Discontinuous Meshing Method in Surface Integral Equation

Yu Tai[1, a], Jianmei Zhou[2], Kun Jiang[1]

[1]School of Electronic Engineering, Chongqing University of Posts and Telecommunications, Chongqing, 400065, China

[2]School of Electronic and Internet of Things, Chongqing Institute of Engineering, Chongqing, 410065, China

[a]ty0906@163.com

Abstract: A novel method to accurately calculate the current on a complex surface is presented in this paper. It first separates the target surface into fine areas and smooth areas, and uses elements with different sizes to decompose the different areas. Then the current in adjacent border of two different areas is described by redefining base functions and patch forms, which can guarantee the current continuity. This work mainly introduces the applications of the proposed method for two kinds of patches (quadrilateral and triangular patches). The numerical results are given to show the flexibility of the method without loss of accuracy.
Keywords: hierarchical basis functions; phase-extracting basis functions; electromagnetic integral equation.

1. INTRODUCTION

The integral equation method is popular in electromagnetic engineering because it is more accurate and needs fewer unknowns than other methods [1]. Furthermore, the invention of fast algorithms such as AIM [2], FFT [3] and MLFMA [4], which can dramatically improve the efficiency of this method, have promoted its applications especially in solving the problems of scatter and radiation. While, even though the integral equation has so many great merits, it still has some difficulties in engineering calculations. One weakness of it lies in the limited computer resource which can not catch up with the requirements when the working frequency of the calculated target becomes higher and higher. Much work has been done to deal with this confliction. One of the effective ways is to use the elements with large sizes and define the special basis functions, for example, the high order basis functions [5-7] or the phase-exaction basis functions [8-9], etc. Using the elements with larger sizes in target modeling requires less unknowns and computer resources while without the loss of accuracy. So this method causes wide attention in electromagnetic engineering.

Although this method works well when the target has a smooth surface, an obvious disadvantage lies in the difficulty of using large size elements to model the complex targets especially with some fine structures such as slots and holes, which often occur in engineering targets leading to the large size elements seldom used in electromagnetic engineering calculation. A conventional method to solve this problem is to use the larger size elements in the smooth area while smaller ones around the fine structure, which may produce a transition zone between the smooth and slick areas. This method works well when the low order bases are used, but some problems will occur for the higher order bases. For example, it is hard to choose the order of the bases in the transition zone resulting in wasting the number of unknowns or inaccurate current calculation.

In this work, a new method to accurately calculate current is introduced when the large size elements are used. It uses the different size elements in the corresponding areas without introducing any transition zone, and only reconsiders the forms of the elements and basis functions on the adjacent border. As a result, the proposed method can use basis functions efficiently and have high flexibility in meshing of the complex target. Considering the quadrilateral and triangular patches as the most popular elements used in geometrical modeling, the new method applied in the two patches will be introduced in section 2 and 3 respectively. Some numerical examples are given to show the accuracy and flexibility of this method.

2. THE CASE OF QUDRILATERAL PATCH

2.1 The definition

In this part, we will describe this novel method applied in the case of the bases defined on the quadrilateral patch, especially using the high order hierarchical Legendre basis functions [6]. Those bases are defined in the parameter coordinate system on the quadrilateral patch, whose the current expansion can be represented as:

$$\begin{aligned}\boldsymbol{J}_s &= \boldsymbol{J}_{\boldsymbol{u}} + \boldsymbol{J}_{\boldsymbol{v}} \\ &= \frac{1}{\eta_s(u,v)}\left\{\boldsymbol{a}_u \sum_{m=0}^{M_u}\sum_{n=0}^{N_v} b_{mn}^u \tilde{C}_m \tilde{P}_m(u) C_n P_n(v) + \boldsymbol{a}_v \sum_{m=0}^{M_v}\sum_{n=0}^{N_u} b_{mn}^v \tilde{C}_m \tilde{P}_m(v) C_n P_n(u)\right\}\end{aligned} \tag{1}$$

