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Two-Sided Matching Model between the Pension Institution and the Elderly

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Abstract: the rapid development of pension services in our country, makes up for the large gaps that exist in the pension in our country, but as service types of the pension agency become the increasingly rich, they meet the traditional demand of the elderly at the same time, also the pension institutions and the elderly are pursuing a higher satisfaction degree. In order to achieve the optimal two- sided matching according to the information of the pension institutions and the elderly, and to improve efficiency and reduce the cost of the matching, the two-sided matching between the pension institutions and the elderly is studied. Firstly, the two-sided matching issue between the pension institutions and the elderly is described. Secondly, the satisfaction degree of the pension institutions and the elderly is calculated based on the supply and demand information provided by the pension institutions and the elderly, respectly. Furthermore, a multi-objective optimization model maximizing the pension institutions and the elderly satisfaction degree is constructed. A solution model is constructed to determine the corresponding optimal matching alternatives. Finally, a numerical example is given to illustrate the feasibility of the proposed method.

Keywords: pension services, two-sided matching, satisfaction degree, multi-objective optimization model

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1. INTRODUCTION

With the development of pension services, the types of pension services are increasingly rich, and a wide range of pension services are emerging. The elders find it difficult to choose more satisfied services from pension institutions with many different service types, quality and service prices. Otherwise, with the encouragement and support of the government, all kinds of pension institutions in the market rise rapidly [1]. For the pension institutions, facing a large number of older people, how to choose the elderly to make them more satisfied is the key issue that each institution needs to address.

At present, the research on pension in China is mainly from the following aspects: 1) study on pension model. The constant weakening of the traditional self-service family pension

function and the limitation of the government's pension capacity can not meet the needs of the elders in China. The model of social pension has become the development direction of the pension service [2-5]. 2) study on pension pricing. As the trend of aging in China becomes more and more serious, the pricing mechanism of pension institutions is concerned by many scholars. It not only concerns the pension burden of the elderly, but also the fairness and accessibility of the pension services in China. Papers [6-8] constructed pension pricing mechanism for pension agencies from different perspectives, so as to effectively overcome the shortcomings of traditional pricing standards such as obsolescence, lack of cost accounting and overall rating. 3) study on "Internet +" pension service. Literature [9-11] through combining the "Internet +" technology and pension services, the traditional pension service industry is being transformed, commitment to achieve the integration, industrialization and personalization of pension services.

However, regarding the issues of how to improve the satisfaction of the pension service for the elderly in their choice of pension institutions, and when pension institutions receive old people, how to choose among a large number of elderly people to improve the institution's satisfaction have not been solved. Therefore, based on the supply and demand information of the elderly and pension institutions, this paper constructs a bilateral matching model which takes the optimal bilateral satisfaction as the goal, which will improve the matching efficiency and reducing the matching cost, it is a new idea to improve the current problem of pensions.

Bilateral matching was first studied by Gale and Shapley in marriage matching and college admission, and the famous Gale-Shapley algorithm is proposed [12]. After that, Roth proposed the stable matching theory [13]. In recent years, with the development of social service industry, two-sided matching research has been applied in many service industries, such as logistics services, military personnel distribution, commodity trading and others [14-18]. Guo applies two-sided matching to transaction matching between buyer and seller in logistics service, in order to achieve the maximum customer satisfaction, a multi attribute transaction matching model is constructed [14].

Korkmaz, combining AHP method to two-sided matching theory, proposed a decision support system (DSS) and applied to the allocation of military personnel [15]. Jiang Zhongzhong and Yuan Yuan have solved the problem of commodity transaction with quantity discount in electronic mediation, through constructing a optimization model which aims to maximize the matching degree between buyers and sellers [16]. Chen and Song studied the matching of banks and enterprises in the credit market, and drawn the conclusion that the geographical position affects the matching rate between banks and enterprises [17]. Studying the users and resources in wireless communication networks, Jorswieck obtains the related properties of stable matching [18].

Bilateral matching is widely used in many industries, but its application in pension services has not been seen so far. Based on this, the two-sided matching decision-making problem for pension institutions and the elderly were studied. Focusing on the types of services provided by pension institutions and the elderly as well as information on multiple indicators of

evaluation information, the optimization model aiming at maximizing bilateral satisfaction with pension institutions and the elderly is constructed, and then the best matching scheme is obtained through solving.

