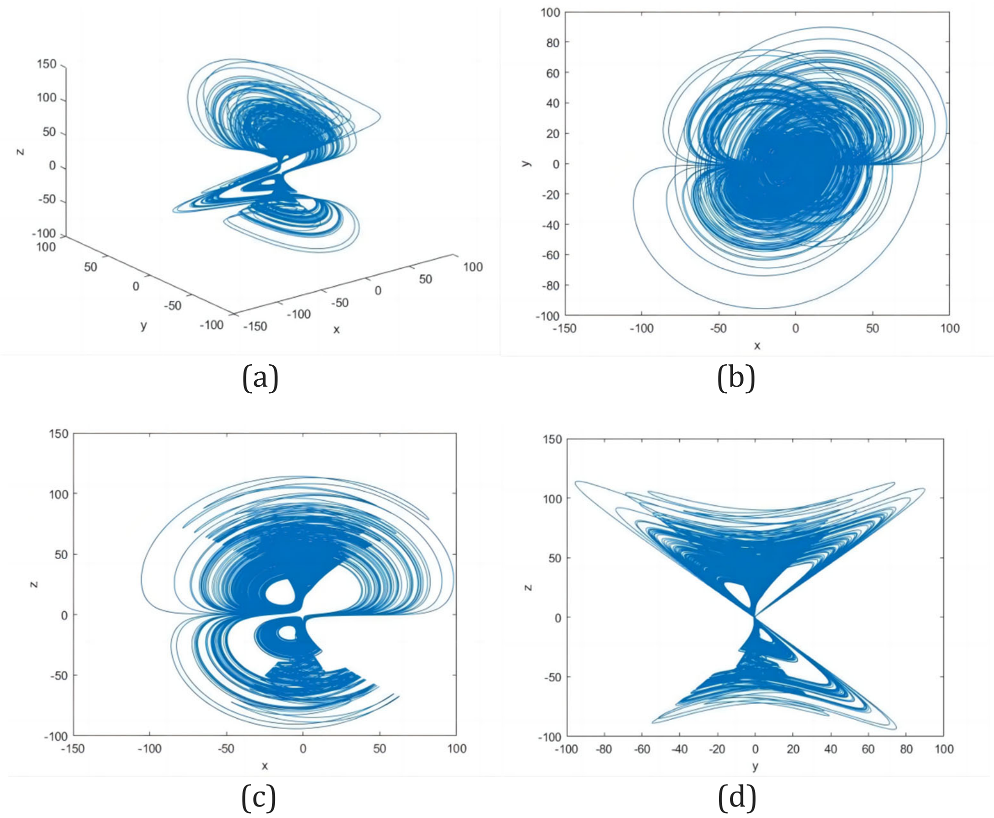

Figure 5. a = 7， b = 25， c = 3.7， d = 0.5 Chaotic attractor of the system.

[4] When c=10.5, a four wing chaotic attractor appears. Figures 6 (a) - (d) show the observation results on different projection planes. Under this parameter condition, the Lyapunov exponent of the system is LE1=2.4033, LE2=0, and LE3=-30.3557.