Where, $\boldsymbol{a}_u$ and $\boldsymbol{a}_v$ are the co-variant unitary vector. $\eta_s(u,v)$ is the Jacobian factor of the quadrilateral modeling. P_n and $\tilde{P}_m$ are the Legendre polynomial and the modified Legendre polynomial, respectively. $\tilde{C}_m$ and C_n are the scaling factor correspond to P_n and $\tilde{P}_m$. M_u, N_v and M_v, N_u are the expanded order of the current along the direction and the transverse direction of current. All of those parameters are the same as in [6]. Those bases can decrease the number of the unknowns greatly and obtain an impedance matrix with well conditions. But an obvious drawback of those bases is that it is hard to use the large element to mesh the complex structure mentioned above. On the other hand, although it is convenient to use the large size elements for modeling even in some place, but the actually physical current may be changed fast in some slick areas such as the edges, vertices, which will lead to increase the order of the bases defined on the elements of those areas and deteriorate the impedance matrix. It is natural to think over that some other bases [10] can be used at the edge to describe the singularity current. However, the authors in [6] have proved that those bases are not easily co-used with others. Because of the above difficulties, using the mentioned bases to compute the electromagnetic response for the general objects is very hard. The proposed method we introduce here plans to deal with the problems.

As we know, any target with complex structures can be separated into the slick area and the subtle area. In the subtle area, the current usually changes fast illustrated in Fig.1(a). Furthermore, it is convenient to mesh the surface of the object with different meshing size in different areas, while using this method should make that a large element in the slick area is adjacent to several small elements in the subtle area. In this case, all of the elements are the quadrilaterals, and the high order basis functions are defined on the corresponding parameter coordinate system based on the elements. In order to explain this method conveniently, only the normal square elements are given, and the case of a larger patch adjacent to four smaller ones is shown in Fig.1. When the quadrilateral patches are used, the current is usually separated into two directions as u and v in the parameter coordinate system described in Eq.1, while using the integral equation method should guarantee the normal continuity of the current on the edge of two adjacent patches. So in Fig.1, when considering the elements around the border, we suppose that the v-direction current pass the border shown in Fig.1 (b), in this moment the u-direction current has no contribution on the normal current. So only the v-direction current should be redefined to satisfy the normal continuity. In conventional ways, the continuity is ensured by defining the base function on the element pairs. In this case, it is impossible to define the element pairs on the border because the large patch is adjacent to several small ones.

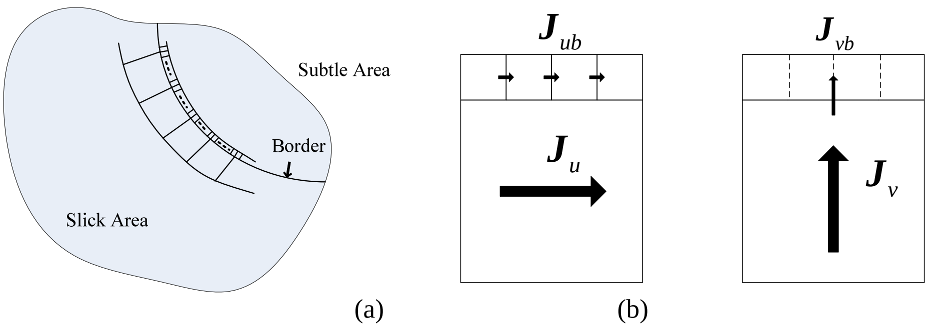


Fig 1. (a): Discrete the surface of complex structure; (b) Reform the elements

So, here we combine a number of small elements adjacent to a large one into a long element in v-direction, and then the base function represented by Eq.1 can be defined on the new long element. In Fig.1 (b), four small elements are combined into a new long element, then, the higher order basis functions as Eq.1 can define on this new long element.

