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Risk Analysis and Hedging of European Put Option Based on Geometric Brownian Motion

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Abstract

On the basis of option pricing model, this paper analyses the sensitivity of option price to several main factors in detail, including the price of the underlying asset, the volatility of the underlying asset price and the maturity date. For each sensitivity index, expounds the corresponding management methods, Such as Delta hedge, Gamma hedge and so on. Finally, this paper takes a stock as the underlying asset, calculate the pricing of its European put options and simulate the trend of stock prices. The Delta value and Gamma value of the option in two states of real value and virtual value are analyzed in detail. And use the hedging method to hedge the corresponding risk, calculate the cost of risk management.

Keywords

European put options, risk analysis, Hedging.

1. INTRODUCTION

With the continuous opening of China's capital market, financial derivatives play an increasingly important role in the market. It provides investors with the option of avoiding risks and promotes the formation of equilibrium prices of basic assets. At present, China's financial derivatives market has been developing continuously in recent years. In April 2010, stock index futures were listed and traded; in 2012, a variety of commodity futures were launched; in 2015, trading open index fund (ETF) options contracts were officially listed on the Shanghai Stock Exchange. This opened the curtain of the development of China's options market. Therefore, the future research on options will become an important topic in China's financial industry.

Options have the effect of avoiding risks and price discovery, and are also a good way to hedge asset risks. This paper aims to analyze the option risk situation and discuss the corresponding management methods.

The pricing of options is the most difficult of all derivatives. It has always been the focus of securities investment research. Western scholars have proposed a variety of option pricing models, but before the emergence of the Black-Scholes option pricing model, these models did not Practical value. At the beginning of the development of China's option market, it is necessary to explore the application of option pricing theory.

This paper will take the geometrical Brownian motion as the basic assumption of stock price, combine the geometric Brownian motion stochastic analysis method and the financial no-arbitrage principle, and quantify the risk faced by the option based on the option pricing model, such as the price of the underlying asset, the volatility of the underlying asset price and the expiration date; and the corresponding risk for the sensitive indicators of the option against each factor Management method.

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2. RISK ANALYSIS AND MANAGEMENT

2.1. Hedging Mechanism

Suppose the number of shares in a portfolio is a, and at the same time sell an option at a price of V, holding a certain amount of cash C. At this point, the value of the portfolio is

$$\prod = aS - V + C$$

When the above formula is converted into a differential form, the cash C strain is rdt, and r is a risk-free rate, so there is (Joseph Stampfli, 2001).

$$d\prod = adS - dV + rCdt \tag{1}$$

Taylor expansion of dV can be obtained,

$$\begin{aligned} dV &= \frac{\partial V}{\partial S}dS + \frac{\partial V}{\partial t}dt + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\left(dS\right)^2 + \cdots \\ &\approx \frac{\partial V}{\partial S}dS + \frac{\partial V}{\partial t}dt + \frac{\sigma^2}{2}S^2\frac{\partial^2 V}{\partial S^2}dt \end{aligned}$$

According to the Black-Scholes equation, there are,

$$dV = \frac{\partial V}{\partial S}dS + rVdt - rS\frac{\partial V}{\partial S}dt$$

Substituting the above formula into equation (1),

$$d\prod = adS - \left(\frac{\partial V}{\partial S}dS + rVdt - rS\frac{\partial V}{\partial S}dt\right) + rCdt$$

Let $a = \frac{\partial V}{\partial S}$, get

$$d\prod = r\left(-V + Sa + C\right)dt = r\prod dt$$

With Delta's hedging, the portfolio is always in equilibrium, which is consistent with the Delta analysis described below, but with different derivations. The risk of financial market comes from uncertainty, and the uncertainty of option price is affected by the underlying assets and execution price. Investors always hope to minimize the losses caused by risks and uncertainties. So people need to understand the risks and manage them. Earlier we discussed the main factors affecting the price of an option, but to understand the risk characteristics of an option, you need to know how sensitive the option price is to these factors. By appropriate means, the sensitivity of the asset option price is reduced to zero, so that hedging can be achieved and the risk managed.

The following is a sensitivity analysis of the main factors affecting the price of the option, including the price of the underlying asset S, the volatility of the underlying asset price σ, the risk rate r and the expiration date T.