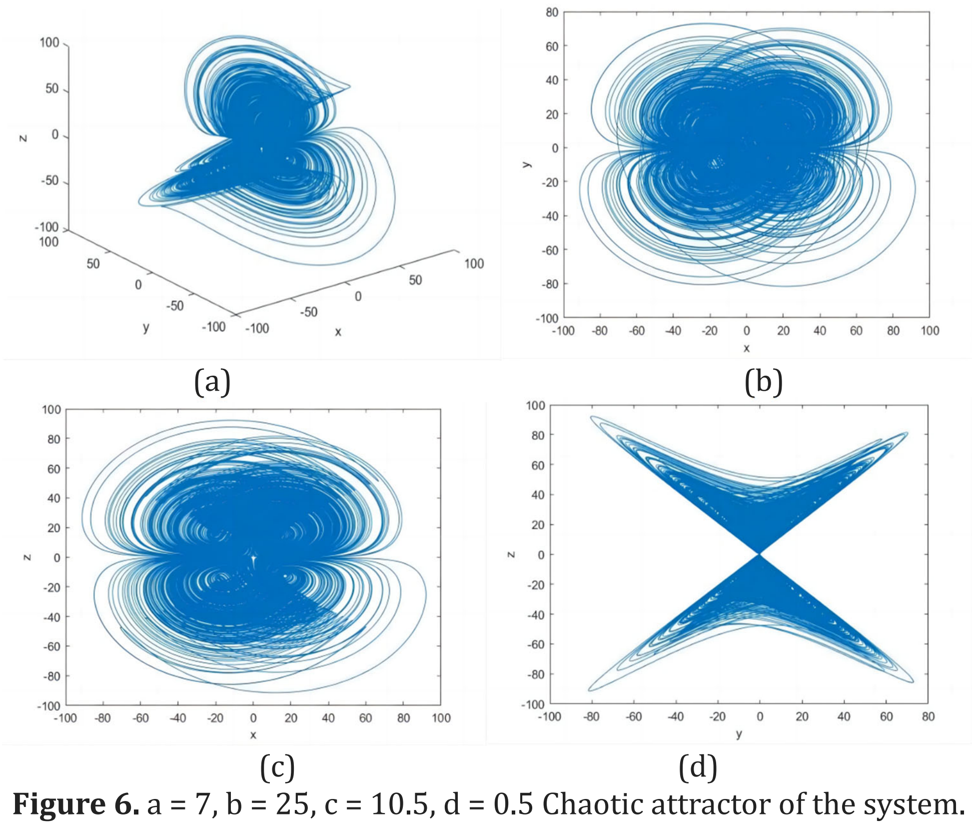

Figure 6. a = 7, b = 25, c = 10.5, d = 0.5 Chaotic attractor of the system.

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From the above figures, we can conclude that the system exhibits complex dynamic characteristics when selecting different values of parameter c and fixing other parameter values.

Similarly, when we set the parameters a=7, b=25, c=3.7, initial values x(0)=1, y(0)=1, z(0)=1, and change *d*, we can obtain different numbers of chaotic attractors on the *x*-*z* plane, as shown in Figures 7 (a) - (d).

