World Scientific Research Journal (WSRJ) ISSN: 2472-3703
www.wsr-j.org

Single-Photon Transport along a One-Dimensional Waveguide Interacting with a Quadratic Optomechanical Cavity

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Abstract: We investigate the single-photon transport in a one-dimensional waveguide side-coupled to a quadratic optomechanical cavity. The effects of the single-photon coupling strength, the coupling strength between the cavity and the waveguide, the cavity dissipation on the single-photon transport properties are discussed. Interestingly, it is found that the coupling strength between the cavity and the waveguide can weaken the optomechanical effect and the influence of the cavity dissipation.

Keywords: single-photon transport, one-dimensional waveguide, quadratic optomechanical cavity

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1. INTRODUCTION

Single-photon transport in waveguide quantum electrodynamics systems coupled with atoms or artificial quantum dots has attracted a great deal of interest for its important applications in quantum information processing and the implement of all-optical quantum devices [1, 2]. Considerable theoretical and experimental works reported the issue of controlling the photon transport in various systems, such as a one-dimensional waveguide or coupled optical cavities both embedded with atoms or semiconductor quantum dots [3], or side-coupled to them [4].

Recently, cavity optomechanics has opened a new route of controlling single-photon transport [5]. It has more advantages and applications over traditional quantum microcavities, such as in ultrasensitive measurements of the position of a mirror allowing for gravitational wave detection, realization of macroscopic quantum objects, and as a fundamental platform for exploring coupling to other quantum systems [5]. With the rapid in optical trapping and microfabrication techniques, cavity optomechanics are approaching the single-photon strong-coupling regime [6]. So it is convenient to control the single-photon transport in a waveguide by means of the optomechanical system within single-photon strong-coupling regime. More recently, some works focus on this topic. Jia'group studied the model of a linear optomechanical cavity with a two-level atom inside coupled to a one-dimensional waveguide [7]. They adopted a real-space approach to obtain the stationary transport properties of the injected photon. The photon transmission spectra showed a multiphonon structure owing to the optomechanical effect. Also, an analogous Rabi-splitting and EIT-like phenomena in the single-photon spectra were observed in different parameter regimes. Ren et al. [8] reported the single-photon spectra in the strong-coupling regime for a single-mode fiber coupled to a linear optomechanical cavity in the absence of atoms. The effects of cavity and mechanical dissipation on the photon transport were explored. In Ref. [9], single-photon emission and scattering in a quadratically-coupled optomechanical system was reported. They used a Laplace transform method to obtain the single-photon emission and scattering spectra. Moreover, it was found the condition under which phonon sideband peaks can be observed in the photon spectra was clarified. However, little attention is paid to a quadratic optomechanical cavity interacting with an optical waveguide. In such a system, there may be new physical phenomena.

In this paper, we study single-photon transport in a one-dimensional waveguide side-coupled to a quadratic optomechanical cavity. The effects of the single-photon optomechanical coupling strength, the coupling strength between the cavity and the waveguide, the cavity dissipation on the single-photon transport properties are investigated. The paper is organized as follows: In Section 2, we describe the Hamiltonian of the considered system and derive the transmission and reflection amplitudes of the single-photon. Section 3 is devoted to the analysis of the influences of the optomechanical system on the single-photon transmission and reflection. Finally, a brief summary is given in Section 4.

2. MODEL

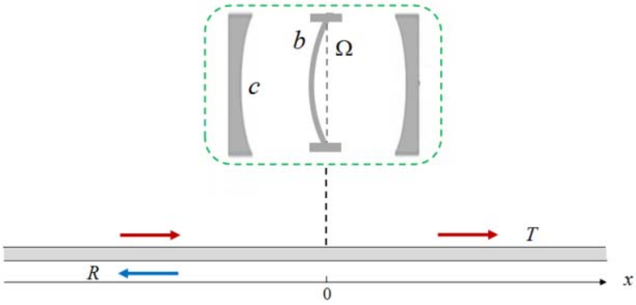

Fig. 1 Schematic diagram of a quadratic optomechanical system with a “membrane-in-the-middle” configuration, interacting with a one-dimensional optical waveguide. The single-photon propagating along the arrow direction in the one-dimensional optical waveguide.

