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Sign relations of the reflection coefficients between the displacement and the normal stress

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Abstract: When analyzing the reflection behavior of a wave, the reflection coefficient is an important physical quantity, which also is the topic of this study. Different from traditional studies, in this paper, we are not interested in the values of reflection coefficients but in the signs of them instead. When an elastic wave impinges upon a barrier, the reflection coefficient of the displacement is opposite in sign to that of the normal stress in certain directions. Such phenomenon is verified by finite difference solutions for the reflected waves in elastic and isotropic media, especially for the case of total reflection. Another work of the study is to relax the preconditions of the sign relation of reflection coefficients, which also is verified by numerical tests.

Keywords: reflection coefficient, displacement, normal stress, elastic wave.

1. INTRODUCTION

The reflection is a physical phenomenon that happens everywhere and every moment, e.g., the visual sense, the radar, the sonar and the imaging. Thus it is observed, analyzed and utilized. And this research is about the reflection of elastic waves.

Since people begun on studying the reflection, plenty of qualitative and quantitative work has been done, especially on the reflection coefficient itself. One can review the fundamental researches from some books [1, 2]. Furthermore, the reflection was studied in many articles for more complex models, such as the anisotropic medium [3], the lossy medium [4] and the curved interface [5]. However, this study is not focused on mode waves and the values of reflection coefficients but on the relations of reflection coefficients between the displacement and the normal stress.

Focusing on such relations has the benefits in two sides. One is avoiding the troublesome derivation about the reflection coefficient itself, and the other is to understand the reflection in a new way. While the similar studies [1, 6] were done, the results were gained in very special situations, such as the incident angle is zero and the dip angle of the interface also is zero.

Recently, Zhang and Hu [7] got an interesting result, i.e., in the certain direction, the signs of some reflection coefficients are opposite.

Unfortunately, Zhang and Hu did not provide enough tests to verify their result. In this study, we will explain such result in more detail and verify it by some numerical experiments. Furthermore, we find that their result also is true even if the total reflection happens.

2. SIGN RELATIONS OF REFLECTION COEFFICIENTS

In Fig.1, the dip angle and azimuth angle of the interface are denoted as β and α respectively. The parameter λ and μ are the Lame constants of the medium in which the incident wave exists. Assumed that the incident angle is θ. And then, the result of Zhang and Hu [7] is as follows.

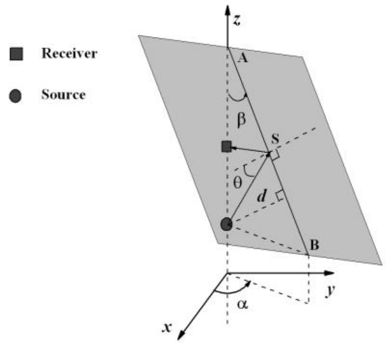


Fig. 1. An interface in three-dimensional space

For all body waves, the relations below are true.

$$R_l^{\sigma} / R_l^{v} < 0 \,, R_l^{\sigma} / R_l^{u} < 0 \,,\ l = x, y \,. \tag{1}$$

where R_l^{σ}, R_l^{u} and R_l^{v} respectively refer to the reflection coefficient of the normal stress $\boldsymbol{\sigma}$, the displacement $\mathbf{u}$ and the particle velocity $\mathbf{v}$ in l-direction. The reflection coefficient is defined as the ratio of reflected wave to incident wave [6].

When the incident and reflect wave are P waves, the reflection coefficients are

$$\begin{cases} R_x^u = -\dfrac{A^1 \cos(\theta-\beta)}{A^0 \cos(\theta+\beta)} = R_y^u \\ R_z^u = \dfrac{A^1 \sin(\theta-\beta)}{A^0 \sin(\theta+\beta)} \end{cases} . \tag{2}$$

$$\begin{cases} R_x^{\sigma} = \dfrac{A^1[\lambda + 2\mu\cos^2(\theta-\beta)\cos^2\alpha]}{A^0[\lambda + 2\mu\cos^2(\theta+\beta)\cos^2\alpha]} \\ R_y^{\sigma} = \dfrac{A^1[\lambda + 2\mu\cos^2(\theta-\beta)\sin^2\alpha]}{A^0[\lambda + 2\mu\cos^2(\theta+\beta)\sin^2\alpha]} \\ R_z^{\sigma} = \dfrac{A^1[\lambda + 2\mu\sin^2(\theta-\beta)]}{A^0[\lambda + 2\mu\sin^2(\theta+\beta)]} \end{cases} . \tag{3}$$

where A^0 and A^1 is the amplitudes of the incident and the reflect wave respectively.