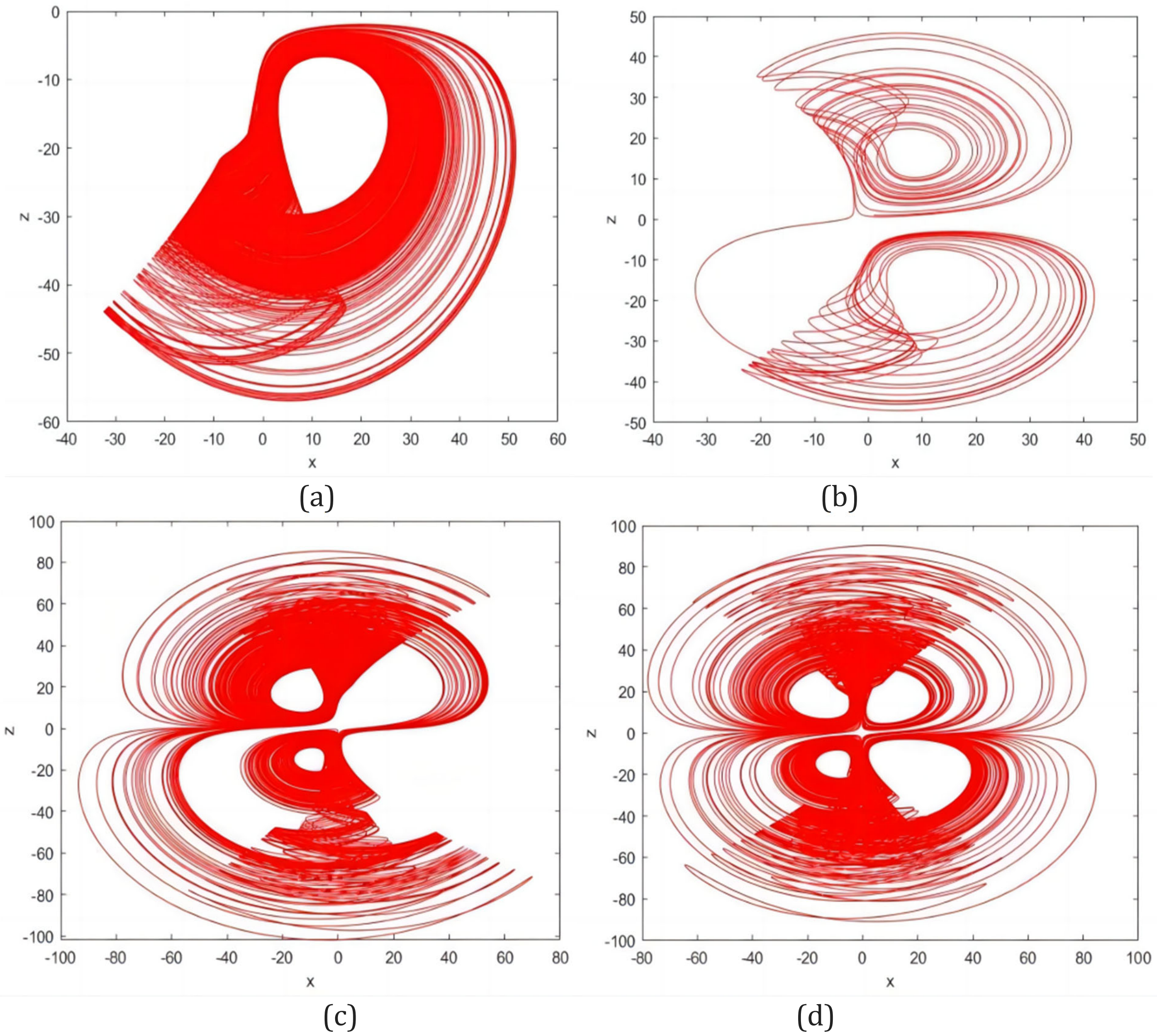

Figure 7. (a) Single wing d=15. (b) Double wing d=17.74. (c) Three wing d=0.1. (d) Four wing d=3.

There are two parameters in the same system, and the system that can achieve one to four wing chaotic attractors by changing either parameter is rarely seen in current research. Therefore, the system proposed in this article has certain research value.

3.2. Constant Lyapunov exponent behavior

constant Lyapunov exponent behavior has also been observed in previously reported chaotic systems with self-excited attractors.[25]when the initial conditions and parameter values involved change within a certain range, the Lyapunov exponent value exhibited by the system remains unchanged. Figures 8(a) - 8(b) show the corresponding Lyapunov exponents and bifurcation diagrams of the state variable d with parameters a=7.4, b=25, c=10, and initial conditions x(0)=1, y(0)=1, and z(0)=1, respectively. From Fig.8(a), it can be clearly seen that the control variable d continuously increases in the range of 0-50, but its maximum Lyapunov

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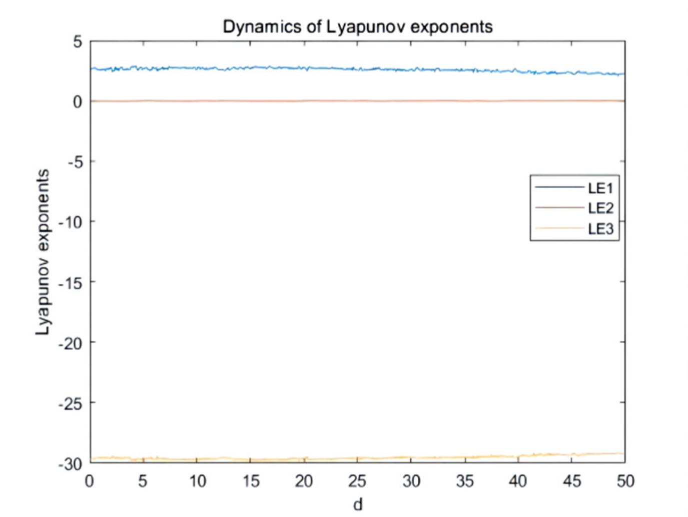
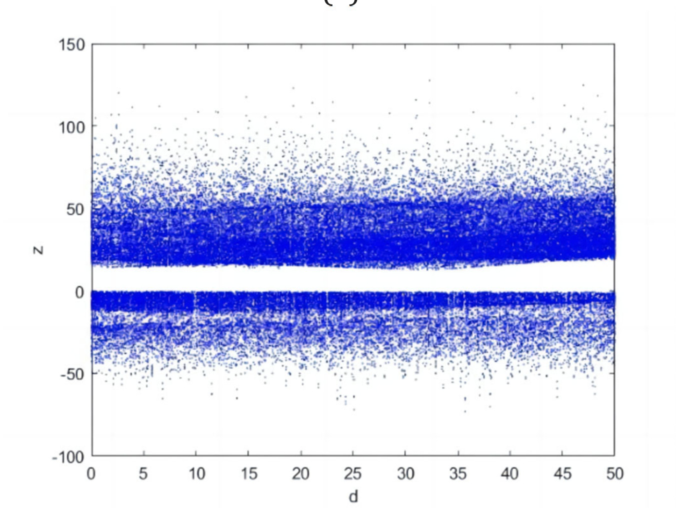
exponent has always been positive and remains basically unchanged. Combined with Fig.8(b), it can be seen that the system has always been in a stable chaotic state, i.e. a robust stable state. Systems in a robust stable state, due to their chaotic characteristics that are not subject to any parameter interference, can be used as a secure communication field based on chaos.[26] From the graph, we can see that the Lyapunov diagram and bifurcation diagram match quite well.