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2.2. Delta Value Analysis

The Delta (Δ) value is usually used to measure the sensitivity of the option price to the underlying asset price change. It reflects the impact of the asset price on the option price. From a mathematical point of view, Delta is equal to the partial derivative of the option price against the underlying asset price. Therefore, according to the above definition and the option pricing formula mentioned above, the Delta value of the European put option with no profitable assets can be obtained.

$$\begin{aligned}\Delta &= \frac{\partial f}{\partial S} = \frac{\partial\left(Xe^{-r(T-t)}N(-d_2) - SN(-d_1)\right)}{\partial S} \\ &= Xe^{-r(T-t)}N'(-d_2)\frac{-\partial d_2}{\partial S} - N(-d_1) - SN'(-d_1)\frac{-\partial d_1}{\partial S}\end{aligned} \tag{2}$$

In the above formula,

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S} = \frac{1}{S\sigma\sqrt{T-t}}$$

So equation (3) can be simplified to

$$\begin{aligned}\Delta &= Xe^{-r(T-t)}N'(-d_2)\frac{-\partial d_2}{\partial S} - N(-d_1) - SN'(-d_1)\frac{-\partial d_1}{\partial S} \\ &= \frac{\left(Xe^{-r(T-t)}N'(-d_2) - SN'(-d_1)\right)}{-S\sigma\sqrt{T-t}} - N(-d_1)\end{aligned} \tag{3}$$

Analyze $N(-d_1)$ and $N(-d_2)$,

$$\begin{aligned}N'(-d_2) &= -\frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2\sigma^2}} = -\frac{1}{\sqrt{2\pi}}e^{-\frac{\left(d_1-\sigma\sqrt{T-t}\right)^2}{2}} = -\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}} \cdot e^{\frac{2d_1\sigma\sqrt{T-t}-\sigma^2(T-t)}{2}} \\ &= -\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}} \cdot e^{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)-\frac{1}{2}\sigma^2(T-t)} = N'(-d_1)e^{r(T-t)}\frac{S}{X}\end{aligned}$$

Substituting the above formula into equation (2) can be simplified.

$$\Delta = -N(-d_1) = N(d_1) - 1, \quad d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

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Since a has $N(d_1)$ value range of $0 \le N(d_1) \le 1$, the delta value of the European put option is between -1 and 0. Conversely, if it is a short position of the European put option, its Delta value is between 0 and 1.

It can be seen from equation (3) that Delta of the option is affected by factors such as asset price, volatility and maturity interval. From the following figure, the relationship between the delta value of the European put option and the underlying asset price is shown as the underlying asset. When the price rises, the absolute value of the Delta value of the European put option decreases.

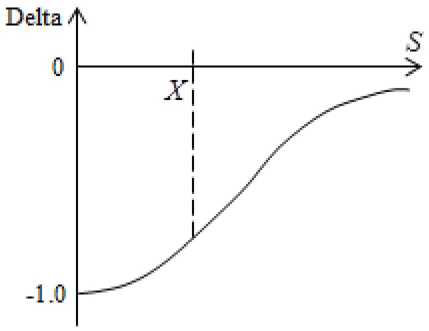

Figure 1. The relationship between the Delta value and the underlying asset price

In addition, according to formula (3), as the risk-free interest rate rises, the delta value of the non-return European put option at the imaginary value, the real value and the flat price also becomes larger, and the trend of the change is shown in the following figure.

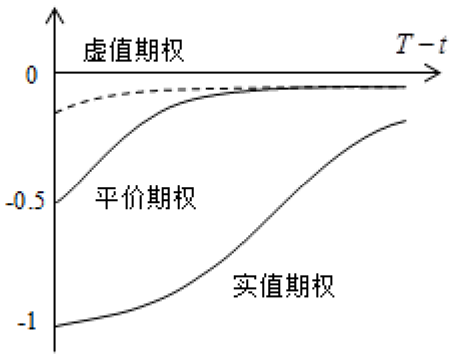

Figure 2. The relationship between Delta value and risk-free interest rate

And the relationship between the imaginary option, the real option, the cheap option and the expiration time interval of the European put option is shown in the following figure.