We consider the model of an optical waveguide side-coupled to a quadratic optomechanical cavity. The Schematic diagram is shown in Fig. 1. Using the linearization of photonic dispersion with a waveguide mode, and within the Markov approximation [3, 4], the Hamiltonian of the considered system could be written as($\hbar = 1$)

$$\begin{aligned} H &= (\omega_c - i\gamma)c^+c + \Omega b^+b + g_0 c^+c(b+b^+)^2 \\ &+ \int dx a_R^+(x)\left(-iv_g \frac{\partial}{\partial x}\right)a_R(x) + \int dx a_L^+(x)\left(iv_g \frac{\partial}{\partial x}\right)a_L(x) \\ &+ V\int \delta(x)[a_R^+(x)c + a_R(x)c^+ + a_L^+(x)c + a_L(x)c^+] \end{aligned} \tag{1}$$

Where $a_R^+(x)$ $\left(a_R(x)\right)$ and $a_L^+(x)$ $\left(a_L(x)\right)$ are the bosonic creation (annihilation) operators of the right- and left-going photons at position *x*, respectively. The photon group velocity is v_g. The imaginary part $-i\gamma$ describes the cavity dissipation. c^+ (b^+) is the photon (phonon) creation operator. ω_c and Ω are the optomechanical cavity and the mechanical mode resonance frequency, respectively. g_0 is the single-photon coupling strength between the cavity and the mechanical mode. *V* denotes the coupling strength between the cavity and the waveguide. $\delta(x)$ indicates that the interaction occurs only at $x = 0$. The decay rate of the photon from the optomechanical cavity guided into the waveguide is given as $\Gamma = V^2/v_g$.

Assuming that a single-photon is injected into a one-dimensional optical waveguide, the stationary eigenstate in single-photon subspace for the whole Hamiltonian can be written as

$$|\psi\rangle = \sum_n \int dx[\phi_R(x,n)a_R^+(x)|\varnothing\rangle|n\rangle_b + \phi_L(x,n)a_L^+(x)|\varnothing\rangle|n\rangle_b] + \sum_n e_n c^+|\varnothing\rangle|\tilde{n}\rangle_b \tag{2}$$

Where $\phi_{R,L}(x, n)$ denotes the wave functions of the right- and left-propagating photons. $|\varnothing\rangle = |0\rangle_k |0\rangle_c$ implies no photon in both the waveguide and the optomechanical cavity. $|n\rangle_b$ is the number state of the mechanical mode. e_n represents the excitation amplitude of the optomechanical cavity. $|\tilde{n}\rangle_b = \exp[\frac{1}{8}\ln(1+\frac{4g_0}{\Omega})(b^2 - b^{+2})]|n\rangle_b$ is the single-photon displaced number state of the mechanical oscillator satisfying the eigenequation

$$[\omega_c c^+c + \Omega b^+b + g_0 c^+c(b+b^+)^2]|1\rangle_c|\tilde{n}\rangle_b = (\omega_c + n\Omega + \delta)|1\rangle_c|\tilde{n}\rangle_b \tag{3}$$

With $\delta = \Omega(\sqrt{1+4g_0/\Omega} - 1)/2$ the photon-state frequency shift caused by a single-photon radiation pressure. $\phi_{R,L}(x, n)$ are given by

$$\phi_R(x,n) = [\theta(-x)\delta_{nn_0} + \theta(x)t_n]e^{i[k+(n_0-n)\frac{\Omega}{v_g}]x} \tag{4}$$

$$\phi_L(x,n) = \theta(-x) r_n e^{-i[k+(n_0-n)\frac{\Omega}{v_g}]x} \tag{5}$$

Where t_n and r_n presents the transmission and reflection coefficients, respectively. $\theta(x)$ stands for the step function. Here, we have assumed that the initial state of the mechanical resonator is in number state $|n_0\rangle_b$. For the sake of simplicity, we let $n_0 = 0$ to get numeric results in the following discussion. According to the eigenequation $H|\psi\rangle = \varepsilon|\psi\rangle$, three coupled equations can be obtained as follows:

$$-iv_g(t_n - \delta_{nn_0}) + V\sum_m e_m U_{nm} = 0 \tag{6}$$

$$-iv_g r_n + V\sum_m e_m U_{nm} = 0 \tag{7}$$

$$\frac{V}{2}(\delta_{nn_0} + t_n + r_n) = \sum_m [\widetilde{\Delta}_c(m) - i\Gamma] e_m U_{nm} \tag{8}$$

Where $\widetilde{\Delta}_c(m) = \Delta_c + (n_0 - m)\Omega - \delta + i\Gamma + i\gamma$, $\varepsilon = v_g k + n_0\Omega$, $U_{nm} = \langle n|_b|\widetilde{m}\rangle_b$, and $\Delta_c = v_g k - \omega_c$ denotes the detuning between the incident photon and the cavity. By solving the above equations, the photon's transmission and reflection spectra could be obtained based on the equations $T = \sum_n |t_n|^2$ and $R = \sum_n |r_n|^2$. What follows, we will discuss the influences of the single-photon optomechanical coupling strength, the cavity decay rate and the interaction amplitude of the cavity and the waveguide on coherent transport properties of the single photon.