(a)



(b)

Figure 8. Constant Lyapunov behavior. (a) Corresponding Lyapunov exponent spectrum. (b) Bifurcation diagram.

3.3. Complex transient transition behavior

In the process of adjusting parameters and initial values to find chaotic attractors, we can discover some wonderful phenomena, such as transient transition behavior. Transient transition behavior,[27] also known as transient chaos, refers to the behavior of a nonlinear system exhibiting a chaotic state for a finite time and then transitioning into a periodic or chaotic state over time. As shown below, when we set the parameters a=7, b=25, c=11.5, d=0.5 and the initial values x(0)=1, y(0)=1, z(0)=1,the phase diagram and the corresponding time sequence diagram are shown in Figures 9(a) - (d). From the overall time sequence diagram in Fig.9(c), we can observe that the system has undergone a period of time from an unstable chaotic state to a stable periodic state. Fig.9(a) corresponds to the state changes of the system within the time range of 0-276 shown in Fig.9(d). At this time, the system presents a four wing attractor, while Fig.9(b) corresponds to the system state after time 276. At this time, the system

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is in a stable periodic state. Research has shown that this transient behavior has high application value.

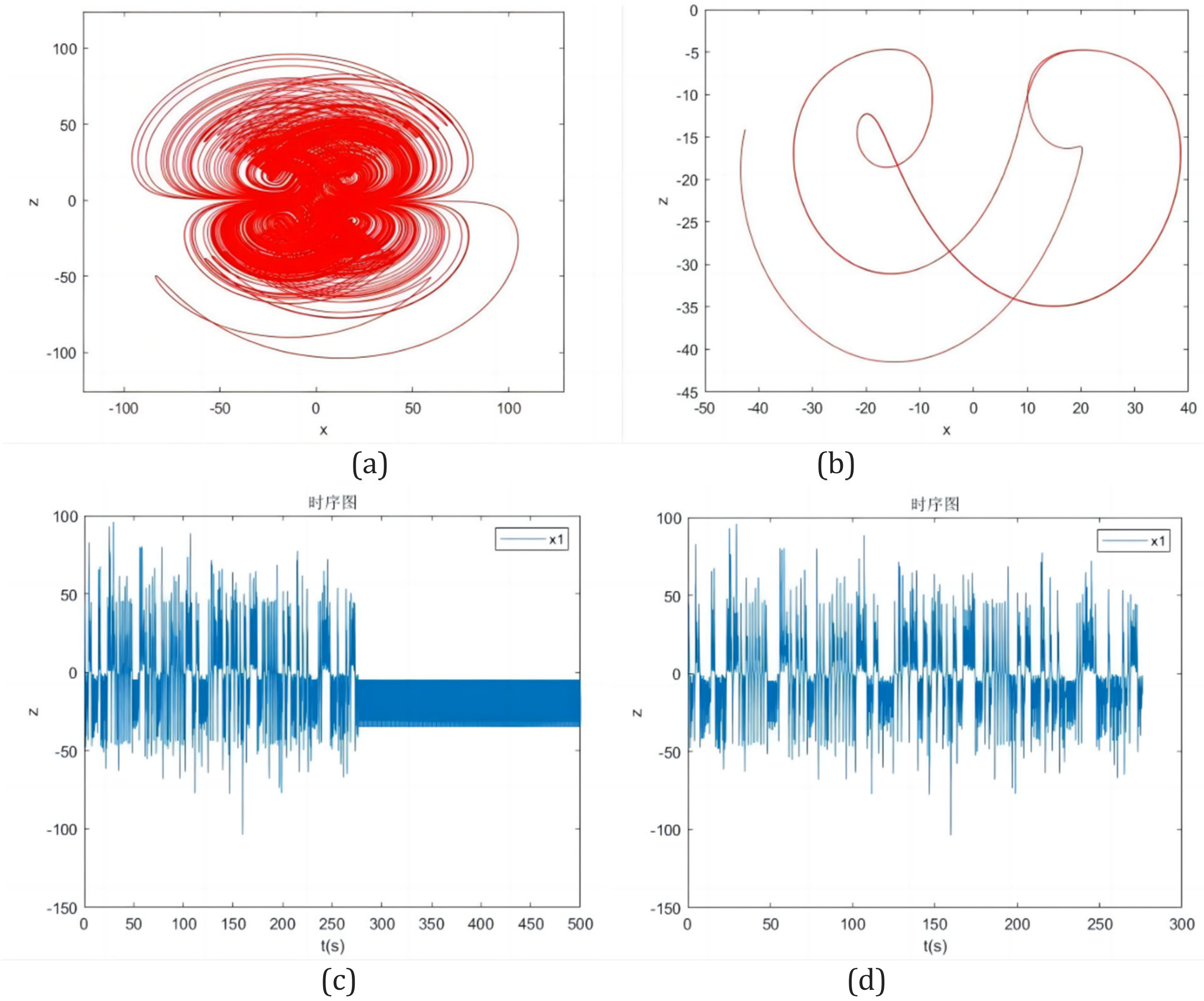

Figure 9. Complex transient transition behavior: (a) Transient chaotic hidden attractor. (b) Periodic attractor. (c) Time-domain waveform. (d) Time domain waveform within a finite time interval of 0 to 276 seconds.

4. CIRCUIT DESIGN AND HARDWARE CIRCUIT EXPERIMENTAL RESULTS

The circuit simulation implementation of chaotic systems is a method for studying their dynamic properties and verifying the feasibility and correctness of chaotic systems.[28]From the perspective of practical applications, the circuit implementation of nonlinear systems plays a crucial role.