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The Best Investment Strategy for Goodgrant Foundation

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Abstract: The project, after defining students' performance and ROI, sets up 3 models to recommend 10 universities for Goodgrant Foundation's investment. To begin with, the provided data sets are processed to filter out the unused data. The censored or missing data are tackled in the following ways: to adapt Maximum Likelihood Estimation for schools with limited value of the indexes; to use Mean Value for schools with a few missing values; to delete the unqualified schools with lots of data missing.

1. INTRODUCTION

Next, the first model –Evaluation Model of the School Ranking– is established to select schools worth investment, namely, the schools with higher student performance as well as heavier tuition burden. Firstly, two evaluating notions, student performance and tuition burden, are defined. The former is evaluated by four indexes: academic performance, future prospects, the school entrance requirements and the diversity of subject. The latter is evaluated by aid, cost and repayment. After the definition, Fuzzy Synthetic Evaluation is used to calculate values for student performance and tuition burden. Based on the calculation, ROI (return on investment) is defined and qualified by a preference coefficient. Thus, Evaluation Model of the School Ranking is established. Furthermore, the sensitivity of the preference coefficient is analyzed, whose different values will determine different ranking patterns. Also, the stability of our model is tested by deleting certain evaluating indexes.Take public schools in California for examples. When preference coefficient is 0.5, the top 5 schools are CAL POLY, UCSD, CAL MARITIME, UCB, and UCLA. However, the ranking pattern will vary with different values of the preference coefficient. Besides, though the omitted indexes will cause some changes, some schools remain in the top 5.

After the schools are selected, an Optimization Model of the School Investment is set up to make the optimal school investment plan. On the basis of the Markowitz Mean-Variance Model in stock investment, the risks and total ROI are defined and a bottom-line ROI value is set according to the mean value of ROI calculated in the first model. Then Optimization Model of the School Investment is established to work out the schools with a given bottom-line ROI value gained at the lowest risks. As a result, the investment plan is made for the public schools in California such as CAL POLY, UCB, UCSD, CSUN, and SDSU-IVC. Later, sensitivity of the bottom-line ROI value is analyzed, with the result that 0.5 is the critical point for safe investment. Meanwhile, the investment risks are quantified in terms of school similarities, which are calculated by various means such as Euclidean distance and Absolute Value of Distance.

In order to devise the strategy for five-year investments, the attenuation coefficient is defined and the change of ROI after one year is measured. Combined with the two models above, we build Model of School Investment with Time Duration. New ROI value is calculated based on the last-year funding strategy to derive the investment plan for the following year. When attenuation coefficient is 0.9, there are14 schools in the five-year plan in total. Schools with the highest time duration (3 years) are CAL POLY, UCD, CAL MARITIME, and UCSB.

Finally, further analysis is discussed and the strength and weakness are also given. The letter to the Chief Financial Officer (CFO) of the Goodgrant Foundation is also presented.

1.1 Background

There exist some challenges in US higher education, such as increasing education costs, growingfinancial pressure on colleges and universities and the widening education gap. In order to tackle the challenges and improve educational performance of undergraduates, The Goodgrant Foundationintends to donate a total of $100,000,000 to an appropriate group of schools.

We face mainly four problems:

Determine the definition of students' performance and return on investment;

Set up the evaluation model to identify a list of schools recommending for investment;

Build the optimization model deciding the investment amount per school and investment time duration;

Analyze the influences of the parameters, and then discuss whether your model couldbe applied into wide fields.

1.2 Previous research

Ourmodels for Goodgrant Foundation investment strategy includeevaluation model and optimization model. As for evaluation model, we adapted Fuzzy Synthetic Evaluation Model and entropymethod. Fuzzy mathematics forms a branch of mathematics related to fuzzy set theory and fuzzy logic. [1]It started in 1965 after the publication of Lotfi Asker Zadeh's seminal work Fuzzy sets. [2]And information entropy is the measurement of disorder degree of a system. When the difference of the value among the evaluating objects on the same indictor is high, while the entropy is small, it illustrate that this indicator provides more useful information.

As for optimization model, we base our work on the Markowitz Mean-Variance Model (EV Model)[3][4], developed by H.M. Markowitz with the aim of selecting a collection of investment assets that has lower overall risk than any other combination of assets with the same expected return.

1.3 Our work

To determine the rankings of schools, we first define two evaluating notions student performance and tuition burden. Fuzzy Synthetic Evaluation is adapted to calculate scores. We define return on investment (ROI) and introduce preference coefficient to quantify ROI.

Based on the Markowitz Mean-Variance Model in stock investment, we introduce the definition of risks and total ROI and determine the bottom-line ROI value.

In order to devise the strategy for five-year investments, we define the attenuation coefficient and measure the change of ROI after one year.

2. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions

The data we collected from the data sets is accurate and reliable. This insures our models are based on the real life and results worked out are meaningful.