3. SINGLE-PHOTON TRANSMISSION AND REFLECTION SPECTRA

3.1 Without cavity dissipation

Firstly, we focus on the case of cavity non-dissipation. We now demonstrate the effects of single-photon optomechanical coupling strength, and the interaction amplitude between the cavity and the waveguide on single-photon transport properties. The group velocity of the photon is chosen as $v_g = 1$ for numerical investigation. Fig.2 shows the influences of the optomechanical coupling strength g_0 on the transmission behavior of the injected photon. As depicted in Fig.2(a), for having no optomechanical effect (i.e., $g_0 = 0$), the transmission and reflection spectra show symmetric line shape, where the resonant photon is completely reflected and meanwhile the transmission amplitude is zero (i.e., R=1, T=0). The results are similar to the case of a two-level atom interacting with an optical waveguide [2]. In this situation, the cavity serves as an optical switch to control the transport behavior of the single photon. With increasing of the single-photon coupling strength, the spectra show a multiphonon process where the transmission (reflection) spectra become asymmetric and some small transmission dips (reflection peaks) next to the main dip (peak) appear, as shown in Figs.2(b)-2(f). It is also seen that the position of the main dip (peak) at around $\Delta_c = 0$ moves in the positive direction along Δ_c axis, which is clearly exhibited in Fig.3, a magnified graph of Fig.2. Furthermore, the minimum of the main transmission dip increases with increasing of the single-photon coupling strength. Specifically, the value increases from 0 to 0.1 when g_0 changes from 0 to 2.

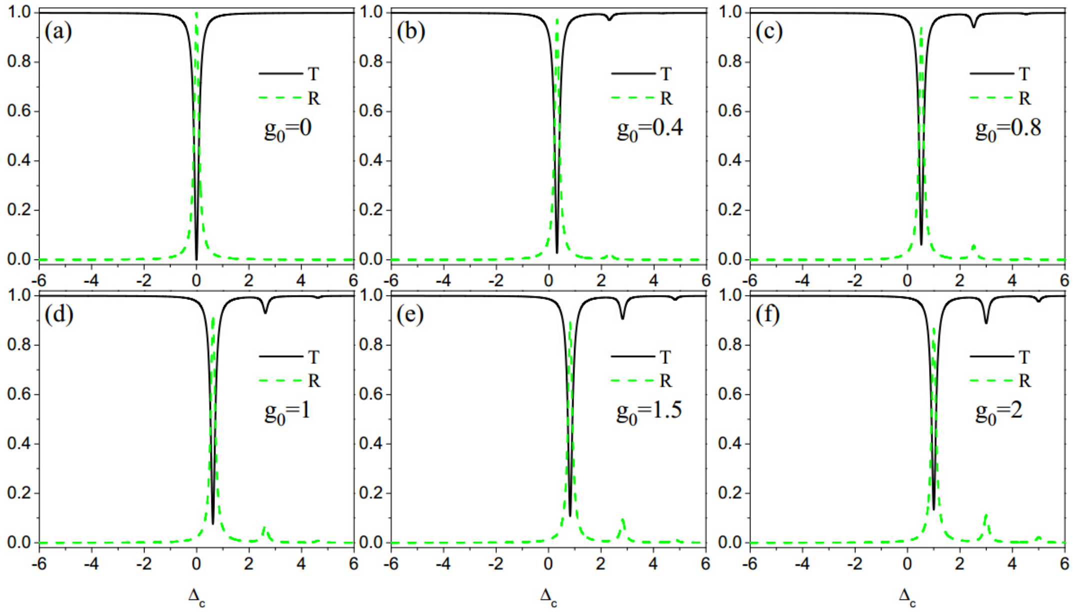

Fig. 2 Single-photon transmission and reflection spectra for different single-photon optomechanical coupling strengths with the parameters γ=0 and Γ=0.1, all the parameters are in units of Ω

Next, we explore how the coupling strength between the optomechanical cavity and the optical waveguide affects the single-photon transport behavior. It is convenient to use Γ to characterize the interaction because it describes the decay rate of spontaneous emission into the one-dimensional waveguide, which is given as $\Gamma = V^2/v_g$. As is shown in Fig.4, the width of the transmission dip at around $\Delta_c = 0$ increases rapidly with increasing of the value of Γ, which is similar for the two cases of the quadratic optomechanical cavity (g_0=0, Fig.4(b)) and a traditional optical cavity including no optomechanical effect (g_0=1.5, Fig.4(a)). However, when a quadratic optomechanical coupling is considered, in contrast to the situation of no optomechanical coupling, the spectra is not symmetry and the small dips become very feeble as the coupling strength gets strong. These results imply that the coupling strength between the optomechanical cavity and the optical waveguide can weaken the optomechanical effect.