[29]In this section, we designed and implemented an analog electronic circuit diagram of the equation shown in system (1) using Multisim simulation software, as shown in Figure 10. It includes some basic electronic components. The circuit has 10 resistors, 3 capacitors, 4 operational amplifiers, and 3 multipliers. The operational amplifiers included in the circuit use TL082CD, with a power supply voltage of ± 15V and a saturation voltage of ± 13.5V. The multiplier uses AD633ANZ, and its output coefficient is 0.1. It should be noted that the state variables included in system (1) are not within the dynamic range, so we need to scale them down to a certain extent.

Based on the equation of system (1), we consider setting the amplitude scaling factor to 10 and setting the parameters a=7, b=25, c=3.4, and d=0.5 to obtain the following equation

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$$\begin{aligned}\dot{x} &= -y - 7(-x) - 10(-y)(-z) - (-0.05)\\ \dot{y} &= -10x(-z) - 25y - (-z)\\ \dot{z} &= -10x(-y) - 3.4z - (-x)\end{aligned} \tag{10}$$

Time scale transformation factor = 1500t,then equation (11) can be written as

$$\begin{aligned}\dot{x} &= -1000y - 7000(-x) - 10000(-y)(-z) - (-50)\\ \dot{y} &= -10000x(-z) - 25000y - 1000(-z)\\ \dot{z} &= -10000x(-y) - 3400z - 1000(-x)\end{aligned} \tag{11}$$

According to Kirchhoff's law, the corresponding circuit equation derived from Figure 10 can be represented as

$$\begin{aligned}\dot{V}_x &= -\frac{1}{R_1C_1}v_y - \frac{1}{R_2C_1}(-v_x) - \frac{1}{10R_3C_1}(-v_y)(-v_z) - \frac{1}{R_4C_1}(-v_1)\\ \dot{V}_y &= -\frac{1}{10R_5C_2}v_x(-v_z) - \frac{1}{R_6C_2}v_y - \frac{1}{R_7C_1}(-v_z)\\ \dot{V}_z &= -\frac{1}{10R_8C_3}v_x(-v_y) - \frac{1}{R_9C_3}v_z - \frac{1}{R_{10}C_3}(-v_x)\end{aligned} \tag{12}$$

Among them, V_x , V_y and V_z represent the voltages on capacitors C1, C2, and C3, respectively. Capacitance C1=C2=C3=10n, V0=1V. By comparing equations (12) and (13), the corresponding resistance values can be calculated,such as R1 = R7 = R10 = 100kΩ, R3 = R5 = R8 = 1kΩ, R2 = 14.3kΩ, R4 = 2MΩ, R6 = 4kΩ, R9 = 29.4kΩ.