In the Optimization Model of the School Investment, the return on investment of each school obeys normal distribution. This guarantees the usability of Markowitz Mean-Variance Model and we can build our model based on this theory.

In the Model of School Investment with Time Duration, the investment strategy of last year only influence the schools being funded and do not affect other schools. This assures the correctness of method we adopt to calculate the return of investment of schools on the following year, thus insures the next year investment plan reasonable.

2.2 Notations

System for Evaluation Model of the School Ranking:

Table 1. Definitions and symbols

Symbol	Definition
Ap	Academic performance
Fp	Future prospects
Ser	The school entrance requirement
Ds	The diversity of subjects
Aid	Aid
Co	Cost
Rp	Repayment
$\mathbf{X}$	fuzzy matrix of student performance
$\mathbf{Y}$	fuzzy matrix of tuition burden
Sp	student performance
Tb	tuition burden
w_j	the weight for each evaluation grade
ROI	Return on investment
α	preference coefficient

System for Optimization Model of the School Investment:

Table 2. Definitions and symbols

Symbol	Definition
R	the total return on the investment
w_i	the money invested on school
d_{ij}	The Euclidean distance of betweenschools
σ_{ij}	the correlation coefficient
ζ	risk
x_i	proportion of total investment
ρ	the bottom-line of ROI the foundation required

System for Model of School Investment with Time Duration:

Table 3. Definitions and symbols

Symbol	Definition
x_i	proportion of total investment on the i^{th} school
k	the improvement of burden with unit funding

3. DATA COLLECTING AND PREPROCESSING

3.1 Data collecting

Based on the data sets provided by the problem, we begin by filter schools not on "IPEDS UID for Potential Candidate Schools" out of "Most Recent Cohorts Data (Scorecard Elements)" data set. The number of school considered is declined to 850.

By using MATLAB.

3.2 Data preprocessing

Censored or missing data preprocessing

After analyzing the missing data in provided data sets, we divide them into four categories and tackle them with different approaches.

As for the index has a limited amount of values, we adapt Maximum Likelihood Estimation to produce the most likely data by the set of existing index data.

Forexample, the value of index HBCU (Flag for Historically Black College and University) is either 0 or 1. The value for Career Academy of Hair Design-Rogers is missing, and existing values in HBCU is most probably to be 0, so we put 0 as the value for this school.

As for schoolsthat only lose a small amount of values, for each school's missing value, we could calculate the mean value of schools that school belongs to and substitute the mean value for the missing data.

For example, the missing data of New Beginning College of Cosmetology in the index gt_25k_p6 (Share of students earning over $25,000/year 6 years after entry) could be filled with the mean value of schools in Alaska in this index.

As for the school has lots of missing data, we consider thisschool would be unqualified in our investment.

4. DEFINITIONS OF STUDENT PERFORMANCEANDROI

4.1 The definition of student performance

Student performance could be measured by both schools' factors and students' factors. So we consider evaluating from four aspects: academic performances, future prospects, the diversity of subjects and the school entrance requirement.

Academic performance: Students' performance on academic undoubtedly account for a large proportion of evaluation for the student performance. According to the "Most Recent Cohorts Data (Scorecard Elements)" data set, we choose total six indexes to indicate academic performance.

Four-year institutions:

First-time, full-time student retention rate at four-year institutions;

First-time, part-time student retention rate at four-year institutions;

150% completion rate for four-year institutions.

Less-than-four-year institutions:

First-time, full-time student retention rate at less-than-four-year institutions;

First-time, part-time student retention rate at less-than-four-year institutions;

200% completion rate for less-than-four-year institutions.

Future prospects: The graduates' prospects assess students 'performance after graduation. The more students earn in work, the better performance would be. Similarly, we choose two indexes in scorecard data set to indicate future prospects:

Median earnings of students working and not enrolled 10 years after entry;

Share of students earning over $25,000/year (threshold earnings) 6 years after entry.

The school entrance requirement: Generally, the higher requirement the school set, the better quality of students would be. This could indicate the student performance to some extent. In the scorecard data set, the admission score directly reflect schools 'requirement, so we choose following indexes:

SAT scores:

Midpoint of SAT scores at the institution (critical reading);

Midpoint of SAT scores at the institution (math);

Midpoint of SAT scores at the institution (writing);

ACT scores:

Midpoint of the ACT cumulative score;

Midpoint of the ACT English score;

Midpoint of the ACT math score;

Midpoint of the ACT writing score.

The diversity of subjects: With subjects more specified, schools could offer a larger range of courses to students, enabling students learn deeper and wilder knowledge and thus improve student performance. So we count the number of degrees the school could offer in the data set to indicate the diversity of subjects

4.2 Definition of ROI

Our aim is to relieve brilliantly performed students of tremendous tuition burdens. Investments would be most effective if we donate money to the students who perform excellently while cope with high pressure of tuition fees and loans.