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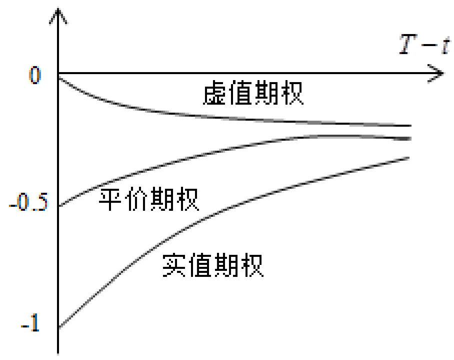

Figure 3. Relationship between Delta value and expiration date

Hedging assets based on Delta values allows the portfolio value to be avoided in a short period of time, so that the portfolio value is not affected by changes in the underlying asset price. This is consistent with the previous idea of solving the Black-Scholes differential equation. By operating the corresponding underlying assets and options, the portfolio is brought to a neutral state, which Δ is zero.

From the characteristics analysis of Delta, it can be seen that with the changes of the target asset price, volatility and expiration time interval, the Delta of the option is also in dynamic change, so the hedge can only be in a short moment. Realization, the position of each asset also needs constant adjustment.

3. CASE ANALYSIS

The above analysis of the risk sources of European put options, including the underlying stock price, time, volatility, etc., and the hedging management methods of related risks are described. In the following, we will use Siyuan Electric (002028) as the underlying stock to analyze the risk of its European put option and discuss how to hedge it.

When using the Black-Scholes model to price an option, the following data is required

Current underlying stock market price S

Option expiration time T

Contracted transaction price X

Risk-free continuous compounding r

Standard stock yield standard deviation σ

This paper collects the trading data of the underlying stock Siyuan Electric in the past six months, which is derived from the Flush software, starting from September 1, 2016, to February 28, 2017, and using the closing price on February 28 as the current stock market price.

Assume that the option expiration interval is 120 days; the assumed delivery price is 17; the risk-free rate is calculated based on the benchmark interest rate of the financial institution's RMB one-year time deposit; the target stock yield can be calculated from the transaction data;

$$
\begin{aligned}
&S = 14.80 \\
&T - t = 0.328767 \\
&X = 17 \\
&r = \ln(1 + r_0) = \ln(1 + 1.5\%) \approx 1.49\% \\
&\sigma = 0.0147
\end{aligned}
$$

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3.1. Stock Price and Option Price

According to the parameters obtained above, the stock price estimation formula can be used to calculate the stock price change within the expiration time. When the current stock market is known, its price change formula is

$$S_t = S_{t-1} + \mu S_{t-1} t + \sigma S_{t-1} \varepsilon \tag{4}$$

Among them,

$$\mu = \bar{R} = \frac{1}{T}\sum_{t=1}^{T} \ln R_t = 0.00258$$

However, since the value of ε is a random variable, the trend of the stock price is not likely to be determined. It changes with the value of ε at each time point. Using R software to simulate it can get a series of stock price changes, the following figure shows one of the changes.

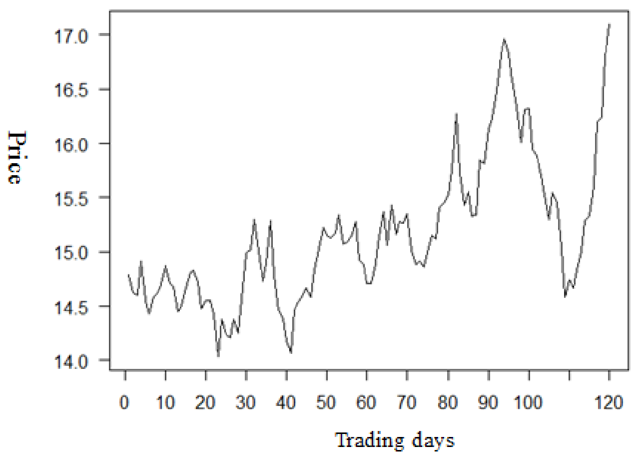

Figure 4. Stock price change

Although the price of the stock price at each time point cannot be determined, the price of the European put option at each time point is determinable due to the neutrality pricing principle of the utilization risk. According to the known parameters, the first step can be calculated