When the incident and reflect wave are S waves, the reflection coefficients are

$$R_x^u = R_y^u = \frac{A^1 \sin(\theta-\beta)}{A^0 \sin(\theta+\beta)},\ R_z^u = -\frac{A^1 \cos(\theta-\beta)}{A^0 \cos(\theta+\beta)}, \tag{4}$$

$$R_x^\sigma = -\frac{\sin(\theta-\beta)\cos(\theta-\beta)A^1}{\sin(\theta+\beta)\cos(\theta+\beta)A^0} = R_y^\sigma = R_z^\sigma . \tag{5}$$

When the incident and reflect wave are SH waves, the reflection coefficients are

$$R_x^u = R_y^u = A^1 / A^0,\ R_x^\sigma = -\frac{\cos(\theta-\beta)A^1}{\cos(\theta+\beta)A^0} = R_y^\sigma , \tag{6}$$

2.1 Notes for the results

Firstly, such sign relation is only true in *x*- and *y*-direction. The reason is that the *z*-axis has a special meaning, which is the line from the source to the receiver in Fig.1. The special meaning will be clearer, when the *z*-axis is set as the tangential direction of the interface. And we will study this case later on.

Secondly, there are four preconditions for formula (1):

A. The media are elastic and isotropic.

B. $\lambda \geq 0$.

C. The incident wave is a plane wave and the interface is a plane.

D. The reflection coefficients are real.

Thirdly, formula (1) shows the sign relations of the reflection coefficients between different physical quantities for just one wave.

2.2 The reflection in special cases

It must be pointed out that the reflection itself will do not change even if the coordinate systems change. In the following, we will analyze the reflection in some special cases, which will display the essence of the reflection more clearly and concisely.

Firstly, let the dip angle be zero. Thus, all the reflection coefficients considered only relate to the amplitude ratio, which can be seen from equations (2)-(6). Furthermore, formula (1) is true here.

Secondly, let the incident angle be zero. And then, all the terms of trigonometric functions in the expressions of the reflection coefficients disappear.

Thirdly, consider the reflection in the liquid or gas. Since only P waves exist and $\mu = 0$, equation (3) can be simplified as such

$$R_x^\sigma = R_y^\sigma = R_z^\sigma = R^p = A^1 / A^0 . \tag{7}$$

In the first case, because the *z*-direction is the tangential direction of the interface, the propagation direction of the wave does not change in that direction and then the sign relations shown in equation (1) are only true in *x*- and *y*-direction, which are the normal directions of the interface. In the second case, equation (1) is always true for all media if the incident angle is zero. In the third case, the reflection coefficient of pressure is always opposite in sign to that of the displacement in *x*- and *y*-directions whatever the medium and the incident angle are.

3. DISCUSSIONS

In this section, we will firstly discuss the preconditions. Specifically, precondition C and D can be relaxed.

For precondition C, the assumption of incident plane waves will be expanded to the incidence of sphere waves. Thus, equation (1) is true for nearly all waves. The essence of the sphere waves is that the amplitude is not a constant but a function related to the distance. Note that the reflection only occurs at a certain point on the interface. Then, at that point, the amplitude of the incident sphere wave is a constant and will have no difference from the incident plane wave with the same amplitude. In the next section we will examine such judgment by numerical simulation.

For precondition D, it will be removed when precondition A is satisfied. Sometimes the reflection coefficients are complex numbers [8]. From the expressions of the reflection coefficients, e.g., equation (3), we find that only A^1 can be complex, because other parameters are all real when precondition A is satisfied. In equation (1), the amplitudes, A^1 and A^0, do not appear, because just the ratio between R_l^{σ} and R_l^{u} is considered. On the other hand, a complex A^1 means that the phase shift [2, 7] happens for the reflected wave, which is difficult to be observed except the receiver is near the interface enough.

From the above, the preconditions for equation (1) are as follows:

1) The media are elastic and isotropic.

2) $\lambda \geq 0$.

3) The interface is a plane.

Next, we briefly state the theoretical significant of equation (1). The relations described by equation (1) connect the reflection coefficients of different observed quantities with each other. Moreover, such relations do not relate to the incident angle, the dip angle and the azimuth angle, and then they actually show the unchanged relations between the incident wave and its reflected wave.

Finally, we point out the meaning of the reflection coefficient's sign in observing. A negative reflection coefficient will reverse the waveform after reflection, while a positive reflection coefficient will not do so. With such features, the numerical results will be analyzed in the next section.

4. NUMERICAL TEST

In this section, with FDTD (finite-difference time-domain) procedures, the wave fields of two-dimensional models, as shown in Fig.2, are simulated and then our results are examined. A monopole source is used to verify that the results are available for the incidence of sphere waves. The parameters of the media are shown in Table 1. Since the results are just the sign relations of reflection coefficients, the real amplitudes of the waveforms will not be shown. Additionally, as a monopole source is used to excite the wave field, the S waves are converted waves and can be ignored.

