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Direct interpolation of Bezier curves by DDA method

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Abstract: In the CNC system, Bezier curves are interpolated by data sampling interpolation method. This method will inevitably produce trajectory error, and the feed amount must be calculated for each interpolation cycle. The calculation amount is large and the real-time performance is not good. . In order to overcome these problems, based on the de Casteljau algorithm proposed by Paul de Casteljau in 1959, a method to push out an arbitrary-order Bezier curve using only DDA linear interpolation superposition is studied. This method uses a plurality of DDA linear interpolators to perform interpolation at the same time. The output pulse at the previous stage is used to correct the integrator register of the next stage, and the last stage output is used to drive the feed. Finally, a simulation program was written in Visual Studio 2010 to simulate the interpolation, which confirmed its correctness. This new method for processing Bezier curves is more accurate and real-time, and it also provides new ideas for the processing of B-spline and NURBS curves.

Keywords: Bezier curve; direct interpolation; de Casteljau algorithm; DDA method.

1. PREFACE

The interpolation algorithm is the core part of the computer numerical control system. Its running time and calculation accuracy determine the performance index of the entire CNC system. The current CNC system can directly perform interpolation by using the principle of pulse increment interpolation for straight lines and circular arcs, such as the point-by-point comparison method and the digital integration method (DDA method). The incremental interpolation of pulses in straight lines and circular arcs requires only a small amount of addition at each step. The algorithm has high efficiency and good real-time performance. Interpolation of Bezier curve, spline curve, etc. can only use the principle of data sampling interpolation, first coarse interpolation, that is, using a tiny straight line segment to approximate the given contour, and then use pulse increment interpolation method to The straight line segment performs fine interpolation. Data sampling method will certainly produce trajectory error during the process of approximation, and the interpolation procedure is more complicated, which has higher requirements for the data ability of the CNC system and the real-time performance is not good.

In order to avoid the problems caused by the data sampling interpolation method, the idea of direct interpolation of the Bezier curve was initiated. Inspired by the geometric model of the de Casteljau algorithm, the basic idea of recursive interpolation of Bezier curves using DDA method is proposed.

2. DDA LINEAR INTERPOLATION

The DDA method, Digital Differential Analysis, is a method that uses numerical integration to calculate the displacement of the tool along each coordinate axis and moves the tool along the processed curve. The principle of linear interpolation using the DDA method is as follows.

In the XY plane, the AB lines to be interpolated are the coordinates of the starting A point (x_s, y_s), the coordinates(x_e, y_e) of the ending B point, the velocity v of the moving point in the direction of the straight AB line, and the dividing velocity along the X axis and the Y axis are respectively sums, then in Δt time Inside, the minor displacement increments of the moving point in the X axis and Y axis directions are: $\Delta X = v_x \Delta t$, $\Delta Y = v_y \Delta t$

Let the length of line AB be L, then:

$$L = \sqrt{(X_e - X_s)^2 + (Y_e - Y_s)^2} \ , \quad \frac{v_x}{v} = \frac{X_e - X_s}{L} \ , \quad \frac{v_y}{v} = \frac{Y_e - Y_s}{L}$$

and so:

$$v_x = \frac{v}{L}(X_e - X_s), \quad v_y = \frac{v}{L}(Y_e - Y_s)$$

If the speed v is uniform ,Then $\frac{v}{L}$ is a constant,make $\frac{v}{L} = k$,the displacement increment along the axis can be expressed as:

$$\Delta X = k(X_e - X_s)\Delta t, \quad \Delta Y = k(Y_e - Y_s)\Delta t$$

take $\Delta t = 1$,the displacement of each axis is:

$$X_e = k\sum_{i=1}^{n}(X_e - X_s) = k(X_e - X_s)n, \quad Y_e = k\sum_{i=1}^{n}(Y_e - Y_s) = k(Y_e - Y_s)n$$

The interpolation operation is performed by two digital integrators, each of which consists of an accumulator and a integrand register. The integrand register stores the difference between the coordinates of the end point and the starting point,For each pulse Δt, the function value in the integrand register is sent to the corresponding accumulator and added once. When accumulating and exceeding the capacity of the accumulator, the pulse is overflowed. As the feed pulse ΔX or ΔY driving the corresponding axis, the remainder still exists in the integral accumulator.In this way, the two axes are synchronously interpolated, and the overflow pulsation is used to control the feed of the machine to get out of the required linear trajectory.Let the number of integral accumulators be m, then the accumulator capacity is 2^m. When the number of iterations of interpolation is 2^m, the number of overflow pulses for each axis is:

$$X = \frac{1}{2^n}\sum_{i=1}^{2^n}(X_e - X_s) = X_e - X_s,\quad Y = \frac{1}{2^n}\sum_{i=1}^{2^n}(Y_e - Y_s) = Y_e - Y_s$$

at this point, the two axes reach the end at the same time and the interpolation ends.