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = -15.85$$

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = -15.86$$

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The value of the cumulative probability density $N\left(-d_1\right)$ and $N\left(-d_2\right)$ can be obtained by looking up the table, and the price of the European put option with the delivery time of 17 and the delivery price of 17 can be obtained.

$$p = Xe^{-r(T-t)}N\left(-d_2\right) - SN\left(-d_1\right) = 2.12$$

3.2. Risk Analysis and Management

Suppose the trader sells 1 European put option. According to the parameters selected above, the current target stock market price is 14.80, the option expiration time interval is four months, the agreed transaction price is 17, and the risk-free continuous compound interest is 1.49%. The standard deviation of the rate is 0.0146.

At this time, the theoretical pricing of a unit of European put options has been calculated. The corresponding Delta value is calculated by the following formula.

$$\Delta = -N\left(-d_1\right) = N\left(d_1\right) - 1 = -1$$

At this moment, the trader as a short seller of European put options should sell one unit of stock for risk hedging, so that the asset portfolio reaches the Delta neutral state. From the nature of the Delta value, Delta will change with the price of the underlying stock. To maintain the neutral state of the portfolio requires constant adjustment of the held position.

3.2.1 In The Money

For European put options, real-value options mean that the option's maturity execution price is higher than the stock market price, at which point the bulls will choose to execute the option.

Using the formula (4) to simulate the stock price and obtain the stock price change when the real value option is obtained, draw it into the following figure. It can be clearly seen that the stock price on the maturity date is lower than the execution price of 17, and the option long will Execution of the option, at this time the dealer as an option short, in order to prevent the risk of stock price changes, risk hedge can be based on the Delta value.

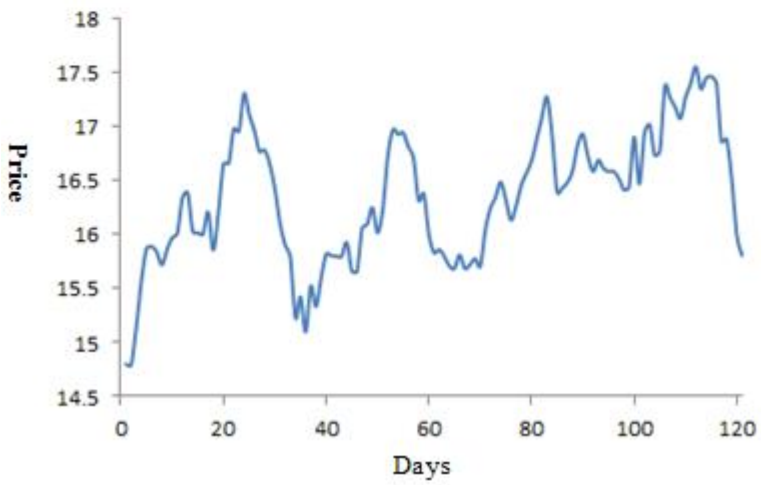

Figure 5. Stock price change

The following table calculates the Delta value within 120 days of the expiration date. In order to hedge the risk, the trader should hold the corresponding Delta shares every day. The negative

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value indicates the number of short-selling stocks, and the positive value indicates the number of shares held. The Delta value less than 1 does not affect the operation of the sale, because this article uses an option as the analysis object, but the actual number of shares in the transaction is often hundreds of thousands, Delta multiplied by the number of shares sold will no longer be less than 1.