So we would begin by evaluating the tuition burden.

The Evaluation of Tuition Burden

Tuition burden could be evaluated from following three aspects: aid, cost and repayment.

Aid: The more aids students appeal, the more likely students in this school are under high tuition burden. So we calculate the total aids provided by the data set and list below:

Percentage of undergraduates who receive a Pell Grant;

Percent of all federal undergraduate students receiving a federal student loan;

Median debt of completers;

Median debt of completers expressed in 10-year monthly payments.

Cost: The students' costs on tuition fee directly measure the tuition burden, so we choose all indexes relating to costs from the data set as below:

Public schools:

Average net price for Title IV institutions (public institutions);

Average net price for $0-$30,000 family income (public institutions)

Average net price for $30,001-$48,000 family income (public institutions)

Average net price for $48,001-$75,000 family income (public institutions)

Average net price for $75,001-$110,000 family income (public institutions)

Average net price for $110,000+ family income (public institutions)

Private schools:

Average net price for Title IV institutions (private for-profit and nonprofit institutions)

Average net price for $0-$30,000 family income (private for-profit and nonprofit institutions)

Average net price for $30,001-$48,000 family income (private for-profit and nonprofit institutions)

Average net price for $48,001-$75,000 family income (private for-profit and nonprofit institutions)

Average net price for $75,001-$110,000 family income (public institutions)

Average net price for $110,000+ family income (public institutions)

Repayment: If the repayment rate is high, students should return more money in the future, which would increase the tuition burden. We use year repayment rate in the data set to indicate the repayment.

ROI

As our aim is to help improve educational performance of undergraduates. Based on the evaluation of student performance and tuition burden, ROI could be defined as

$$ROI = Sp^{\alpha} \cdot Tb^{1-\alpha}$$

Where, Sp denotes the score of student performance and Tb denotes the scores of tuition burden, α denotes preference coefficient ($\alpha \in [0,1]$).When $\alpha > 0.5$, we put more emphasis on student performance.

5. EVALUATION MODEL OF THE SCHOOL RANKING

The Fuzzy Synthetic Evaluation provides a comprehensive and rational framework for list of candidate school.

5.1 Calculation rule for evaluations

For each indexlistedabove, we calculate the average number as the evaluation score.

5.2 Membership functions

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1]. [1]

Let x_{ij} denotes the x_j value for the i^{th} school and $x_{j\max}$ denotes the maximum x_j value for all schools.

We use the normalization function as membership function

$$\mu(x_{ij}) = \frac{X_{ij}}{X_{ij\max}}$$

After calculating $\mu(x_{ij})$ for each x_{ij}, we could combine them into fuzzy matrix X for student performance and Y for tuition burden

$$X = \begin{bmatrix} Ap_j & Fp_j & Ser_j & Ds_j \end{bmatrix};$$
$$Y = \begin{bmatrix} Ai_j & Co_j & Rp_j \end{bmatrix}.$$

5.3 Weights by entropy method

Information entropy is the measurement of disorder degree of a system. When the difference of the value among the evaluating objects on the same indictor is high, while the entropy is small, it illustrate that this indicator provides more useful information [2].To use entropy method, there are mainly five steps:

Calculate the characteristic weight p_{ij} for i^{th} school's j^{th} evaluation grade X_{ij} based on the normalized fuzzy matrix X

$$p_{ij} = \frac{X_{ij}}{\sum_{i=1}^{n} X_{ij}}$$

Calculate the entropy for the j^{th} evaluation grade:

$$e_j = -\frac{1}{ln(M)} \sum_{i=1}^{M} p_{ij} \ln(p_{ij})$$

Calculate the diversity factor for the j^{th} evaluation grade:

$$g_j = 1 - e_j$$

Determine the weight for each evaluation grade:

$$w_j = \frac{g_j}{\sum_{j=1}^{M} g_j}$$

5.4 Evaluation scores

After obtaining the weight metrics W_1 W_2 and fuzzy metrics X Y , the final score for student performance can be calculated as

$$Sp = W_1 \circ X$$

Similarly, the final score for tuition burden is

$$Tb = W_2 \circ Y.$$

Normalizing the value of Sp Tb and then combing them together, we have

$$ROI = S_p^{\alpha} \cdot T_b^{1-\alpha}$$

5.5 Results and analysis

We choose total 161public schools in California from the “IPEDS UID for Potential Candidate Schools” list as examples.

$$w_1 = \begin{bmatrix} 0.0197 & 0.0822 & 0.8289 & 0.0697 \end{bmatrix}$$

$$w_2 = \begin{bmatrix} 0.2468 & 0.4182 & 0.3350 \end{bmatrix}$$

Here are the results for the top 10 schools. ($\alpha = 0.5$)