2. PROBLEM DESCRIPTION

This paper is concerned with the two-sided matching problem with a pension institution and many elders. A pension institution can provide services for many old people, but an old man only has a choice to enter one of many old-age pension institutions. Let $A = \{A_1, A_2, \dots, A_m\}$, A is composed of m pension services, where A_i represents the pension service rank i;$B = \{B_1, B_2, \dots, B_n\}$, a collection of components for n elders}, where B_j represents the elder rank j;. Let H^k represent the k types of services. Let $\boldsymbol{h}_i = (h_i^1, h_i^2, \dots, h_i^l)$, $\boldsymbol{h}_i$ is service vector possessed by the pension agency A_i, $h_i^k = 0$ or 1. $h_i^k = 1$ represents the pension institution A_i can provide the k service type; Otherwise $h_i^k = 0$. This paper consider the pension institution can provide a number of service types. Let $\boldsymbol{g}_j = (g_j^1, g_j^2, \dots, g_j^l)$, $\boldsymbol{g}_j$ is the service type vector required by the B_j of the elderly. $g_j^k = 0$ or 1, $g_j^k = 1$ represents the elderly B_j that need the k service type provided by the pension institution ; Otherwise $g_j^k = 0$. This paper considers that the elderly only have a single service demand, therefore, only a vector in $\boldsymbol{g}_j$ is 1, the others are 0. Further, let $E = \{E_1, E_2, \dots, E_p\}$, For the elderly, in the choice of pension institutions, they also consider a collection of other indicators in addition to the type of service. E_k represents the elderly consider the k indicator. $\boldsymbol{w}_j = (w_{j1}, w_{j2}, \dots, w_{jp})$ as index weight vector is given by the elderly B_j, Where w_{jk} denotes the weight index E_k given by the elderly B_j, $0 \le w_{jk} \le 1$, $\sum_{k=1}^{p} w_{jk} = 1$. $\boldsymbol{a}_{jk} \in (a_{jk}^1, a_{jk}^2, \dots, a_{jk}^m)$ represents all pension institutions strict preference vector sequence given by the elderly B_j under E_k index, $\boldsymbol{a}_{jk} = \{1,2, \dots, f_j\} \cup \{\varphi\}$, wherein, f_j represents the total number of the elderly B_j interested in pension institutions, $0 \le f_j \le m$.If the elderly B_j is interested in the pension agency A_i, and then $\boldsymbol{a}_{jk} \in (a_{jk}^1, a_{jk}^2, \dots, a_{jk}^m)$, a_{jk}^i represents the elderly B_j is interested in the pension agency A_i, and $A_i$rank a_{jk}^i in all pension institutions the elderly B_j is interested in. If the elderly B_j is not interested in the pension agency A_i, $a_{jk}^i = \varphi$.

$U = \{U_1, U_2, \dots, U_q\}$ represents that when pension institutions chooses the elderly, they also consider a collection of other indicators in addition to the service content. U_v represents the pension agency consider the v index. $\boldsymbol{o}_i = (o_{i1}, o_{i2}, \dots, o_{iq})$, index of weight vector for pension institutions given by A_i, o_{iv} represents the index weight of U_v given by the pension institution A_i,$0 \le o_{iv} \le 1, \sum_{v=1}^{q} o_{iv} = 1.\boldsymbol{b}_{iv} \in (b_{iv}^1, b_{iv}^2, \dots, b_{iv}^n)$,for all elders strict preference sequence vector under U_v given by the pension institution A_i, $b_{iv}^j \in \{1,2, \dots, t_i\} \cup \{\varphi\}$, wherein, t_i represents the total number of elderly the pension institution A_i interested in, $0 \le t_i \le n$. If the pension institution A_i is interested in the elderly, then $\boldsymbol{b}_{iv} \in (b_{iv}^1, b_{iv}^2, \dots, b_{iv}^n)$, Among them,

b_{iv}^{j} represents that under the k index, the elderly B_j ranked No. b_{iv}^{j} is given by the pension institution A_i at all interested in the elderly; if the pension institution A_i is interested in the elderly B_j, then $b_{iv}^{j} = \varphi$.