Table 1 Parameters of the media

Parameter	Layer I	Hard Layer	Soft Layer	Water	Alcohol
Density（kg/m^3）	2600	3000	2000	1000	800
V_p (m/s)	2773	3162	2121	1483	1060
V_s (m/s)	1240	1527	1000	/	/

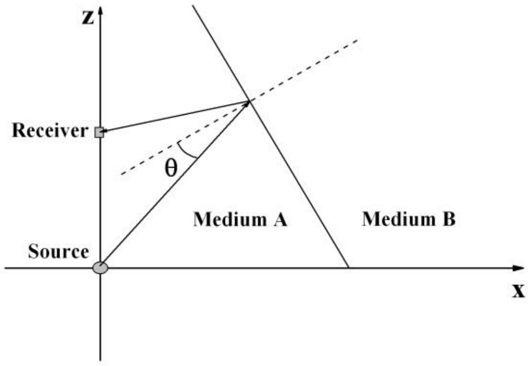


Fig.2 A two-dimensional model for observing the reflection.

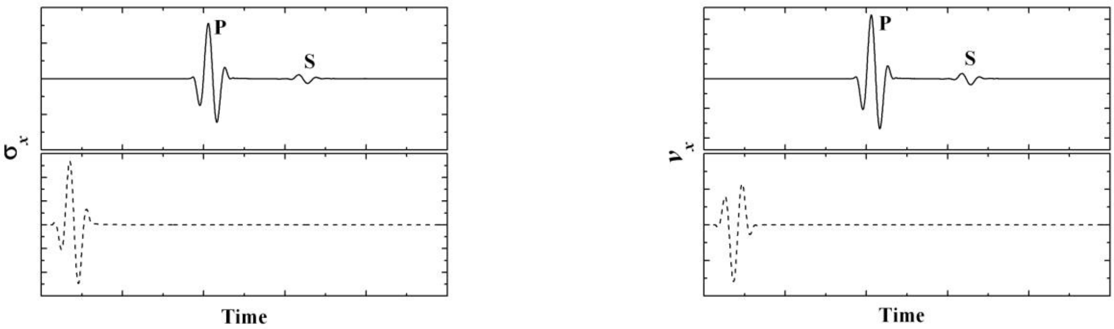
To calculate different wave fields, we will change some settings of the model in Fig.2, such as the interface equation and the parameters of the media. Nevertheless, these things are never changed that the source is on the origin of the coordinate system, the incident medium is medium A and the reflected medium is medium B.

For the wave field excited by a monopole, all the outgoing waveforms of pressure or σ_x have the same shape and so does that of v_x if the outgoing waves propagate towards the positive *x*-direction. Then the outgoing wave recorded at (1m, 1m) has the same waveforms in shape as the real incident waves on the interface, whose waveforms are drawn by dotted lines in the following pictures as the incident waveforms.

4.1 Different reflected media

In this part, the wave fields of two models are simulated numerically. And the only difference between these two models is whether medium B is hard or soft. For model 1, medium B is Hard Layer in Table 1, and the reflected waveforms are displayed in Fig.3; for model 2, medium B is Soft Layer in Table 1, and the reflected waveforms are displayed in Fig.4. In model 1 and model 2, the equations of the interfaces are both *x*=5m and medium A is Layer I in Table 1.

In Fig. 3, two waveforms on σ_x are reversed in shape with each other, which means R_x^σ is negative; two waveforms on v_x are similar in shape with each other, which means R_x^v is positive. In Fig. 4, two waveforms on σ_x are similar in shape with each other, which means R_x^σ is positive; two waveforms on v_x are reversed in shape with each other, which means R_x^v is negative. Then, one can conclude that the relations expressed by equation (1) do not depend on the impedance of the medium.



a) b)