Results

Table 4. The results of ROI

Ranking	School’s name	ROI	Sp	Tb
1	California Polytechnic State University-San Luis Obispo	0.829291	0.909262	0.756353
2	University of California-San Diego	0.78021	0.835291	0.72876
3	California Maritime Academy	0.76378	0.713002	0.818174
4	University of California-Berkeley	0.761554	0.893376	0.649183
5	University of California-Los Angeles	0.755196	0.837107	0.6813
6	University of California-Santa Barbara	0.747624	0.800658	0.698102
7	University of California-Davis	0.742808	0.779432	0.707906
8	University of California-Santa Cruz	0.716857	0.690298	0.744438
9	University of California-Irvine	0.710947	0.731585	0.690892
10	Sonoma State University	0.710446	0.659728	0.765064

From the table, we can arrive at the following conclusions:

The highest ranking school, California Polytechnic State University-San Luis Obispo, does not rank highest either in student performance evaluation or in tuition burden evaluation; however, when taking these two aspects into consideration, this school rank highest.

The schools ranking loweris due to the fact that its student performance score or tuition burden score is close to 0, thus the overall score is relatively low.

5.6 Sensitivity analysis

Next, sensitivity analysis of preference coefficient is made to analysis the influence on the ranking.Here are the results:

Table 5. The result of sensitivity analysis

Ranking	α=0	α=0.5	α=1
1	California Maritime Academy	California Polytechnic State University-San Luis Obispo	California Polytechnic State University-San Luis Obispo
2	Beaumont Adult School	University of California-San Diego	University of California-Berkeley
3	Sonoma State University	California Maritime Academy	University of California-Los Angeles
4	Humboldt State University	University of California-Berkeley	University of California-San Diego
5	California Polytechnic State University-San Luis Obispo	University of California-Los Angeles	University of California-Santa Barbara

Some conclusions can be made through analysis and observing the table:

When preference coefficient α alters, the ranking result would be different. When α increases, the ranking of California Polytechnic State University-San Luis Obispo goes up.

$ROI = S_p{}^{\alpha} \cdot T_b{}^{1-\alpha}$. When $\alpha = 0$, the ROI is determined by tuition fee.

When $\alpha = 1$, the ROI is determined by student performance.

5.7 Stability Analysis

In order to test the stability of the ranking, we cross out two indexes future prospects (Fp) and the diversity of subject (Ds) of student performance. (α=0.5)

Table 6. Result of stability analysis

Ranking	original	Without Fp	Without Ds
1	California Polytechnic State University-San Luis Obispo	University of California-San Diego	University of California-San Diego
2	University of California-San Diego	California State University-Channel Islands	California State University-Channel Islands
3	California Maritime Academy	California Polytechnic State University-San Luis Obispo	California Polytechnic State University-San Luis Obispo
4	University of California-Berkeley	University of California-Davis	San Diego State University
5	University of California-Los Angeles	San Diego State University	University of California-Davis

From the table above, the following conclusions can be obtained:

After crossing out some indexes, the ranking has some changes, but California Polytechnic State University-San Luis Obispo and University of California-San Diego remain in the top five. It shows that our model has certain stability.

Comparing the changes between “Without Fp ”and “Without Ds ”, the overall number of schools undergoing changes is both three. It shows that these two indexes have equal impact on the ranking.

6. OPTIMIZATION MODEL OF THE SCHOOL INVESTMENT

From the school ranking model, we have evaluated and ranked schools selected from the Potential Candidate Schools list. Now, we would make the optimal school investment strategy determining 1 to N optimized and prioritized schools and investment amount per school.

6.1 Analysis of the Question

Let R denotes the total return on the investment and suppose we fund n schools. R is defined as

$$R = \sum_{i=1}^{n} ROI_i \cdot w_i$$

Where, w_i is the money invested on the i^{th} school.

Apparently, the schools with high ROI value should be the priority for the investment. But the fund should not all give to the school with the highest ROI score when considering risks. Like in stock investment, in order to avoid risks, investors often choose to buy several stocks. In this problem, the foundation is facing the similar risk as stock inventors.

So, our optimized investment strategy is taking the low risk as optimization target under the guarantee of a certain ROI value.

6.2 Establishment of School Investment Model

Based on Markowitz Mean-Variance Model (EV Model) [3] [4], we make some adjustments to establish our School Investment Model.

Definition of Correlation Coefficient

Some schools may share several common characteristics and have the cascading effect. This means when one school is suffered from exogenous events, the same type school may also be influenced. In order to reduce the risk, we should avoid investing the correlated schools.

We determined the correlation coefficient based on the two indexes student performance and tuition burden in the Ranking Model.

The Euclidean distance of between i^{th} and j^{th} school is defined as

$$d_{ij} = \sqrt{(Sp_i - Sp_j)^2 + (Tb_i - Tb_j)^2}$$

If the distance value is small, the two schools are more similar on these two aspects.