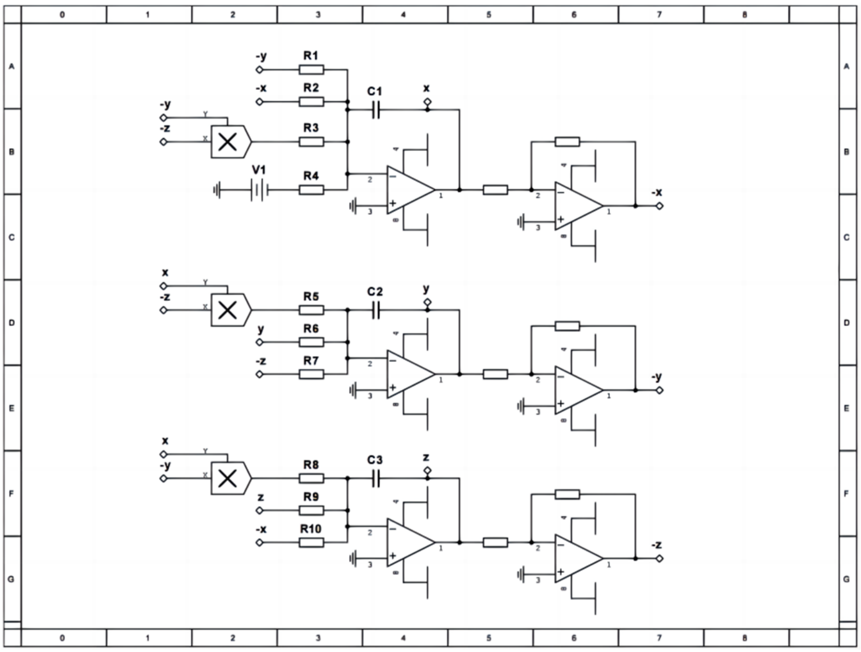

Figure 10. Circuit implementation of system (1)

In this case, we can observe the single wing attractor shown in Fig.11(a). Similarly, we fix the remaining resistors and only change the resistance values to obtain the other three types of chaotic attractors proposed in the second part of the article. The specific images are shown in

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Figures 11 (b) - (d). From the image, we can see that the results of using Multisim to build the circuit simulation are basically consistent with those of using Matlab simulation.