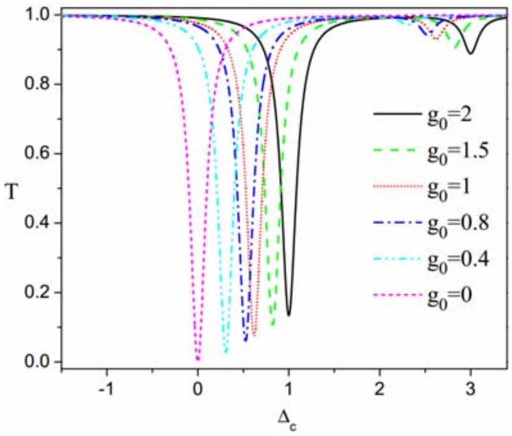

Fig. 3 Single-photon transmission spectra for different single-photon optomechanical coupling strengths with the parameters γ=0 and Γ=0.1, all the parameters are in units of Ω

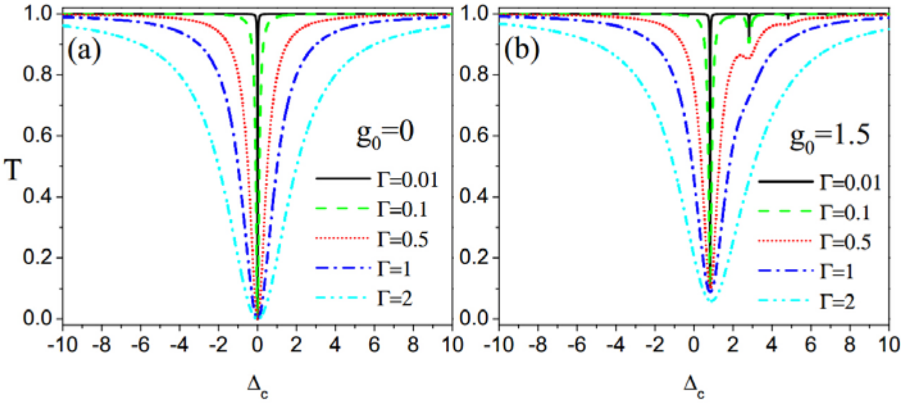

Fig. 4 Single-photon transmission and reflection spectra for different cavity-waveguide coupling strengths with the parameter γ=0. (a) g_0=0, (b) g_0=1.5, all the parameters are in units of Ω

3.2 With cavity dissipation

In this section, we discuss the influences of cavity dissipation on the single-photon properties. For a real physical system, the unavoidable loss always leads to the leakage of photons into non-waveguide channels. Therefore, the sum of the transmission and reflection probabilities should be less 1. Figs.5(a) and 5(b) show discrepancies in two cases of cavity dissipation and non-dissipation. Obviously, the total value of R+T in these transmission dips and reflection peaks is less 1. It is also found that the values gradually decrease with increasing the cavity loss. In particular, for the situation of γ=0.05, the dissipation effect get very noticeable, where the sum of R+T is only 0.6.

In addition, one can observe that as the coupling strength between the optomechanical cavity and the optical waveguide increases, the effect of cavity dissipation is reduced and even eliminated, as depicted in Fig.6. Especially for Fig.6(f) with Γ=1.5, the sum of the transmission and reflection probability gets close to 1.