Normalize the distance and we get the correlation coefficient

$$\sigma_{ij} = 1 - \frac{d_{ij}}{d_{\max}}$$

In this case, the higher σ value is, the higher similarity between the two schoolshares.

Definition of Risk

Based on the Markowitz Mean-Variance Model, we define risk ζ as

$$\zeta = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$$

Where, x_i denotes proportion of total investment on the i^{th} school.

When $i = j$, $\sigma_{ii} = 1$ $\zeta = \sum_{i=1}^{n} x_i^2$, ζ is the risk for inventing the school itself;

When $i \neq j$, $\sum_{i=1}^{n}\sum_{j\neq i}^{n}\sigma_{ij}x_i x_j$ is the risk of cascading effect.

School Investment Model

To conclude, we have the School Investment Model as follows:

$$\zeta = \min \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$$

$$s.t.\begin{cases} \sum_{i=1}^{n} ROI_i x_i \geq \rho \\ \sum_{i=1}^{n} x_i = 1 \qquad (i = 1, 2, ..., n) \\ 0 \leq x_i \leq 1 \end{cases}$$

Where, ρ is the bottom-line of ROI the foundation required.

x_i denotes proportion of total investment on the i^{th} school.

6.3 Results and conclusions

Determine the value for ρ

Based on the list provided in the ranking model, we calculate the mean value of ROI is 0.2530. So we choose $\rho = 0.2530$ in order to gain the average return.

Solve the nonlinear programming problem

By using LINGO programming, we obtain the lowest risk value is $\zeta = 0.4376$ and the investment strategy is listed below:

Table 7. The investment strategy of $\zeta = 0.4376$

ROI Ranking	School's name	Proportion
1	California Polytechnic State University-San Luis Obispo	0.3286
4	University of California-Berkeley	0.3547
11	San Diego State University	0.1245
27	California State University-Northridge	0.1074
33	San Diego State University-Imperial Valley Campus	0.0848

Some conclusions can be made through analysis and observing the table:

There are total five schools recommended for the investment. University of California-Berkeley accounts for the largest proportion of funding, representing 35.47%, outnumbering the number of California Polytechnic State University-San Luis Obispo, the NO.1 school in ROI Ranking.

Some schools ranked behind in ROI Ranking, such as San Diego State University-Imperial Valley Campus (33^{th}), are included in the recommending list. This is because these schools have low correlation coefficient value, thus have fewer risks.

6.4 Sensitivity Analysis

Next, sensitivity analysis of bottom-line of ROI value ρ and correlation coefficient σ Are made to analysis the influence on the optimized investment strategy.

Sensitivity Analysis of ROI Bottom-line ρ

The variation of the risk with different values of ρ is illustrated in the figure as follows:

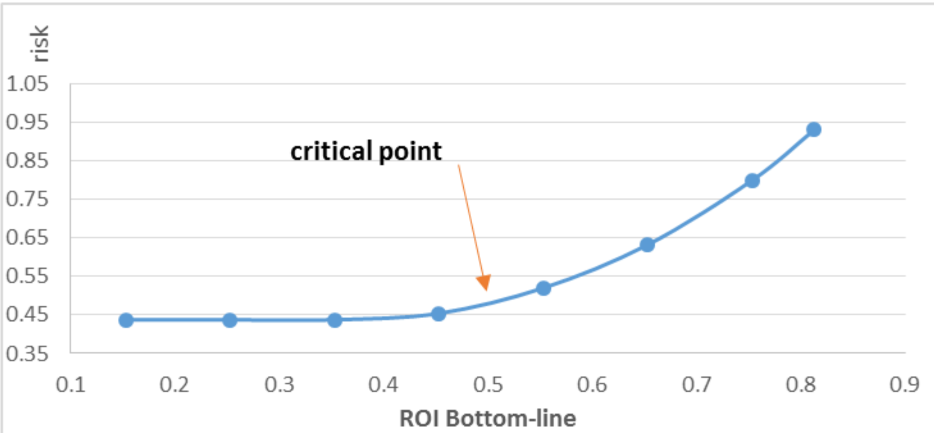

Figure 1. The variation of the risk with different values of ρ is illustrated

From the pictureabove, the following conclusions can be obtained:

The value of risk has a lower limit, that is no matter how small the expect return is, the risk is at least 0.4376.

When ρ exceeds 0.5, the risk begins to increase dramatically, thus $\rho = 0.5$ is a relatively safe expected return value.