On the other hand, the v-direction current should be reconsidered because the new generated edge is too long, while the small elements in u-direction still use low order base functions to describe the u-direction current. But in the new long element, the v-direction current should be expanded by low order base functions while the high order base functions for the u-direction, which obviously guarantees the current continuity and the consistency of the electric charge. In i-th patch, the v-direction current will be rewrite as:

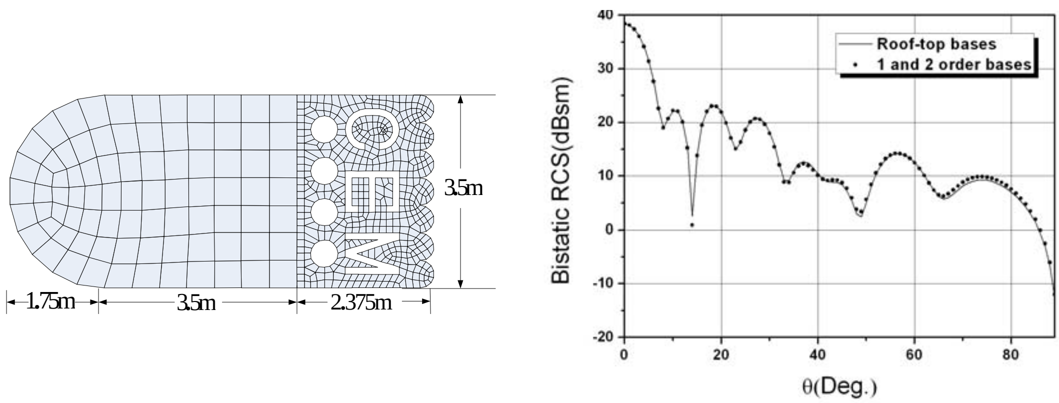
$$
\begin{aligned}
&\boldsymbol{J}_{vi}=\frac{1}{n\eta_s(u,v)}\boldsymbol{a}_v\sum_{m=0}^{M_v}\sum_{n=0}^{N_u}b_{mn}^{v}\tilde{C}_m\tilde{P}_m(v_i^{'})C_nP_n(u),\quad i=1,N \\
&u=-1\sim 1,\quad v=-1\sim 1,\quad v_i^{'}=\left[v+1+2(i-1)\right]/(N-1)
\end{aligned}
\tag{2}
$$

Where, N is the number of the sub elements, i is the rank of the sub-patch. In Fig. 1(b), N=4. In order to depict the v-direction current accurately, n should be larger than m.

In this method, only the elements on the border is reconsidered, which leads to convenience in modeling due to use different size elements illustrated in Fig.1. As a result, any complex structure can be meshed into the quadrilateral patches by the method, and the unique restriction is that the small elements adjacent to a large element in the subtle area must be combined into a long element described in Fig.1 (b), which can be solved easily by controlling the meshing procedure.

2.2 Numberical results

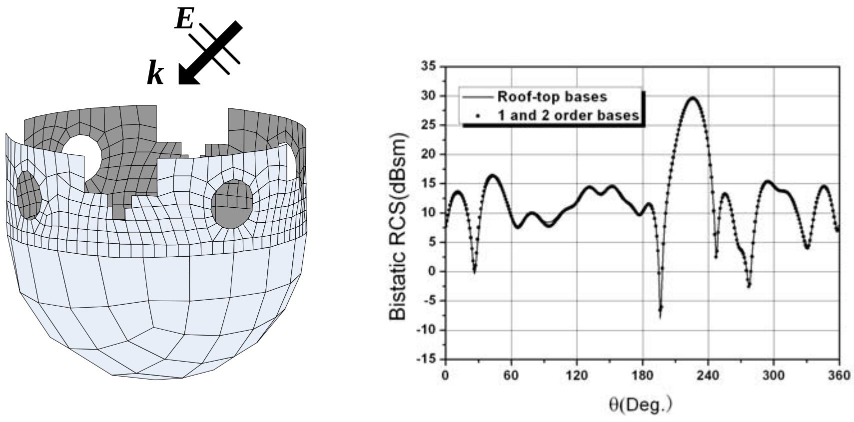
This part gives some results to show the effect of the novel method. First, the introduction of the method is to solve the problem that a target cannot be meshed by the large quadrilateral patches, so an example is given to compare the traditional method using small quadrilateral patches with the proposed method. The first target, a plate shown in Fig.2 (a), is impossible to use large elements for meshing. In this example, the right side is the subtle area and cannot be meshed by the large elements, while the left part is perfect to be meshed by the large elements and use high order base functions. As a result, only the method by using the Roof-top bases with small elements and the proposed method in this paper can be used. The Fig.2 (b) represents the two results on the condition of the incident angle $\theta=90^o,\phi=0^o$ of the incident wave, 300MHz frequency and HH polarization.