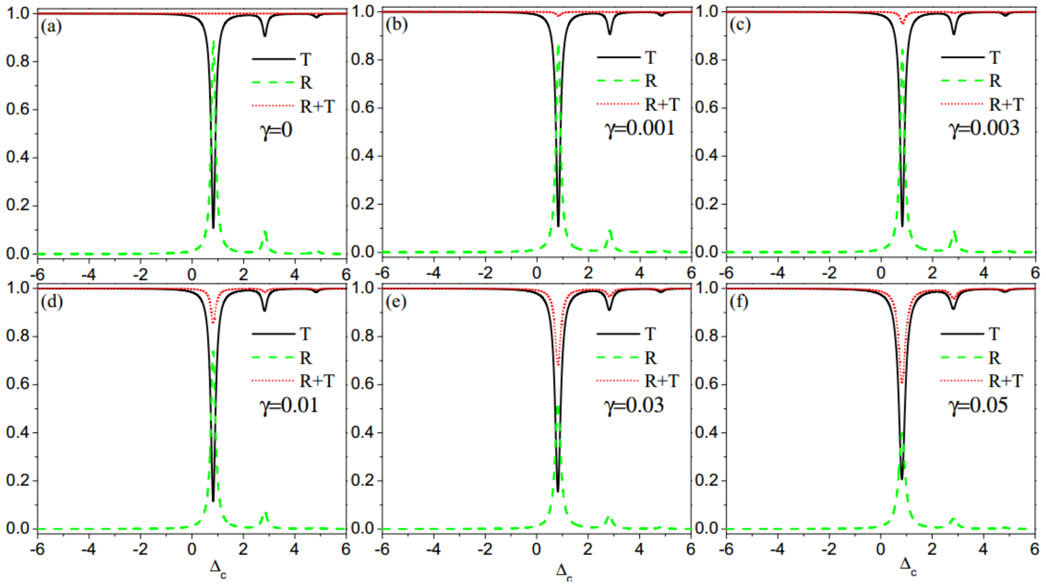

Fig. 5 Single-photon transmission and reflection spectra for different cavity decay rates with the parameters g_0=1.5 and Γ=0.1, all the parameters are in units of Ω

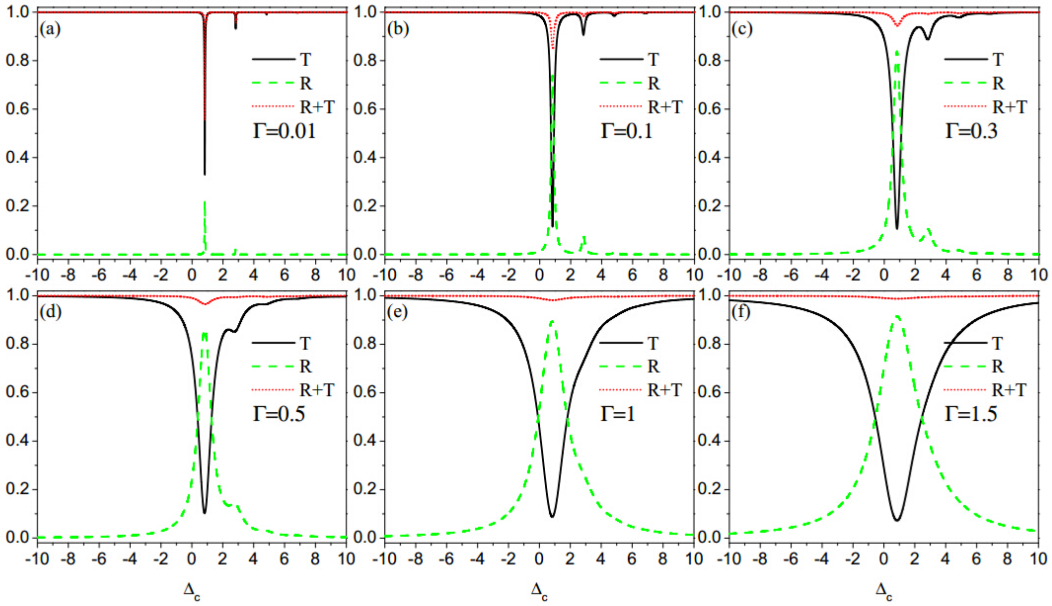

Fig. 6 Single-photon transmission and reflection spectra with cavity dissipation for different cavity-waveguide coupling strengths with the parameters g_0=1.5 and γ=0.01, all the parameters are in units of Ω

4. CONCLUSION

We have studied the single-photon transport along a one-dimensional waveguide interacting with a quadratic optomechanical cavity. The effects of the single-photon coupling strength, the coupling strength between the cavity and the waveguide, the cavity dissipation on the single-photon transport properties are analyzed. In the presence of the optomechanical effect, the transmission spectra show a multiphonon structure and become asymmetry. We also find that the coupling strength between the cavity and the waveguide can weaken the optomechanical effect and reduce the influence of the cavity dissipation.

ACKNOWLEDGEMENTS

This paper was supported by the Scientific Research Fund of Hunan Provincial Education Department (Grant No.15C0539).

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