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Solution Analysis and Administrative Application of Prisoner's Dilemma Based on Judgement Game Theory

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Abstract

In traditional game, all participants will take action, however, the article is discussing a new kind of game model: a brand-new "judge", he will not participate the game, nor set up game rules. At the end of game, the judge will praise the winner. This kind of game is judgement game. The article discusses the prisoner's dilemma based on judgement game and describes the general solution of prisoner's dilemma under judgement game. The article thinks the prisoner's dilemma is not a two-participant game, but a three-participant game, including the police, and is a type of judgement game. Meanwhile, prisoner's dilemma is not a Nash Equilibrium of two participants, it is a brand-new type of equilibrium. The article analyzes the prisoner's dilemma based on judgement game, and also discusses its application in management.

Keywords

Judgement Game, Game Theory, Nash Equilibrium, Equilibrium of Judgement Game.

1. INTRODUCTION

Prisoner's dilemma is an important example in game theory, is widely discussed in many articles and applied in management, economy and other fields.

It describes the following case:

Two conspirators are arrested by police, and they are separated, waiting for interrogation. They are told that if anyone confesses, while the other refuses, the one will be set free, and the other will have a 3-year prison life. If both of them confess, they will face 2-year prison life each. If both of them refuse, they will be sent into prison for 1 year respectively.

Till now, all the papers of game theory think it is a dilemma, which troubles the two conspirators. According to the survey of author, all the documents of game theory think it is a Nash Equilibrium, both of the two will confess, meanwhile, all the researches insist it is a game only for two participants.

The author of the article thinks it is not a game for only two participants, but three. In addition, it is not a Nash Equilibrium, but a judgement equilibrium based on judgement game. Prisoner's dilemma is not a difficult game to solve, but a typical case of judgement game. The application of this game is not merely to solve prisoner's dilemma, but also to fully discuss the benefit of judge, then applied to management, politics, economy and other fields. The application of this game is epoch-making change, which will lead to a complete thinking on the typical game model, and then applied to the territory of related subjects. It will be a theoretical breakthrough in management, economy and politics.

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2. TYPES OF PRISONER'S DILEMMA

Why the prisoner's dilemma is not a game for only two participants?

Because in prisoner's dilemma, there is also a police officer besides two conspirators. All the game theory articles believe that the game only contains two participants, and the final result belongs to Nash Equilibrium with two prisoners confess. Then after confession of two prisoners, who is the person with maximized benefit? Absolutely the police. Because only when the prisoners confess, the police will gain the maximized benefit. Therefore, there is a third party who owns largest profit, how to say it is only a game for two participants? It is obvious a game of three participants. In this game, police set the rules, so we think he is the judge, at the end of game, he will announce the result. This game has two participants and one judge, so it is a judgement game.

Why the prisoner's dilemma is not a Nash Equilibrium for two people?

Based on the theory of Nash Equilibrium, the premises are all the participants are rational, and all the participants will gain maximized benefit.

Due to the interference of police, it is not a two-participant game, but three. In this condition, Nash Equilibrium changes a lot. In three-participant game, because two prisoners are rational, the result will like this: two prisoners are rational, pursing maximized benefit. They are clear that rule-maker is the police, who will gain the maximized benefit. Therefore, two prisoners are able to realize that the police need their confession. Each of them knows the purpose the police and think they will have the same idea. In this case, one prisoner will refuse, and he is sure that the other will do the same. Thus, the police will not gain the maximized benefit, however, the two prisoners will get the maximized benefit, 1-year prison life each.

In this game, there are three participants, the police only set rules, does nothing else, just waiting for the strategies of two prisoners, so the police is judge. When taking strategies, two prisoners will consider the other's strategy, police's purpose, but not police's, because the police will do noting besides setting rules. In this cirumstance, the final result should be, two prisoners do not confess. If they choose confession, that is to say, they do not condiser the third party (police), it is not rational. It is obvoius that the result is in contradcition with the assumption of Nash Equilibrium. Therefore, prisoner's dilemma is not a Nash Equilibrium but a brand-new equilibrium: judgement equilibrium.