To solve this problem: On the basis of are multi index evaluation information of $\boldsymbol{a}_{jk}$ and$\boldsymbol{b}_{iv}$ and $\boldsymbol{w}_j$ and $\boldsymbol{o}_i$weight vector given by the elderly and pension institutions, then based on the full consideration of the pension institutions with the service types and the service needed by the elderly, this paper use a decision-making method to gain a high satisfaction matching scheme for the elderly and pension institutions as possible as it can.

3. BILATERAL MATCHING METHOD

The matching process in the actual pension institutions and the elderly needs to consider two factors: Service types and multi index evaluation information. First, only when the pension service A_i has the service skills meet the elder B_j requirements, the two sides could match. As a pension institution tend to have multiple types of services, so a pension institution can provide services for many elderly people. Therefore, the elderly pension institutions and the two-sided matching model, considering the service type constraint, the bilateral matching model of pension institutions and the elderly is constructed in this paper after fully taking into account the constraints of service types.

Second, the multi index evaluation information of the elderly and pension institutions need considering, the old manB_j have much greater satisfaction with A_i on multi index evaluation information, the old manB_j will be more likely to match with pension services A_i; Similarly, the old man A_i have much greater satisfaction with B_j, A_i will be more inclined to form a matching pair with B_j[19]. Therefore, based on the maximum satisfaction of and satisfaction of the old man and the pension agency, the two-sided matching model was constructed.

3.1 Satisfaction calculation

First, calculate the satisfaction degree α_{jik} of the elderly B_j with A_i about the index E_k. o The smaller the ordinal value of α_{jik}, the greater the satisfaction of the elderly B_j with A_i; On the contrary, the greater the ordinal value of α_{jik}, the lower the satisfaction of the elderly B_j with A_i. In this paper, the α_{jik} formula of the elderly B_j to the pension institution A_i about the index E_k is given as follows,

$$\alpha_{jik} = \begin{cases} e^{\frac{1-a_{jk}^{i}}{f_j}}, a_{jk}^{i} \in \{1,2,\dots,f_j\}; \\ -M,, a_{jk}^{i} = \varphi. \end{cases} \quad (1)$$

Wherein, $j = 1,2, \dots, n;\ i = 1,2, \dots, m;\ k = 1,2, \dots, p;$ M is a large enough positive number. Further consider the index weight given by B_j, we can draw the conclusion that the overall satisfaction degree of the elderly B_j with the pension institution A_i is

$$\alpha_{ji} = \sum_{k=1}^{p} a_{jik}\omega_{jk}. \tag{2}$$

Similarlly, the overall satisfaction degree β_{ijv} of the pension institution A_i with the elderly B_j about the index U_v is

$$\beta_{ijv} = \begin{cases} e^{\frac{1-b_{iv}^{j}}{t_i}}, b_{iv}^{j} \in \{1,2, \dots, t_i\}; \\ -M,, b_{iv}^{j} = \varphi. \end{cases} \tag{3}$$

Wherein, $v = 1,2, \dots p$. Further consider the index weight given by A_i, we can draw the conclusion that the overall satisfaction degree of the pension institution A_i with the elderly B_j is

$$\beta_{ij} = \sum_{v=1}^{q} \beta_{ijv} o_{iv}. \tag{4}$$

3.2 Construction and solution of model

Let x_{ij} be a decision variable of type 0-1, $x_{ij} = 1$ means that the pension institution A_i matches with the elderly B_j; otherwise $x_{ij} = 0$. According to the satisfaction degree β_{ij} of the pension institution to the elderly and the satisfaction degree α_{ji} of the elderly to the pension institutions, we can construct the two-sided matching optimization model with the greatest satisfaction of the elderly and the maximum satisfaction of the pension institutions:

$$\max Z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ji}\, x_{ij}, \tag{5}$$

$$\max Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \beta_{ij} x_{ij}, \tag{6}$$

$$\text{s.t.} \left(h_i^k - g_j^k\right) x_{ij} \geq 0\ k = 1,2, \dots, l, \tag{7}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1, \tag{8}$$

$$\sum_{j=1}^{n} x_{ij} \leq n, \tag{9}$$

$$x_{ij} = 0 \text{ or } 1. \tag{10}$$

In the model, formula (5) and (6) are the objective function, formula (5) represents the elderly makes the maximum satisfaction with the pension institution; formula (6) represents the pension institution makes the maximum satisfaction with the elderly. In the constraint condition, formula (7) indicates the type of service pension institutions have to be able to meet the needs of the elderly; formula (8) indicates the elderly can each matched with a pension agency; formula (9) says each pension agency can match n old men.