3. DE CASTELJAU ALGORITHM

The Bezier curve was originally developed by Paul de Casteljau in 1959 using the de Casteljau algorithm to derive a Bezier curve in a stable numerical method.The de Casteljau algorithm can calculate the point C(u) , $u \in [0,1]$ on the Bezier curve.Therefore, by giving a set of u values, the coordinate sequence on the Bezier curve can be calculated and the Bezier curve can be drawn.The basis of the de Casteljau algorithm is to select a point C on the vector AB such that the C sub-vector AB is $u : 1-u$ (is, $|AC| : |AB| = u$).This procedure is referred to below as taking a Bezier point on a line segment (AB here).Given the coordinates of point A, B and the value of $u(u \in [0,1])$, the coordinates of point C are.as shown in picture 2.

$$C = A + (B - A) * u = (1-u) * A + u * B$$

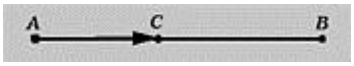

Fig. 2 Bezier point C on line

The control point number defining the Bezier curve is ij, where i denotes the iteration number and j denotes the control point number in each iteration.The idea of the de Casteljau algorithm is as follows:in order to calculate the point C(u), $u \in [0,1]$ on the nth Bezier curve, the control points are first connected in sequence to form a fold line 00-01-02... 0(n-1)-0n.Using the above method, a point 1k on each line segment 0k-0(k+1) in the polyline is calculated so that the ratio of the line segment 1k to the line segment is $u : 1-u$ .The algorithm is then called recursively on the polyline 10-11-12...1(n-2)-1(n-1), and so on.Finally, find the last point n0. Paul de Casteljau proved that point n0 must be a point on the curve.For example, the fifth-order Bezier curve formation process is as follows:

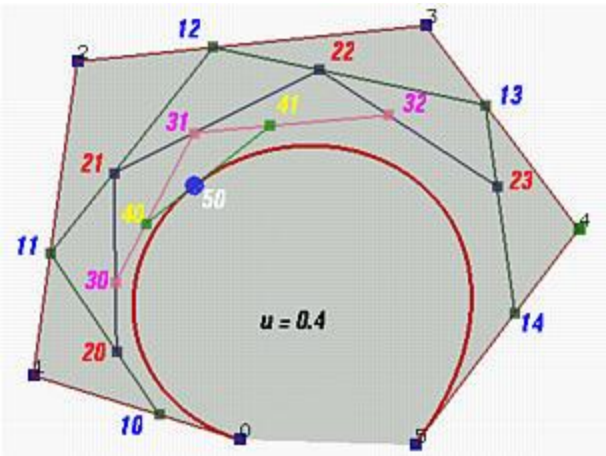

Fig. 3 The process of forming the fifth-order Bezier curve

Curve control points are 0, 1, 2, 3, 4, and 5. Line 0-1 takes a point of 10, 10 points is the ratio of the line segment, similarly takes points 11, 12, 13, and 14,Then, in the second iteration, the point 20 is taken on line 10-11, and the point 20 points is the ratio of the line segment, similarly to the points 21, 22, and 23. Then for the next iteration,and so on, until the last point on the line 40-41 to take points 50,50 is the only last point, that is, the point on the curve.

4. CONCLUSION

This method innovatively applies the de Casteljau algorithm to the interpolation process of numerically-controlled machine tools, and studies the theory of iteratively deriving Bezier curves using the DDA method, which provides a new, more accurate and real-time process for Bezier curve processing. The method has even provided new ideas for the processing of B-spline, NURBS and other curves.This type of non-circular curve is widely used in practice, especially in complex contour parts, almost all professional modeling software provides such a curve modeling tool, so this method will be important for numerical control processing technology influences.

Of course, the methods discussed in this article are not yet mature, and there are still some problems that need to be solved. For example, in the Bezier curve interpolation process of the second-order and second-order or higher, the feed speed is generally non-uniform; in the simulation interpolation results, occasionally the path of the tool path and the theoretical contour do not completely close.

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