Fig. 3. Signals reflected by a hard layer

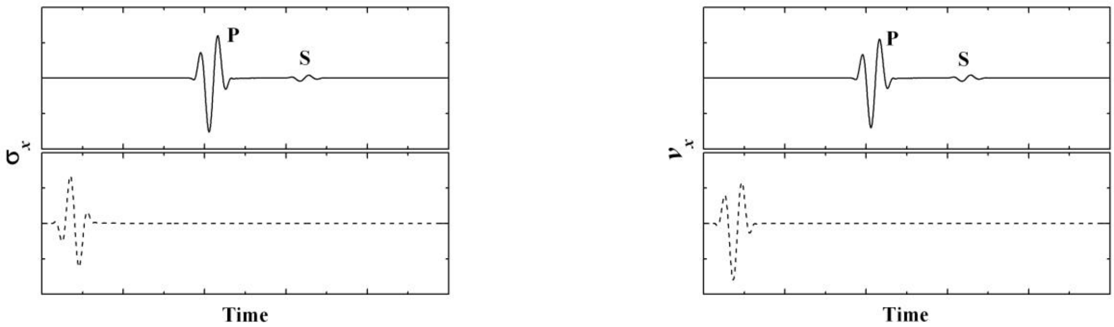


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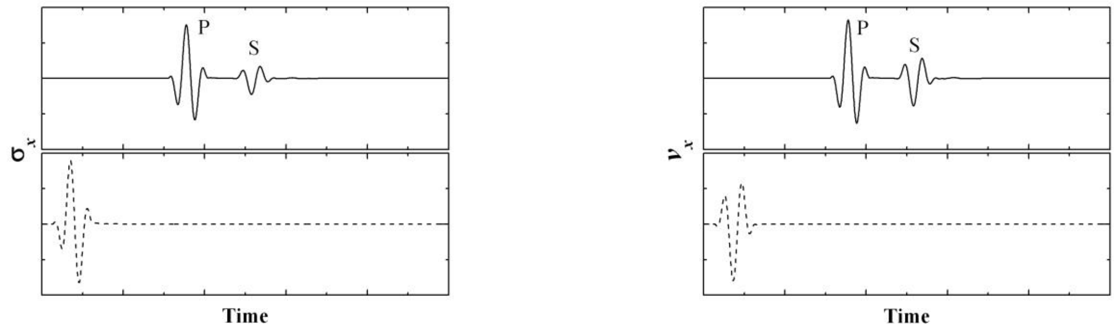
Fig. 4. Signals reflected by a soft layer

4.2 Different dip angles



a) b)

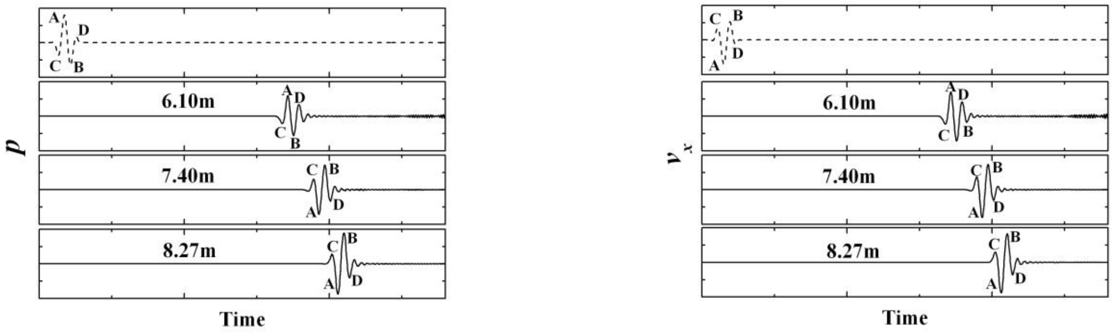
Fig.5 Signals reflected by an interface with the dip angle $\beta = \pi / 4$

Model 3 is different only in the dip angle of the interface from model 1. Specifically, the interface equation is changed from x=5m into x+z=7m, which means the dip angle β is changed from 0 into $\pi / 4$. The reflected waves are shown in Fig.5.

In Fig. 5, two waveforms on σ_x are similar in shape with each other, which means R_x^σ is positive; two waveforms on v_x are reversed in shape with each other, which means R_x^v is negative. With such, equation (1) is verified to be true for different dip angles.

4.3 The total reflection

As mentioned in section 3, the total reflection will not influence the relations in equation (1) and what it influences is the reflected waveforms. To observe the total reflection, the distance from the interface to the *z*-axis must be close enough. And then the waves, such as the reflected P wave, the converted SV wave and the surface wave, will be superposed together. Considering such, we simulated a special model named as model 4. In model 4, medium A is alcohol, medium B is water and the interface equation is *x*=3m. The parameters can be found in Table 1 and the signals received are displayed in Fig.6. With the parameters, it is true that the critical angle of incidence is 45.65°, and then the total reflection will be observed at least at *z*=6.13m on the *z*-axis.



a) b)

Fig. 6. Signals recorded when the total reflection occurs

As the phase shift occurs after the total reflection, some important phase positions are marked for checking the relation between two waveforms. From Fig.6a, it is true that the waves recorded at *z*=6.10m, 7.40m and 8.27m, respectively correspond with the positive, negative and negative reflection coefficient. In Fig.6b, at the same locations, the reflection coefficients of v_x are negative, positive and positive, respectively. Thus, equation (1) is available even when the total reflection happens.

5. CONCLUSION

In this paper, the rule of reflection coefficients in sign is analyzed and verified. The preconditions of the rule can be relaxed as follows.

(1) The media are elastic and isotropic.

(2) The Lame constant λ is not negative.

(3) The interface is a plane in three-dimensional space.

The numerical experiment indicates that the rule is still true even if the reflection coefficient is complex.

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