Model (5) ~ (10) is a double objective programming model with the decision variable of type 0-1. The model is solved by using weighted method based on membership function, then the two membership functions [20] are

$$\mu_{Z_1} = 1 - \frac{Z_1^{max} - Z_1}{Z_1^{max} - Z_1^{min}} \tag{11}$$

$$\mu_{Z_2} = 1 - \frac{Z_2^{max} - Z_2}{Z_2^{max} - Z_2^{min}} \tag{12}$$

Among them, Z_1^{min} and Z_1^{max} are the minimum and maximum values of the objective function Z_1; $Z_2^{min}$and Z_2^{max} are the minimum and maximum values that only consider the objective function Z_2, $0 \le \mu_{Z_1}, \mu_{Z_2} \le 1$.According to formula (11) and formula (12), the double objective optimization model (5) ~ (10) is transformed into the formula (13) as the objective function, which the single objective function optimization model with constraints of (5) ~ (10).

$$\max Z = \omega_1 \mu_{Z_1} + \omega_2 \mu_{Z_2} \tag{13}$$

Wherein, ω_1 and ω_2 represent the importance of the objective functions Z_1 and Z_2, respectively, and $0 \le \omega_1, \omega_2 \le 1$， $\omega_1 + \omega_2 = 1$. To ensure the fairness of the matching between the elderly and the pension institutions, take $\omega_1 = \omega_2 = 0.5$. The transformed optimization model is a single objective programming model with the decision variable of type 0-1, when the number of elderly and pension institutions is small, branch and bound algorithm can be used to solve the problem; when the number of elderly and pension institutions is large, the optimization software packages (such as Cplex, Lingo, etc) or the intelligent optimization algorithms (such as simulated annealing algorithm, genetic algorithm, etc.) can be used to solve this problem.

4. A NUMERICAL CASE

A platform registered basic information of 3 pension institutions $\{A_1, A_2, A_3\}$,and 5 elders $\{B_1, B_2, B_3, B_4, B_5\}$. Consider the service type set for $H = \{H^1, H^2, H^3, H^4, H^5\}$, wherein H^1 represents medical services, such as health care, medical rehabilitation, physical examination,

emergency services; H^2 represents service life, such as food, housing, logistics and other daily services; H^3 represents help services, such as personal health care services, health care, personal living room nursing services; H^4 represents travel services, such as route planning services, location services; H^5 represents emotional service, such as sports and leisure services, tourism services, cultural learning services, entertainment and leisure services, communication services etc. And the elderly considering the index set of $E = \{E_1, E_2, E_3, E_4\}$, where E_1 represents the price; E_2 on behalf of accommodation; E_3 on behalf of accommodation environment; E_4 on behalf of service content.

The pension agency to consider in choosing the old index set of $U = \{U_1, U_2, U_3\}$, where U_1 represents the age, U_2 represents physical condition, U_3 represents the representative type of disease. The relevant information of the elderly and pension institutions is given in table 1~6. According to the formula (1) ~ (4), calculated the elderly and pension institutions satisfaction as shown in Table 7 and table 8, according to the formula (11) ~ (13), calculated both satisfaction by weighted shown in table 9. Among the satisfaction table, for the convenience of calculation, take$-M = -1$.