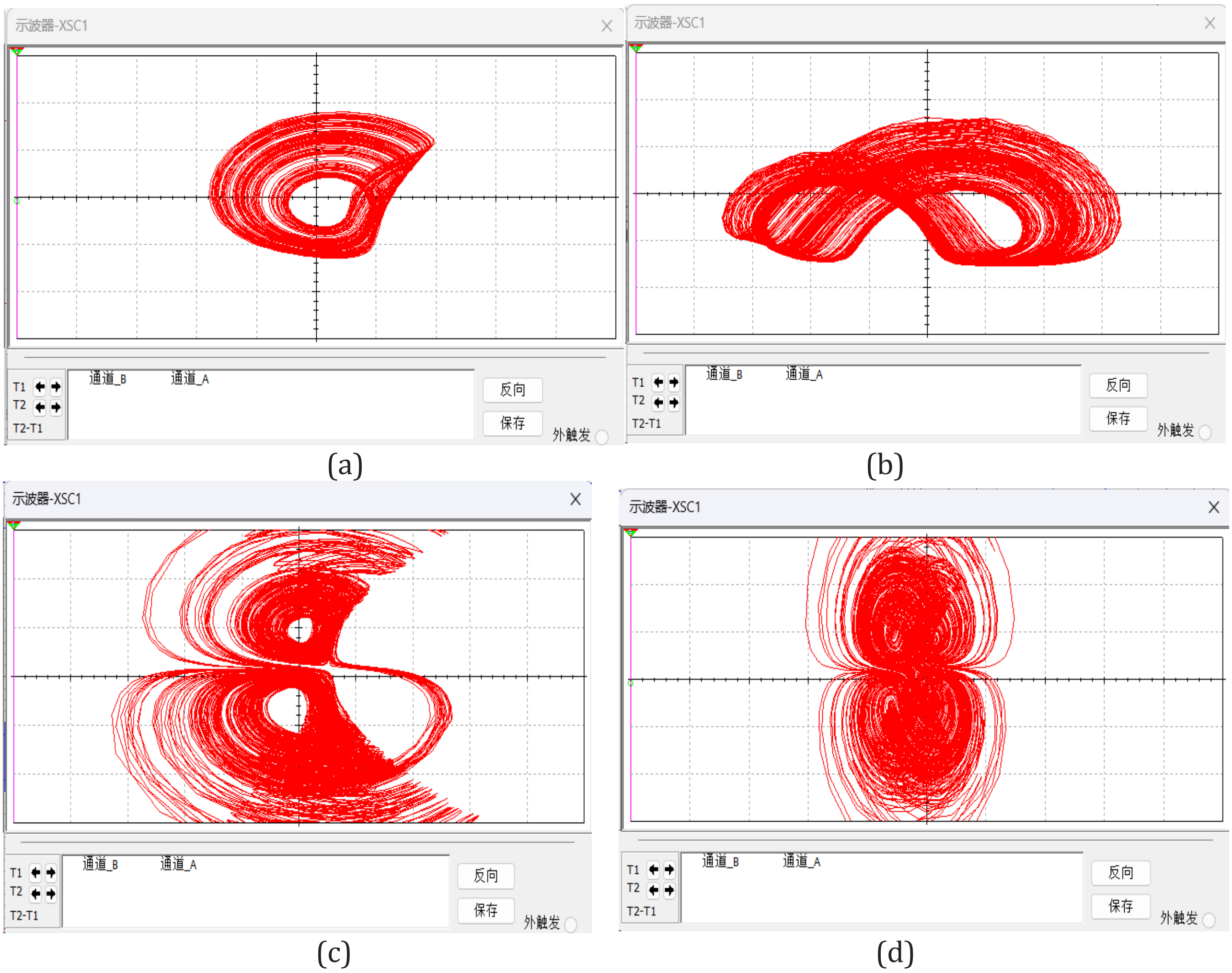

Figure 11. (a) One wing chaotic attractor (R_9=29.4kΩ). (b) two wing chaotic attractor (R_9=5kΩ). (c) three wing chaotic attractor (R_9=27kΩ). (d) four wing chaotic attractor (R_9=9.5kΩ).

In the above article, we designed and demonstrated the chaotic system on the NT Multisim platform. However, it should be noted that circuit simulation software such as Multisim did not consider various disturbances that may be encountered in actual situations. Therefore, the simulation results have certain limitations and sometimes lead to incorrect conclusions. In this case, we conducted hardware circuit experiments.

The actual environmental application of electronic circuit simulation is implemented on broadband as shown in Fig.12. Observe the experimental results in XZ mode using a digital oscilloscope (Tektronix TDS 3032B) (see Fig.13). Obviously, the hardware experimental results are in good agreement with the Matlab simulation results in Figures 3-6 and the Multisim simulation results in Figure 11, proving the correctness and feasibility of the designed circuit.[31]