Table 1. Delta value change

ΔT	Delta	ΔT	Delta	ΔT	Delta	ΔT	Delta	ΔT	Delta
120	-1.00	96	-0.08	72	-1.00	48	-1.00	24	-1.00
119	-1.00	95	-0.40	71	-1.00	47	-1.00	23	-1.00
118	-1.00	94	-0.90	70	-1.00	46	-1.00	22	-1.00
117	-1.00	93	-0.88	69	-0.98	45	-1.00	21	-0.92
116	-1.00	92	-0.99	68	-0.43	44	-1.00	20	-1.00
115	-1.00	91	-1.00	67	-0.59	43	-1.00	19	-0.78
114	-1.00	90	-1.00	66	-0.54	42	-1.00	18	-0.29
113	-1.00	89	-1.00	65	-0.91	41	-1.00	17	-1.00
112	-1.00	88	-1.00	64	-0.99	40	-0.86	16	-1.00
111	-1.00	87	-1.00	63	-1.00	39	-0.09	15	0.00
110	-1.00	86	-1.00	62	-1.00	38	0.00	14	0.00
109	-1.00	85	-1.00	61	-1.00	37	-0.78	13	0.00
108	-1.00	84	-1.00	60	-1.00	36	-1.00	12	-0.03
107	-1.00	83	-1.00	59	-1.00	35	-1.00	11	0.00
106	-1.00	82	-1.00	58	-1.00	34	-1.00	10	0.00
105	-1.00	81	-1.00	57	-1.00	33	-1.00	9	0.00
104	-1.00	80	-1.00	56	-1.00	32	-0.97	8	0.00
103	-1.00	79	-1.00	55	-1.00	31	-0.76	7	0.00
102	-1.00	78	-1.00	54	-1.00	30	-1.00	6	0.00
101	-0.98	77	-1.00	53	-1.00	29	-1.00	5	0.00
100	-0.98	76	-1.00	52	-1.00	28	-1.00	4	-1.00
99	-0.35	75	-1.00	51	-1.00	27	-1.00	3	-1.00
98	-0.42	74	-1.00	50	-1.00	26	-1.00	2	-1.00
97	0.00	73	-1.00	49	-1.00	25	-1.00	1	-1.00
								0	0.00

According to the change of the Delta value, the risk hedging operation is not required every day for 120 days. For example, the risk hedging can be realized only by short selling within 102-120 days of the expiration date. This is because the Delta value is not after the hedging. Change again, the previous shares owned or sold are sufficient to hedge the risk. The table below calculates the time points at which risk hedging is required and the corresponding number of shares traded. It can be calculated that in 120 days, only 82 hedging operations are required. On the expiration date, since the stock price is lower than the execution price, the option longs will choose to execute the option, and at this time, buy 1 unit of stock.

Before the option expiration date, the asset value will change due to the stock price and the hedging operation. The following table shows the changes in the asset value during this period. At the beginning, due to the debts borne by the stocks, the value of the assets is at a high level. As the exercise date approaches, the stocks that have been short-selled need to be closed, and the value of the assets will decrease. At the expiration date, the value of the asset is -0.24, which is the cost of the hedging risk.

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Table 2. Hedging operation

ΔT	Number of shares	ΔT	Number of shares	ΔT	Number of shares	ΔT	Number of shares
120	-1.0000	95	-0.3144	47	0.0000	26	0.0000
116	0.0000	94	-0.5056	46	0.0000	25	0.0000
115	0.0000	93	0.0191	45	0.0000	24	0.0000
114	0.0000	92	-0.1050	43	0.0000	23	0.0000
112	0.0000	91	-0.0117	42	0.0000	21	0.0820
111	0.0000	90	0.0000	41	0.0003	20	-0.0820
110	0.0000	89	0.0000	40	0.1387	19	0.2190
109	0.0000	73	0.0000	39	0.7664	18	0.4944
108	0.0000	72	0.0000	38	0.0945	17	-0.7134
107	0.0000	71	0.0000	37	-0.7797	16	0.0000
106	0.0000	70	0.0000	36	-0.2201	15	1.0000
105	0.0000	69	0.0186	35	0.0000	14	0.0000
104	0.0000	68	0.5533	34	0.0000	13	0.0000
103	0.0000	67	-0.1653	33	0.0000	12	-0.0341
102	0.0000	66	0.0487	32	0.0271	11	0.0342
101	0.0182	65	-0.3634	31	0.2176	10	0.0000
100	0.0005	64	-0.0832	30	-0.2445	4	-1.0000
99	0.6290	63	-0.0086	29	-0.0002	3	0.0000
98	-0.0692	62	0.0000	28	0.0000	2	0.0000
97	0.4196	61	0.0000	27	0.0000	0	1.0000
96	-0.0804	48	0.0000				