Table 1 Service types of pension institutions

The pension institution	A_1	A_2	A_3
h_i	(1,1,1,1,0)	(0,1,1,1,1)	(1,0,1,1,0)

Table 2 Service types required by the elderly

the elderly	g_j
B_1	(1,0,0,0,0)
B_2	(0,1,0,0,0)
B_3	(1,0,0,0,0)
B_4	(0,0,1,0,0)
B_5	(1,0,0,0,0)

Table 3 Preference information given by pension institutions

thepension institution	index	B_1	B_2	B_3	B_4	B_5
A_1	E_1	3	7	5	φ	2
	E_2	2	4	8	φ	6
	E_3	4	6	2	φ	8
	E_4	6	3	4	φ	7
A_2	E_1	3	7	φ	4	2
	E_2	5	1	φ	3	7
	E_3	1	5	φ	7	3
	E_4	3	4	φ	2	5
A_3	E_1	φ	7	1	4	φ
	E_2	φ	3	4	5	φ
	E_3	φ	1	3	2	φ
	E_4	φ	6	7	8	φ

Table 4 Weights of Indices given by pension institutions

the pension institution	E_1	E_2	E_3	E_4
A_1	0.2	0.4	0.3	0.1
A_2	0.3	0.3	0.2	0.2
A_3	0.1	0.5	0.3	0.1

Table 5 Preference information given by the elderly

the elderly	index	A_1	A_2	A_3
B_1	U_1	5	3	1
	U_2	4	2	1
	U_3	2	6	1
B_2	U_1	2	φ	3
	U_2	4	φ	1
	U_3	5	φ	2
B_3	U_1	4	2	6
	U_2	5	6	3
	U_3	1	2	3
B_4	U_1	1	2	3
	U_2	6	3	1
	U_3	3	1	6
B_5	U_1	5	4	φ
	U_2	1	2	φ
	U_3	4	5	φ

Table 6 Weights of Indices given by the elderly

the elderly	B_1	B_2	B_3	B_4	B_5
U_1	0.5	0.4	0.2	0.3	0.5
U_2	0.2	0.4	0.6	0.3	0.2
U_3	0.3	0.2	0.2	0.4	0.3

Table 7 The satisfaction of the elderly to pension institutions

the pension institution	B_1	B_2	B_3	B_4	B_5
A_1	0.77	0.61	0.62	$-M$	0.56
A_2	0.75	0.67	$-M$	0.68	0.65
A_3	$-M$	0.79	0.72	0.68	$-M$

Table 8 The satisfaction of the pension institution to the elderly

the elderly	A_1	A_2	A_3
B_1	0.63	0.66	1
B_2	0.64	$-M$	0.83
B_3	0.63	0.60	0.66
B_4	0.72	0.87	0.69
B_5	0.59	0.57	$-M$

Table 9 The satisfaction weighted by two sides weighted

satisfaction	A_1	A_2	A_3
B_1	0.930	0.933	0.000
B_2	0.888	0.466	0.989
B_3	0.888	0.428	0.924
B_4	0.460	0.969	0.921
B_5	0.861	0.880	0.000

According to the satisfaction of the old man and the pension agency, construct a multi-objective optimization model (5) ~ (10), and by formula (11) ~ (13), converts it into a single objective model, through the software solution of Lingo11.0, gain the matching results: $A_3 \leftrightarrow B_2, A_2 \leftrightarrow B_4, A_2 \leftrightarrow B_1, A_3 \leftrightarrow B_3, A_2 \leftrightarrow B_5$.

5. CONCLUSION

(1) In view of the problem of two-sided matching decision-making between the elderly and the pension institutions, this paper puts forward the corresponding decision analysis method. In this method, we consider the service types of the pension institutions and the elderly and the 2 aspects of multi index evaluation. In order to determine the bilateral matching scheme that the pension institutions and the elderly are satisfied as much as possible, the two-sided matching optimization model is constructed with the maximum satisfaction of both sides as the goal.

(2) Compared with the previous research, this paper uses mathematical modeling method to build a bilateral mechanism to match the pension institutions and the elderly, not only can the satisfaction of the pension institution and the elderly be the result of the matching, but also it can improve the shortcomings of the low satisfaction caused by the subjective comparison. It can also improve the matching efficiency and reduce the cost of looking for the elderly and the pension institution.

(3) The method proposed in this paper can not only solve the two-sided matching decision problem between the elderly and the pension institution, but also can be extended to solve other two-sided matching decision problems, such as the matching of multi skilled technicians and the company, the matching between the educational institutions and the recruitment of students in the educational institutions etc.

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