(a) (b)

Fig 2. (a) The structure of a plate with “CEM” hole; (b) The RCS.

When the Roof-top bases are used, 3422 unknowns are obtained with 67 iterations, the corresponding execution time of filling the impedance matrix and solving the matrix equation are 833.9s and 80.9s respectively. The new method uses large elements and two order bases in the left side, and small elements and Roof-top bases for the right side, which leads to 1466 unknowns with 83 iterations and the corresponding time of filling the impedance matrix and solving the matrix equation are 65.6s and 17.2s.



(a) (b)

Fig 3. (a) The structure of a 3D object; (b) The RCS

The second example shown in Fig.3 (b) is a 3D target with some subtle structures, whose bottom part can be meshed by large elements but not for the top part. Figure 3(b) shows the comparison of two results based on the Roof-top bases and the method proposed in this paper. When Roof-top bases are used, 3064 unknowns are obtained. It takes 581.5s to fill the impedance matrix, and 66.06s to solve the matrix equation with 65 iteration steps. Figure 3(a) shows the meshing map of the method proposed in this paper. Roof-top bases and 2 order bases are used in smaller elements and larger elements respectively. 1376 unknowns are obtained. It takes 66.2s to fill the impedance matrix, and 14.7s to solve the matrix equation with 81 iteration steps. Obviously, the proposed method not only solves the problem that the large patch is hard to model the complex target, but also reserves the high efficiency and accuracy of high order bases.

3. THE CASE OF TRIANGULAR PATCH

3.1 The definition

The most popular element for modeling is the triangular patch, therefore we will concentrate on triangular elements in this part.

Similar to quadrilateral patches, the surface of the target should be discreted as slick area and subtle area, which generates a border as shown in Fig.4. Then, triangular elements of different sizes are used to model different areas. As shown in Fig.4, the large patch in the slick area is adjacent with several smaller triangles in the subtle area. For the sake of explanation, Fig.4 only shows the plane patch and discusses Fig.4 (a). Of course, the most common mesh is shown as Fig. 4(b), but it can be transformed into Fig. 4(a) by deleting the dashed line part in Fig. 4(b).

We assumed that a large element is adjacent with N smaller patches, and the length of the common edge is l, which is divided as $l_1, l_2, \ldots, l_{N-1}, l_N$ by every smaller triangle. $\sum_{i=1}^{N} l_i = l$.

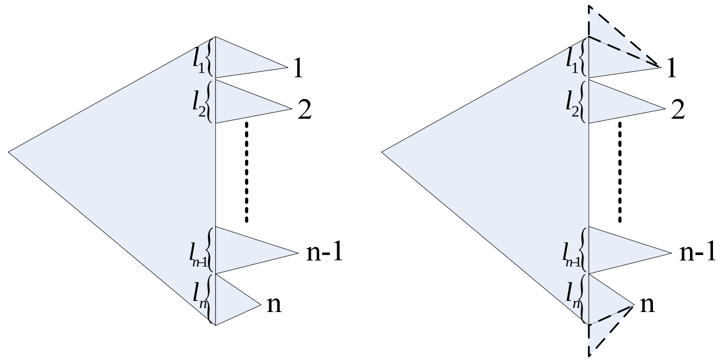


Fig 4. The discontinues meshing method with triangular patch.