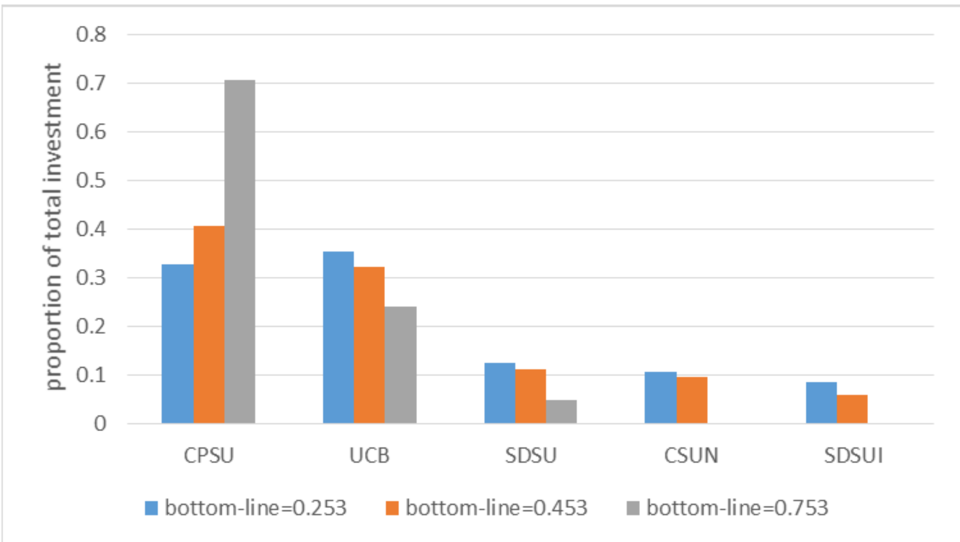

Figure 2. The variation of the proportion with different values of ρ is illustrated

Some conclusions can be made through analysis and observing the chart:

When the requirement for ROI is relatively low, schools with high ROI value do not occupy a significant investment advantage and schools ranked behind are also included in the recommendation.

As ρ increases, the focus of investment turns to schools with high ROI value. When $\rho = 0.753$, the latter two schools are longer included in the recommendation list. Only schools with high ROI value can reach the high expectation, which conforms to the common sense and thus verifies the correctness of the model.

6.5 Other understanding of the best strategy

We build another model of the investment strategy targeted at providing funds to more schools, with the constraints of ROI values and risks.

$$\max \sum_{i}^{n} k_i$$

$$st.\begin{cases} \sum_{i=1}^{n} ROI_i x_i k_i \geq \rho \\ \sum_{i=1}^{n} x_i = 1 \\ 0 \leq x_i \leq 1 \\ \zeta = \min \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j k_i k_j \leq \varepsilon \end{cases} \qquad (i = 1, 2, ..., n)$$

Where, k_i is a Binary value, when the school is invested, the value is 1; otherwise, the value is 0;

ρ is the bottom-line of ROI the foundation required;

ε is the risk limit;

x_i denotes proportion of total investment on the i^{th} school.

7. RESULTS

We determine $\rho = 0.2530, \varepsilon = 0.4376$ and the result is listed below:

Table 8. the result of $\rho = 0.2530, \varepsilon = 0.4376$

Schools 'name	Proportion
California Polytechnic State University-San Luis Obispo	0.17843
California State Polytechnic University-Pomona	0.04717
California State University-Chico	0.04121
University of California-Berkeley	0.1555
University of California-Davis	0.04228
University of California-Irvine	0.06754
University of California-Los Angeles	0.121
University of California-Riverside	0.05225
University of California-San Diego	0.1261
University of California-Santa Barbara	0.1019
University of California-Santa Cruz	0.06168
California Maritime Academy	0.0098

8. CONCLUSION

Compare with the results in Optimization Model of the School Investment and California Polytechnic State University- San Luis Obispo and University of California-Berkeley stay in the top 5 list.

The total number of the schools got funded is 12, while the number is 5 in the Optimization Model of the School Investment.

The higher school ranks in ROI, the more likely it would get funded.

Sensitivity Analysis of Correlation Coefficient σ

Similarly, we change the definitionofdistance from Euclidean distance into Absolute value of distance, that is $d_{ij}^{*} = |Sp_i - Sp_j| + |Tb_i - Tb_j|$.

The variation of optimized investment strategy with different definitionof distancecan also be illustratedin the figure, there follows:

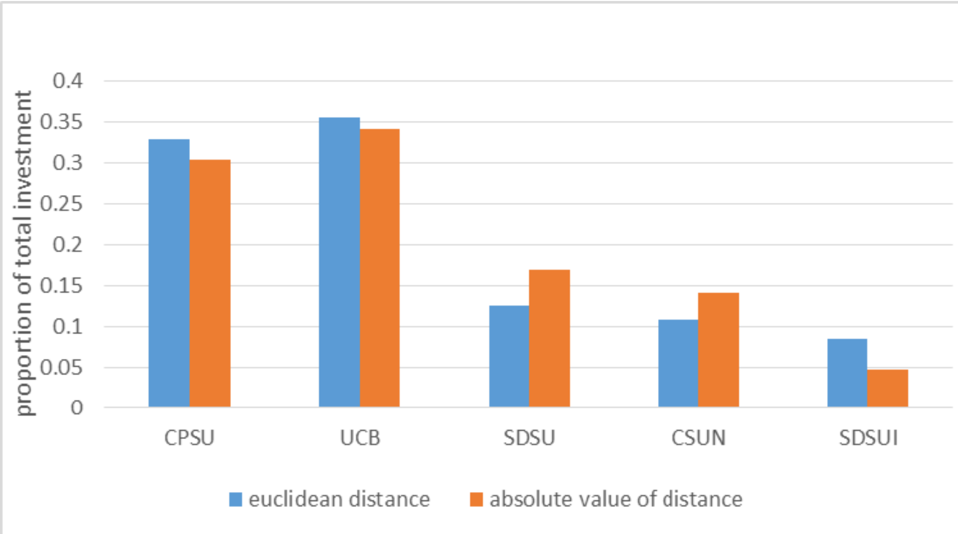

Figure 3. Thestrategy with different definitionof distance

Conclusion: Vary the definition of distance and it turns out that the proportion of each school remains unchanged. It shows that the optimized strategy is not sensitive with the distance definition changes, indicating our model is stable.

9. MODEL OF SCHOOL INVESTMENT WITH TIME DURATION

We have obtained the optimized investment strategyfor the first year in Optimization Model of the School Investment, deciding to give fun to the following five schools: California Polytechnic State University-San Luis Obispo, University of California-Berkeley, San Diego State University, California State University-Northridge and San Diego State University-Imperial Valley Campus.

Now we consider the investment plan for next four years and the time duration of each school's investment.

9.1 Measure the change of ROI

After first year investment, ROI value of schools mentioned above would change. As ROI is defined as $ROI = Sp^{\alpha} \cdot Tb^{1-\alpha}$, we consider the change from these two aspects: student performance and tuition burden. Considering improvement of student performance is a long-term process and could not be measured only after one year, we only consider the improvements on tuition burden.

We assume that the more funds invested, the greater improvements tuition burden would have. So the change on tuition burden is expressed as

$$Tb_i^{t+1} = k \cdot x_i \cdot Tb_i^t \quad (0 < k < 1)$$

Where, Tb_i^t and Tb_i^{t+1} denote the burden value of the i^{th} school in the year of t and $t+1$.

x_i denotes proportion of total investment on the i^{th} school.

k is the attenuation coefficient.

According to the expression above, we could calculate the ROI value for the next year as

$$ROI_i^{t+1} = Sp^{\alpha} \cdot (k \cdot x_i \cdot Tb_i^t)^{1-\alpha} .$$

As the ROIvalue of one school on the potential list decreased, this school's ranking would go down and may not be selected investing in the second year.

9.2 Steps of investment with time duration model

Step 1: Apply Evaluation Model of the School Ranking and obtain the value of student performance (Sp), tuition burden (Tb), ROI and Correlation Coefficient (σ). Determine the ROI Bottom-line (ρ) and use Optimization Model of the School Investment to get the optimized strategy of the first year.

Step 2: Based on the strategy of the first year, calculate the new value for tuition burden, ROI and Correlation Coefficient. Then work out the optimized strategy of the next year by applying Optimization Model of the School Investment.

Step 3: Repeat step 2 until find out the optimized strategy for the fifth year.

Step 4: Calculate the time duration of investment for each school.

9.3 Results and analysis

We choose k=0.9. Follow the steps above and we have results as below:

Conclusions:

The five-year plan for investment includes 14 schools and the time duration ranges from one year to three years.

California Polytechnic State University-San Luis Obispo, University of California-Davis, California Maritime Academy and University of California-Santa Barbara. These four schools' time duration are both three years and their original ROI Ranking are 1,7,3,6 respectively. This indicates that in the long-term investment, schools with high ROI value occupy a position of prominence.

Figure 9. The result of investment with time duration

college name	share				
	2016	2017	2018	2019	2020
California Polytechnic State University-San Luis Obispo	0.3286	0.1912	0.0411	0	0
University of California-Berkeley	0.3547	0.0817	0	0	0
San Diego State University	0.1245	0	0	0	0
California State University-Northridge	0.1074	0	0	0	0
San Diego State University-Imperial Valley Campus	0.0848	0	0	0	0
University of California-San Diego	0	0.2912	0.1208	0	0
University of California-Davis	0	0.2895	0.1926	0.0766	0
San Francisco State University	0	0.1464	0.0218	0	0
California Maritime Academy	0	0	0.3095	0.2584	0.1817
University of California-Santa Barbara	0	0	0.3142	0.2018	0.1591
University of California-Irvine	0	0	0	0.1804	0
University of California-Santa Cruz	0	0	0	0.2828	0.2012
University of California-Los Angeles	0	0	0	0	0.2914
California State University-Monterey Bay	0	0	0	0	0.1666

San Diego State University, California State University-Northridge, San Diego State University-Imperial Valley Campus, University of California-Irvine, University of California-Los Angeles, California State University-Monterey Bay. These six schools ‘original ROI Ranking are11, 27, 33, 18, 9, and 20. This means that in the short-term investment, schools with low ROI value could be considered investing out of consideration for risk.