3. JUDGEMENT GAME MODEL

Now look at our model:

The first judgement game model:

For example, there are three people, A, B and C. The game is between A and B, C is the judge. C will set up rules based on his own benefit, and ask A and B to participate the game, C will not. During the game between A and B, C does not make any strategy, only at the end of the game, C announce the result.

The model is judgement game, prisoner's dilemma meets all the characteristics of this model.

Then, how to solve this game model? A wins, or B wins, or C wins?

4. MODEL ANALYSIS AND SOLUTION

Judgement game has three results: the result based on maximized benefit of judge, based on maximized benefit of two prisoners, and based on maximized benefit of one prisoner.

First result: maximized benefit of judge.

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This result is the common result in all articles concerning the dilemma, two prisoners confess. Premise of result: prisoners do not consider the existence of police, following two-participant game.

Table 1. Result of prisoner's dilemma ignoring the judge----usually Nash Equilibrium

		Prisoner 2	
		Confess	Refuse
Prisoner 1	Confess	2,2	1,4
	Refuse	4,1	3,3

Although the result is commonly recognized in all documents, the author thinks it is against basic assumption of game theory: all the participants are rational, and purse maximized benefit. Therefore, when making decisions, the two prisoners do not consider the strategy and benefit of all participants, as well as the police. Although the police does not take any strategy, he set up rules before the game start and actually, he participates the game, in addition, the rules will maximize the benefit of the police.

Second result: based on maximized benefit of two prisoners.

Because all the participants are rational, pursing maximized benefit.

As a judge, the police will firstly consider his own benefit. In order to maximize the benefit, he must set up rules, which will benefit himself most. if anyone confesses, while the other refuses, the one will be set free, and the other will have a 3-year prison life. If both of them confess, they will face 2-year prison life each. If both of them refuse, they will be sent into prison for 1 year respectively. For prisoner, in order to maximize the benefit, we must fully consider the benefit and strategy of the other prisoner and the police.

Table 2. Game result of prisoner's dilemma of three people----judgement equilibrium

		Prisoner 2	
		Confess	Refuse
Prisoner	Confess	2,2 Police, Max	1,4 Prisoner 1, Max
	Refuse	4,1 Prisoner 2, Max	1,1 Prisoners, Max Police, Min

Because when two prisoners choose the strategy, the police will not take any strategy, the two prisoners should not consider the counterbalanced strategy of police, meanwhile, two prisoners trust each of them will make rational choice to minimize the benefit of police and maximize the benefit of themselves. At this time, the result of game is two prisoners refuse to confess. It is also an equilibrium, named judgement equilibrium.

Third result: based on maximized benefit of one prisoner.

In order to get the solution, we make the following assumption.

1. A is our part; B is the opponent.

2. In the game, A must win, and gain maximized benefit. B must lose. If A gains maximized benefit, B must have minimized benefit. In addition, we will not consider the consequence of B.

Based on above assumption, in order to gain maximized benefit, and ensure A will win, we should attack firstly when B does not take any action or strategy. Therefore, the process of

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gaming is what strategy A will take to attack B, when A attacks firstly, we will not care what action B will take. After first action, when B will take corresponding action, A will consider the counterbalanced measures.

Simple model of judgement game:

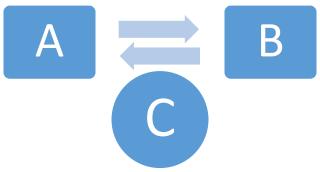

Figure 1. Judgement game model

According to judgement model, C is not the participant, because we do not know the action or strategy of B, initiative action or strategy is needed for A. Thus, how should A take initiative action or strategy? We thinks A should take the following steps first, and then take further action or strategy.

First step: list all the people or things related to A, B and C.

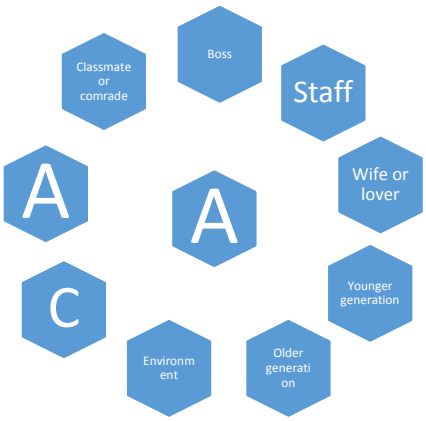

Figure 2. Extension of judgement model

Now, it is changed to the following:

Now the game is changing, participant of game is not only A and B, but all the elements related to A, B and C. That is A and B are not the participants of game, the elements related to A, B and C will be the participants.