Then, a compatible basis functions defined on those triangles should be chosen. In order to decrease the number of unknowns, we use the CRWG-based phase-extracting bases (PE bases), which could be easily obtained through multiplying a factor of the phase (which is usually expressed as $\exp(i\boldsymbol{k}\cdot\boldsymbol{r})$) with CRWG [9]. The definition of PE bases could be used to extract the fast phase variation of the induced current, and it could also lead to only several unknowns with high accuracy of per square wavelength on a very large patch. But, in some areas, the PE bases will not work well as the current changed faster than $\exp(i\boldsymbol{k}\cdot\boldsymbol{r})$ (for example, on the edge, vertex or any other place with singularity). Of course, there is no basis function which could perform well in any case. And the simplest and most effective useful method is to increase the mesh density in those areas, which might lead to the problems mentioned above. So the discontinues meshing method is introduced in this paper to deal with those problems when the triangular patches are used.

First, assume that the basis function defined in triangle is $\boldsymbol{f}$, which will be expressed as $\boldsymbol{f}^+, \boldsymbol{f}^-$ in the positive and negative patch respectively. Then, the PE-CRWG bases will be represented as

$$\boldsymbol{f}_{pe} = \begin{cases} \boldsymbol{f}_{pe}^+ = \boldsymbol{f}^+ \cdot \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \ , & \boldsymbol{r} \in S^+ \\ \boldsymbol{f}_{pe}^- = \boldsymbol{f}^- \cdot \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \ , & \boldsymbol{r} \in S^- \end{cases} \tag{3}$$

Where, the $\boldsymbol{f}^{\pm}$ is the CRWG [12] and it is defined as

$$\begin{cases} \boldsymbol{f}^+ = \dfrac{l_n}{2A_n^+}\left[(u-u_+)h_u\hat{\boldsymbol{u}} + (v-v_+)h_v\hat{\boldsymbol{v}}\right] , & \boldsymbol{r} \in S^+ \\ \boldsymbol{f}^- = \dfrac{l_n}{2A_n^-}\left[(u-u_-)h_u\hat{\boldsymbol{u}} + (v-v_-)h_v\hat{\boldsymbol{v}}\right] , & \boldsymbol{r} \in S^- \end{cases} \tag{4}$$

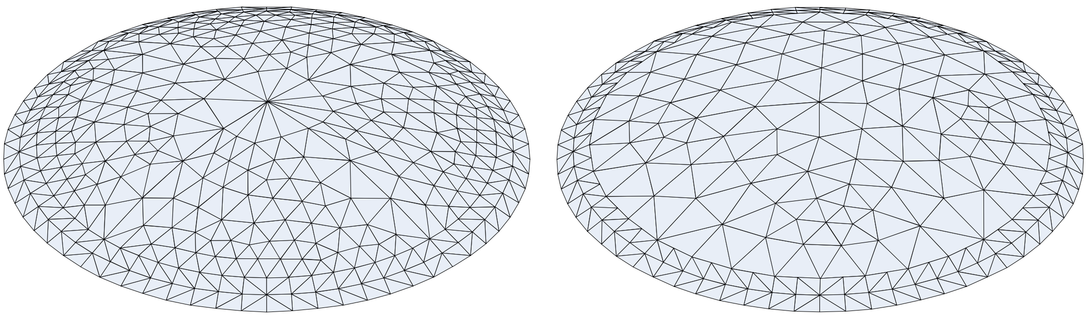
Where, $(u_{\pm}, v_{\pm})$ is the parameter coordinate of the vertex in the corresponding positive or negative patches. It could be observed from Eq.4 that the normal current pass the common edge is a constant which is related to length of the edge l_n. The PE-CRWG bases performed well in the slick area with no strong coupling is because it has extracted the fast variation phase. However, in this paper, the base function would not define on a pair of triangles, but a larger and several smaller ones as shown in Fig.5. Without loss of generality, assume that the larger patch is the positive element, and the smaller elements are the negative, then the basis can be represented as