Table 3. Asset change

ΔT	Asset	ΔT	Asset	ΔT	Asset	ΔT	Asset	ΔT	Asset	ΔT	Asset
120	16.92	99	5.94	78	16.67	57	16.60	36	16.34	15	-1.29
119	16.92	98	7.11	77	16.67	56	16.60	35	16.34	14	-1.29
118	16.92	97	-0.15	76	16.68	55	16.60	34	16.34	13	-1.29
117	16.92	96	1.23	75	16.68	54	16.60	33	16.34	12	-0.71
116	16.92	95	6.56	74	16.68	53	16.60	32	15.89	11	-1.30
115	16.92	94	15.04	73	16.68	52	16.60	31	12.20	10	-1.30
114	16.92	93	14.72	72	16.68	51	16.61	30	16.29	9	-1.30
113	16.92	92	16.47	71	16.68	50	16.61	29	16.30	8	-1.30
112	16.92	91	16.66	70	16.68	49	16.61	28	16.30	7	-1.30
111	16.92	90	16.67	69	16.37	48	16.61	27	16.30	6	-1.30
110	16.92	89	16.67	68	6.98	47	16.61	26	16.30	5	-1.30
109	16.92	88	16.67	67	9.78	46	16.61	25	16.30	4	15.56
108	16.92	87	16.67	66	8.95	45	16.61	24	16.30	3	15.56
107	16.93	86	16.67	65	15.06	44	16.61	23	16.30	2	15.56
106	16.93	85	16.67	64	16.46	43	16.61	22	16.30	1	15.56
105	16.93	84	16.67	63	16.60	42	16.61	21	14.92	0	-0.24
104	16.93	83	16.67	62	16.60	41	16.61	20	16.27		
103	16.93	82	16.67	61	16.60	40	14.27	19	12.56		
102	16.93	81	16.67	60	16.60	39	1.18	18	4.14		
101	16.63	80	16.67	59	16.60	38	-0.46	17	16.08		
100	16.62	79	16.67	58	16.60	37	12.73	16	16.08		

From the previous Gamma analysis, there is a certain error in Delta hedging, the size of which can be measured by the gamma value, and the formula is calculated according to the gamma value.

$$Gamma = \frac{\Delta Delta}{\Delta S}$$

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Gamma can be calculated for each adjustment. Although the Gamma value can not represent the actual error, it can reflect the error. To some extent, the following figure shows the change process of Gamma. It can be seen that the greater the Gamma value is near the execution price of 17.

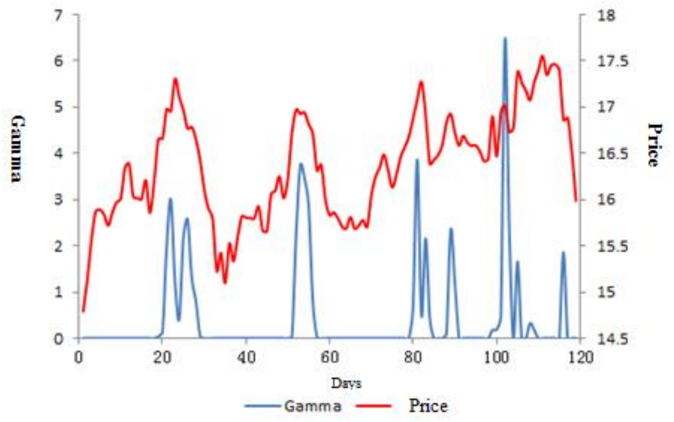

Figure 6. Gamma Value Change

3.2.2 Out of the Money

The virtual value option is the opposite of the real value option. For the European put option, the option execution price is lower than the stock market price, and the bull will not execute the option.

Using the formula (4) to simulate the stock price and obtain the stock price change when the virtual value option is obtained, draw it into the following figure. It can be clearly seen that the stock price on the maturity date is lower than the execution price of 17, and the option long will Execution of the option, at this time the dealer as an option short, in order to prevent the risk of stock price changes, risk hedge can be based on the Delta value.