10. FURTHER STUDY

10.1 Application

Investors may prefer investing a certain type of universities, such as colleges of arts and science or schools of science and technology.

So we would first conduct the Cluster analysis:

Step 1: Determine the distance function. The minimum distance is chosen as the criterion for measurement

$$d(Uni_l, Uni_m) = | v_{1l} - v_{1m} |$$

Where,

$d(Uni_l, Uni_m)$ Is the distance of clusters between university l and university m ,

$| v_{1l} - v_{1m} |$ Is the distance of the school’s type,

Step 2: Use MATLAB to calculate of each object and draw the Dendrogram. We could classify the schools into two large categories: comprehensive schools and non-comprehensive ones.

We take the public comprehensive schools as examples.

According to the characteristics of the comprehensive schools, there is no need in studying the“diversity of majors” index.So we could fix the evaluation of student performance by deletingthe “diversity of majors” index.

The process of investment strategy is illustrated as below:

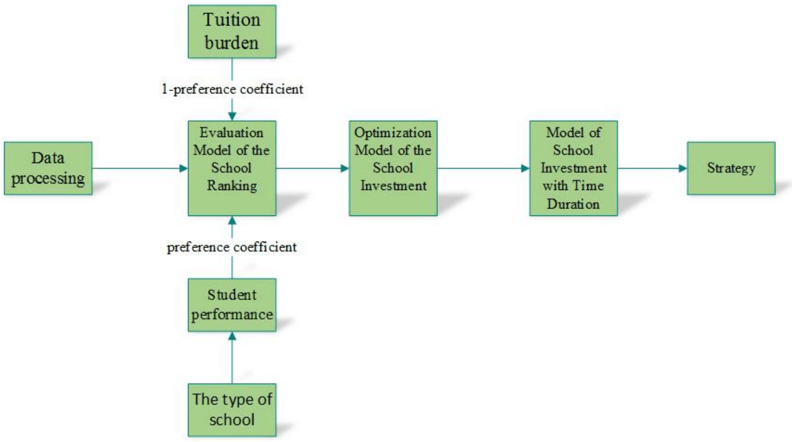

Figure 4. The strategy of comprehensive schools

10.2 Strategies

Step 1: We use Cluster analysis to classify the potential schools and choose one specified type to invest money. Then we preprocess the data and make evaluation for student performance and tuition burden. By using Evaluation Model of the School Ranking, we work out the rankings of potential candidate schools.

Step 2: Based on the theory in stock investment, we quantify the risk of investments on schools anddetermine the bottom-line ROI value. By applying Optimization Model of the School Investment, we get the investment plan for the first year.

Step 3:By using Model of School Investment with Time Duration, new ROI value could be calculated based on the funding strategy last year and we get the investment plan for the following year.Repeat this process and we can obtain the investment plan for five years.

11. STRENGTH AND WEAKNESS

11.1 Strength

We first classify schools depending on the categories that the school belongs to, such as public schools and private schools. This would avoid the blindness when we fund different types of schools. When defining the risk index, we refer to the related theory in stock investment. This makes the investment more close to real situations.

Based on the investment plan of the first year, we re-rank schools by renewed ROI value and work out the funding strategy of the following year. This prevents providing the same investment strategy every year.

Subsequent analysis is more complete, and has rich conclusions, can be in line with reality.

11.2 Weakness

There are a variety of ways to quantify ROI, and we only select one of them to establish the model, which is not representative.

When considering the change on ROI in the next year, we only take tuition burden change into consideration.

12. THE LETTER

Dear Mr. Alpha Chiang,

There exist some challenges in US higher education, such as increasing education costs, growingfinancial pressure on colleges and universities and the widening education gap.

In our view, the charity's aim is to relieve brilliantly performed students of tremendous tuition burdens. Investments would be most effective if we donate money to the students who perform excellently while cope with high pressure of tuition fees and loans. The more students under such conditions we help, the more we get back from the investment.

Based on this concept, we start with measuring the student performance and tuition burden by the data given. The student performance can be evaluated from four aspects: academic performances, future prospects, the diversity of subjects and the school entrance requirement. Tuition burden could be evaluated from following three aspects: aid, cost and repayment. The preference coefficient could be changed by the investor.

Reference to the theory of stock investment, we put forward the definition of the risk of the investment. When the bottom-line of return is guaranteed, the lower risk is, the more effective this investment would be.

Here are the results recommending for the first year investment:

college name	share
 California Polytechnic State University-San Luis Obispo	0.3286
 University of California-Berkeley	0.3547
 San Diego State University	0.1245
 California State University-Northridge	0.1074
 San Diego State University-Imperial Valley Campus	0.0848

Yours,

Team 47755

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