$$\boldsymbol{f}_{pe}^{de} = \begin{cases} \boldsymbol{f}_{pe}^{de+} & , \quad r \in S^{+} \\ \dfrac{l_1}{l} \boldsymbol{f}_{pe}^{de1-} & , \quad r \in S_1^{-} \\ \dfrac{l_2}{l} \boldsymbol{f}_{pe}^{de2-} & , \quad r \in S_2^{-} \\ \vdots & \\ \dfrac{l_N}{l} \boldsymbol{f}_{pe}^{den-} & , \quad r \in S_N^{-} \end{cases} \tag{5}$$

The positive part of Eq.5 is the same as Eq.3, while, the negative parts should multiply with l_i/l to modify the normal component of the current and guarantee the continuity of current.

3.2 Numberical results

A cake-like object is used to test the validity of this new method when the triangular patches are used, as shown in Fig. 5. This object is a crown of sphere with the radius of 2m, the bottom radius of 1m, and the corresponding height of 0.267m. The coupling between the top and bottom of the object is strong, resulting to the rapid change of induced current on the edge.



(a) (b)

(a) by traditional method; (b) by this method

Fig 5. The meshing map of the cake-like object

In this example, the plane wave incidents at the angle of $\theta = 45^o, \phi = 0^o$ with 600MHz frequency and HH polarization. The far-field is scanned at plane $\phi = 0^o$. Conventionally, in order to describe the distribution of current precisely and guarantee the continuity of current, it is necessary to mesh with smaller elements on the edges and large ones in the middle of it, and the different sizes of elements should transit in accordance with Fig. 5(a) [11]. While, the mesh in Fig. 5(b) is the result of the method proposed in this work. In this figure, smaller elements are only used on the edges while large patches are in the middle, which leads to the adjacency of some large patches and two smaller elements. Of course, the traditional methods could not ensure the normal current continuity on those patches, while, the method proposed here could achieve that easily. Furthermore, fewer elements and more regular shapes are obtained with this method, as shown in Fig. 5. It will greatly affect the accuracy of the results, as shown in Fig 6. Figure 6 shows the results of three far-fields. The first is the result of dense mesh approach with $\lambda/8$ mesh size and 5631 unknowns. The second line is the result of method proposed in reference [11] with 1451 unknowns and its meshing map is shown in Fig. 5(a). The last line is the result of the method proposed in this work, with 1249 corresponding unknowns and its meshing map is shown in Fig. 6(b). Obviously, the last two results decrease the number of unknowns, and the result of the third line is more accurate than that of the second line, which means the method proposed in this work will get more accurate result and result to fewer unknowns compared with the traditional method. This example aims to show the validity of this method, so only the results of the edges are given, while complex structures are missing in this work. However, it’s not hard to find out that this method is very suitable for subtle structures such as the vertex, slot, connect structure, etc, which appear frequently in complex objects.