Solution rules of judgement game:

1. If A wants to gain maximized benefit, the best way is not gaming. That is to say, if B will quit, A will obtain the maximized benefit directly. In actual situation, we will use extra elements related to A, B and C to force B quit. Due to changed condition, B is willing to quit. Another case is B's death, if this happens, it is the limit of minimized benefit of B. The case will happen in official circles. Therefore, we can infer that the greatest game is no game.

2. A is able to make full use of all related elements to weaken the elements related to B, so B will not gain maximized benefit.

3. A is able to make full use of all related elements to strengthen the elements related to A, so A will gain maximized benefit.

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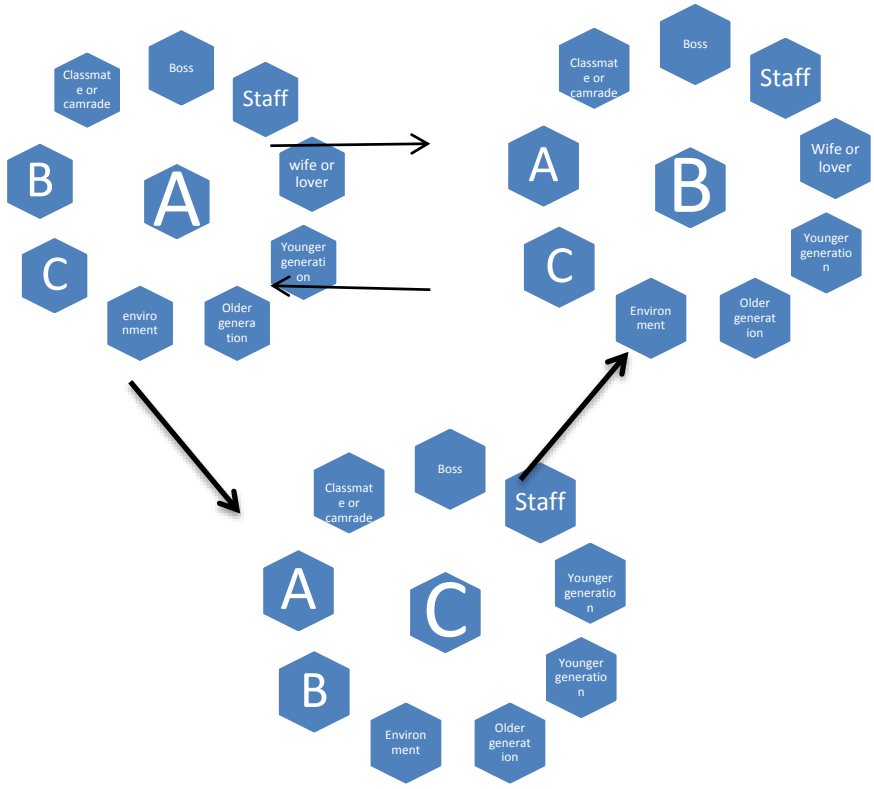

Figure 3. New state of extended judgement model

4. A is able to alienate all participants in order to gain maximized benefit. Alienation is one strategy of A to arouse contradiction between B and C.

5. Establish new game model. There are many ways to build a new game model, we can ask C to participate or ask all the elements without any relations to A, B and C to participate the game. For example, the element is D, which aims to game with B.

For prisoner's dilemma, based on judgment game, we assume the two prisoners are A and B, A is our part, B is the opponent. According to the rules of judgement game, according to the rules of judgement game, all the solutions are as following:

1. The best game is no game. To realize this, we should use all the elements related to A, B and C (police), and force B to quit. There are two ways to force B to quit: prove B is not guilty, B dies, B has opposite identity. If the police can prove B is not guilty, it is a way for B to quit. If we can kill B, B will quit directly. The third method is there exists a fourth party to prove that B is the hero, then B will not participate crime game, and he will hold his identity.