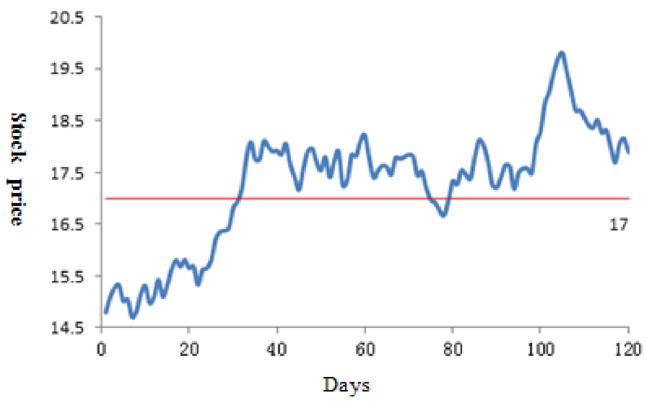

Figure 7. Stock Price Change

The following table calculates the Delta value within 120 days of the expiration date. In order to hedge the risk, the trader should hold the corresponding Delta shares every day. The negative value indicates the number of short-selling stocks, and the positive value indicates the number of shares held. It can be seen from the Delta value change that the Delta value is basically 0 in the last 22 days, which is related to the nature of Delta. The Delta value will tend to 0 when the virtual value option approaches the exercise date.

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Table 4 Delta value change

ΔT	Delta	ΔT	Delta	ΔT	Delta	ΔT	Delta	ΔT	Delta
120	-1	96	-1	72	-1.7E-11	48	-7.6E-10	24	0
119	-1	95	-1.00	71	-7.1E-08	47	-0.01	23	-1.0E-15
118	-1	94	-1.00	70	-2.6E-14	46	-0.46	22	0
117	-1	93	-1.00	69	-1.4E-05	45	-0.71	21	0
116	-1	92	-1.00	68	-2.2E-12	44	-0.99	20	0
115	-1	91	-0.84	67	0	43	-1.00	19	0
114	-1	90	-0.48	66	0.00	42	-0.60	18	0
113	-1	89	-0.02	65	0.00	41	-1.1E-05	17	0
112	-1	88	-1.3E-11	64	0	40	0.00	16	0
111	-1	87	0	63	0	39	-5.2E-12	15	0
110	-1	86	-1.7E-11	62	0	38	-1.3E-09	14	0
109	-1	85	-2.5E-11	61	0	37	-2.8E-07	13	0
108	-1	84	0	60	-3.0E-15	36	0	12	0
107	-1	83	0	59	-6.6E-06	35	0	11	0
106	-1	82	-1.1E-15	58	-5.5E-09	34	0	10	0
105	-1	81	0	57	-1.3E-11	33	0	9	0
104	-1	80	-3.4E-14	56	-3.3E-11	32	-4.6E-05	8	0
103	-1	79	0	55	-2.1E-07	31	0.00	7	0
102	-1	78	-7.1E-10	54	0	30	-2.2E-09	6	0
101	-1	77	-3.2E-05	53	0	29	0	5	0
100	-1	76	-0.03	52	0	28	0	4	0
99	-1	75	-2.5E-08	51	0	27	-0.00	3	0
98	-1	74	0	50	0	26	-3.8E-14	2	0
97	-1	73	0	49	-1.2E-07	25	0	1	0

According to the change of Delta value, it is only necessary to short-sell one stock within 95-120 days from the expiration date to realize the risk hedging. In the last 22 days, only 9.99201E-16 stocks need to be bought to realize the hedging. The table below calculates the time points at which risk hedging is required and the corresponding number of shares traded. It can be calculated that only 61 hedging operations are required in 120 days, where a negative value indicates the number of shares sold and a value of the number of shares purchased.

Table 5. Hedging operation

ΔT	Number of shares	ΔT	Number of shares	ΔT	Number of shares	ΔT	Number of shares
120	-1.0000	79	0.0000	59	0.0000	39	0.0002
95	0.0000	78	0.0000	58	0.0000	38	0.0000
94	0.0000	77	0.0000	57	0.0000	37	0.0000
93	0.0000	76	-0.0262	56	0.0000	36	0.0000
92	0.0000	75	0.0262	55	0.0000	32	0.0000
91	0.1626	74	0.0000	54	0.0000	31	-0.0008
90	0.3585	72	0.0000	49	0.0000	30	0.0009
89	0.4611	71	0.0000	48	0.0000	29	0.0000
88	0.0178	70	0.0000	47	-0.0051	27	-0.0015
87	0.0000	69	0.0000	46	-0.4550	26	0.0015
86	0.0000	68	0.0000	45	-0.2462	25	0.0000
85	0.0000	67	0.0000	44	-0.2839	23	0.0000
84	0.0000	66	-0.0029	43	-0.0096	22	0.0000
82	0.0000	65	0.0028	42	0.4008		
81	0.0000	64	0.0001	41	0.5989		
80	0.0000	60	0.0000	40	-0.0002		