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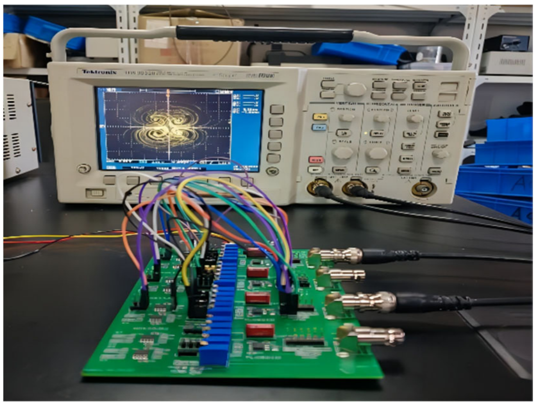
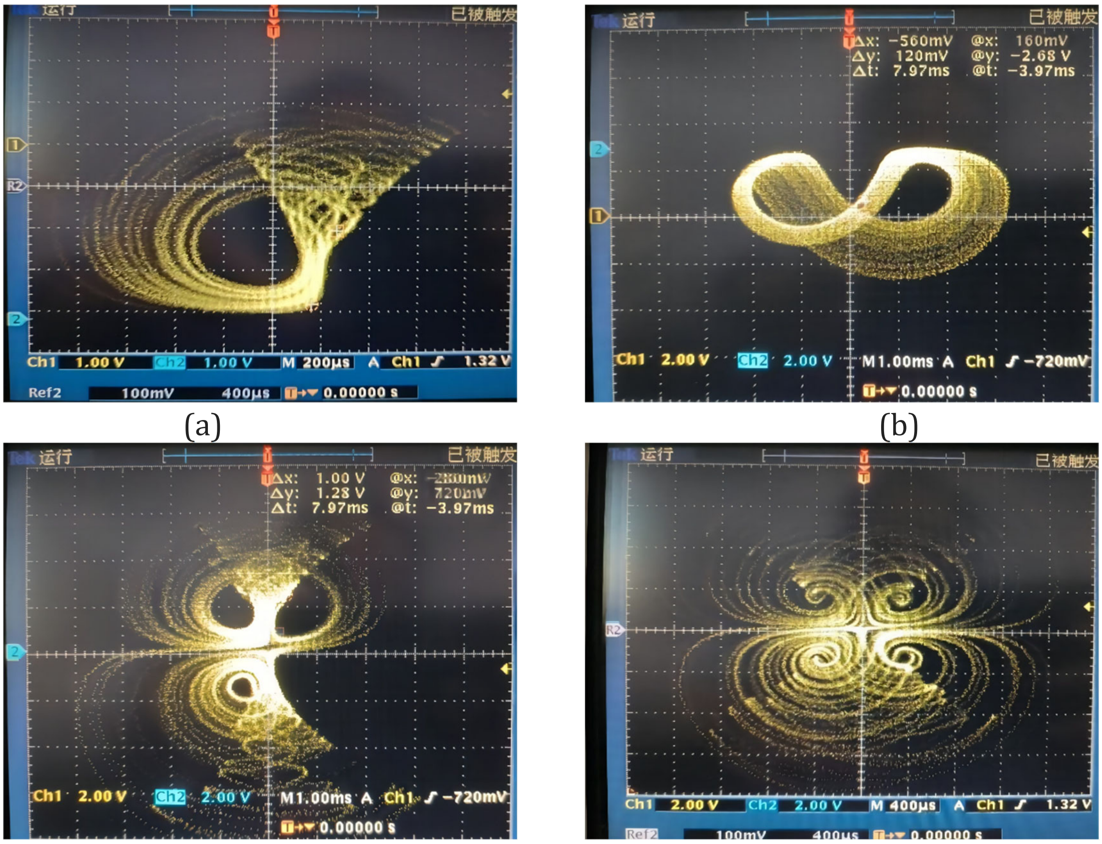


Figure 12. Application of Digital Oscilloscope Images in Real Environment.



(a) (b)

(c) (d)

Figure 13. Screenshots of the x-z plane variable wing attractor digital oscilloscope: (a) Single wing chaotic attractor. (b) Double wing chaotic attractor. (c) Three wing chaotic attractor. (d) Four wing hidden chaotic attractor.

5. DISCUSSION

This article is presented in Ref [10] on the basis of this, a novel 3D controllable multi wing chaotic system with constant Lyapunov exponent behavior was proposed,. The newly proposed system has five equilibrium points and the system equation includes three nonlinear terms, which is compared to Ref [10] in terms of the chaotic system proposed in the article, the new system achieves the transformation between single wing, double wing, three wing, and four wing by changing a single parameter using only three-dimensional equations, and there is more than one such parameter. More interestingly, by adding a constant control term d to the first equation, the system exhibits a constant Lyapunov exponent over a wide range, which is known as a robust stable state. This phenomenon enables the system to be better applied to secure

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communication based on chaos. We also conduct theoretical and numerical simulations and analysis of other complex chaotic phenomena that occur in the system through the use of phase diagrams, bifurcation diagrams, Lyapunov diagrams, etc.

On the basis of theoretical and numerical research, we designed and simulated the circuit modules, and the actual hardware circuits constructed also showed good agreement with the simulation results on the Multisim platform. Due to the existence of controllable multi wing chaotic attractors, constant Lyapunov exponent behavior, and complex transient transition phenomena in the new three-dimensional autonomous system designed in this article, the system constructed in this article can be well applied to engineering applications based on chaos, such as image encryption based on chaos.

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AUTHOR CONTRIBUTIONS STATEMENT

All authors contributed to the study conception and design. D.B.Y. Conceptualize the structure and write the content of the article, L.X.Y. Collaborate with D.B.Y. to conduct actual circuit

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simulation experiments and analyze the experimental results. All authors have reviewed the manuscript.

STATEMENTS AND DECLARATIONS

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