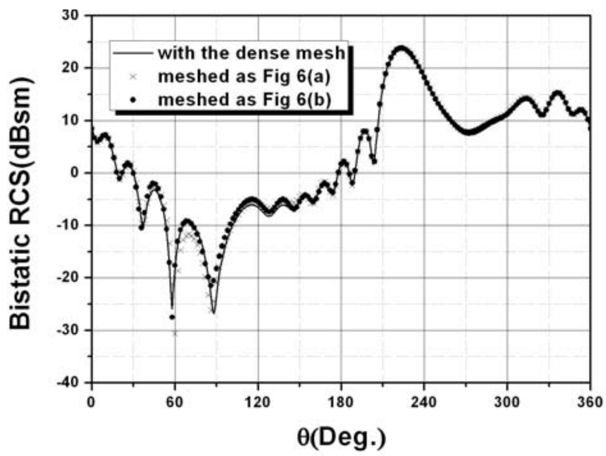


Fig 6. The RCS of the cake-like object

4. CONCLUSION

A novel method to model objects with complex structures based on the electromagnetic surface integral equation is proposed in this work. This method overcomes the difficulty of modeling complex objects on large patches, which would be introduced in two cases, triangular patches and quadrilateral patches. This method ensures the continuity of current and obtains more regular meshing elements, which increase the accuracy of its result. Moreover, the purpose of this work lies in solving the problem that it’s hard to use basis functions defined on the large patches in the complex structure. Therefore, this method has many advantages as the basis function defined on large patches has fewer unknowns, and the modeling defined on small patches is very convenient. At last, some numerical examples are given to prove the accuracy and flexibility of this new method.

REFERENCES

[1] Harrington R F, “Field Computation by Moment methods”. New York Macmillan, 1968

[2] Wang, C. F., Li, L. W., Kooi, P. S., Leong, M. S. “Efficient capacitance computation for three-dimensional structures based on adaptive integral method”, Progress In Electromagnetics Research, 30:33-46, 2001.

[3] Rautio, B., Okhmatovski, V. I., & Lee, J. K. “The Unifed-FFT Grid Totalizing Algorithm for Fast O (N Log N) Method of Moments Electromagnetic Analysis with Accuracy to Machine Precision”, Progress In Electromagnetics Research, 154:101-114, 2015.

[4] J. M. Song, C. C. Lu., W.C.Chew. “Multilevel fast multipole algorithm for electromahnetic scattering by large complex objects”. IEEE Trans. Antennas and Propagation. 45(10):1488-1493, 1997.

[5] L. R. Hamilton, P. A. Macdonald, et al, “Electromagnetic scattering computations using high-order basis functions in the method of moments,” IEEE APS Int. Symp. Dig., 1994, 3:2166-2169.

[6] E. Jorgensen, J. L. Volakis, P. Meincke, and O. Breinbjerg, “Higher order hierarchical Legendre basis functions for electromagnetic modeling.” IEEE Trans. Antennas and Propagation. 52(11): 2985-2995, 2004.

[7] Cai, Q. M., Zhao, Y. W., Zheng, Y. T., Jia, M. M., Zhao, Z., & Nie, Z. P. , “Volume integral equation with higher order hierarchical basis functions for analysis of dielectric electromagnetic scattering”, IEEE Transactions on Antennas and Propagation, 63 (11): 4964-4975, 2015.

[8] Kowalski, M.E., Singh, B., “Application of the Integral Equation-Asymptotic Phase (Ie-Ap) Method to Three-Dimensional Scattering”, Journal of Electromagnetic Waves and Applications, 15(7):885-900, 2001.

[9] Nie, Z. P., Yan, S., He, S., Hu, J. “On the basis functions with traveling wave phase factor for efficient analysis of scattering from electrically large targets”, Progress In Electromagnetics Research, 2008, 85: 83-114.

[10] Trintinalia, L.C., Hao Ling. “An improved triangular patch basis for the method of moments”, IEEE Antennas and Propagation Society International Symposium, 2000, 4: 2306-2309.

[11] M.E.Kowalski and B.Singh. “Application of the integral equation-asymptotic phase (IE-AP) method to three-dimensional scattering”, J.of Electromagn. Wave and Appl., 15 (7): 885-900, 2001.

[12] Debra L. Wilkes, Chung-Chi Cha. “Method of Moments solution with Parameteric Curved Triangular Patches”, Proc. IEEE Antennas and Propagation Society Int. Sym., 1512-1515, 1991.