2. Weaken the elements related to B. The method is: we should let B know if he accuses, the people he loved will be hurt, or B will lose more benefit.

3. Strengthen the elements related to A. The method is: the element related to A may directly control the element related to B, so to weaken this element. Or the element related to A is able to control another maximized benefit of B.

4. Alienation method. Arouse contradiction between B and C. For example, A will report some actions of B to damage the benefit of police (C).

5. Establish a new game. Ask the fourth party to participate the game against B. For example, after D appears, if B chooses to report, D will cause much more benefit lose to B.

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5. CASE STUDY AND APPLICATION OF GAME DESIGN AND JUDGEMENT GAME

In actual situation, many games are thought to be a two-participant game, but actually a judgement game. If we think they are three-participant game, or judgement game, maybe we will obtain unexpected solutions, that will be the breakthrough of game theory, and will be the new idea for solving the "dilemma" problems.

1. As the source of supply chain, the supplier should emphasize the management of the relationship of suppliers, as well as how to choose suppliers, which is the foundation for cooperation and also the major section of supply chain management. How to select suppliers and maximize the benefit is the critical problem of enterprise. According to actual situation of enterprise, it is important to set up an index system for selecting and evaluating the supplier in supply chain management. From the perspective of judgement game, if the enterprise is the judge, and sets up the rules, each supplier will game in the fields of product competitiveness, internal competitiveness and reputation. After several rounds of game, the enterprise will select the supplier according to traditional supply chain theory, which is a better guarantee for maximizing benefit.

2. In human resources, when recruiting, the enterprise can set up some rules previously, the applicants should participate the game according to the rules, winners will be recruited as reserve force. In daily HR management, also need some judgement rules to start gaming among administrators and administrators, among staffs and staff, and also among administrators and staffs. Unexpected result will occur.

3. Judgement game can also be used in salary design, set up certain rules to make salary management more effective.

If judgement game is applied in performance assessment, good effect will be gained. For example, in an enterprise, there is one rule in performance management: if sales amount is 30 million-yuan, bonus is 300,000 yuan, over 30 million sales amounts, bonus will increase 100,000 yuan for every 1,000,000 yuan. Based on the rules, each employee just focus the 30 million-yuan sales amounts, there is no game among them, for common salesman, their goal is to exceed 30 million yuan as much as possible. Actually, base number of 30 million yuan maybe very small with less effort. Can a person achieve 50 million sales with hard working?

How should the enterprise do? The enterprise should set up judgement game rules, create a game among all the salesman, gain the maximized sales amount, therefore, the enterprise will gain the maximized benefit. If the rules like this: there is no certain base number, 300,000 yuan for the salesman ranking first plus additional bonus, the bonus is 10% of the difference between the first and second sales. The second will win 200,000 yuan plus bonus (10% of the difference between the first and third sales, deducting 5% of the difference between the first and second). The third ranked will win 150,000 yuan plus bonus ((10% of the difference between the third and fourth sales, deducting 5% of the difference between the first and third). The rules will continue like this. Therefore, the result is: the first ranker will try his best to achieve best performance, the second ranked will chase after the first and widen the gap with the third, the same to the third ranked. The final result is that the sales amount will greatly exceed 30 million yuan. The bonus the enterprise paid may be less than the previous plan, so with the lowest cost can obtain largest sales amount.

The above are the applications of judgement game in enterprise management, thus, we believe the prisoner's dilemma will be applied in more fields in management.

6. CONCLUSION

Above all, we will conclude:

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The prisoner's dilemma is not a two-participant game, but a three-participant game. The prisoner's dilemma is not a Nash Equilibrium with two participants, but a three-participant judgment game, the result of prisoner's dilemma is not inconsistent with normal Nash Equilibrium, but a judgment game with three possible solutions.

When applied in actual situation, we should not just think it is a "dilemma", we should consider the benefit of judge based on judgement game. That is to say, we should design some dilemmas from the perspective of judge, and ask all the participants besides ourselves to game, and our part gain the maximized benefit. For the enterprise, if it will make full use of this game from the perspective of a judge, many hard-tackled problems will be solved effectively.

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