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The table below shows the changes in asset value during this time. Similarly, at the beginning, due to the debts borne by the stocks, the value of the assets is at a relatively high level. As the exercise date approaches, the stocks that have been short-selled need to be closed, and the value of the assets will decrease. At the expiration date, the value of the asset is -0.40, which is the cost of the hedging risk.

Table 6. Asset change

ΔT	Asset	ΔT	Asset	ΔT	Asset	ΔT	Asset	ΔT	Asset	ΔT	Asset
120	16.92	99	16.93	78	-0.12	57	-0.13	36	-0.40	15	-0.40
119	16.92	98	16.93	77	-0.12	56	-0.13	35	-0.40	14	-0.40
118	16.92	97	16.93	76	0.33	55	-0.13	34	-0.40	13	-0.40
117	16.92	96	16.93	75	-0.13	54	-0.13	33	-0.40	12	-0.40
116	16.92	95	16.93	74	-0.13	53	-0.13	32	-0.40	11	-0.40
115	16.92	94	16.94	73	-0.13	52	-0.13	31	-0.39	10	-0.40
114	16.92	93	16.94	72	-0.13	51	-0.13	30	-0.40	9	-0.40
113	16.92	92	16.94	71	-0.13	50	-0.13	29	-0.40	8	-0.40
112	16.92	91	14.20	70	-0.13	49	-0.13	28	-0.40	7	-0.40
111	16.92	90	8.13	69	-0.13	48	-0.13	27	-0.38	6	-0.40
110	16.92	89	0.20	68	-0.13	47	-0.04	26	-0.40	5	-0.40
109	16.92	88	-0.12	67	-0.13	46	7.68	25	-0.40	4	-0.40
108	16.93	87	-0.12	66	-0.08	45	11.85	24	-0.40	3	-0.40
107	16.93	86	-0.12	65	-0.13	44	16.61	23	-0.40	2	-0.40
106	16.93	85	-0.12	64	-0.13	43	16.77	22	-0.40	1	-0.40
105	16.93	84	-0.12	63	-0.13	42	9.98	21	-0.40	0	-0.40
104	16.93	83	-0.12	62	-0.13	41	-0.40	20	-0.40		
103	16.93	82	-0.12	61	-0.13	40	-0.40	19	-0.40		
102	16.93	81	-0.12	60	-0.13	39	-0.40	18	-0.40		
101	16.93	80	-0.12	59	-0.13	38	-0.40	17	-0.40		
100	16.93	79	-0.12	58	-0.13	37	-0.40	16	-0.40		

The error of Delta hedging can be measured by the gamma value, and the formula is calculated according to the gamma value.

$$Gamma = \frac{\Delta Delta}{\Delta S}$$

The Gamma value of each adjustment can be calculated. The following figure shows the change process of Gamma. It can be seen that the greater the Gamma value is near the execution price of 17.

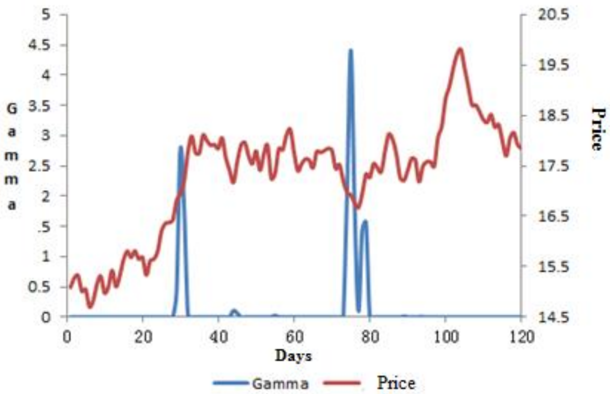

Figure 8. Gamma Value Change

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