Finite-SNR Diversity-Multiplexing Tradeoff for Two-Way Compress-and-forward Relaying

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Abstract

This paper considers two-way relay channels (TWRC) with two-phase compress-and-forward (CF) relaying. For such a scenario, we analyze outage probability and finite-SNR diversity-multiplexing tradeoff (DMT) for single-antenna relaying with equal time-sharing and fair rate allocation. Simulation results comparison the outage performance between different relaying schemes.

Keywords

Two-way relay channels, Compress-and-forward, finite-SNR diversity-multiplexing tradeoff.

1. INTRODUCTION

The two-way relay channel (TWRC), where two source nodes communicate with each other with the help of a relay node, is regarded as fundamental and useful model for the next generation wireless communication systems. According to the processing manners for the received signal at the relay node, the basic relay schemes consist of amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF). In AF protocol, the relay node linear amplifies the received signal and has the simplest processing. However, in half-duplex case, the time-sharing parameter of AF is restricted to 1/2, which severely limits the achievable rate performance. Although DF protocol performs the highest spectral efficiency in some scenarios, the relay node needs to perfectly decode both two sources message. CF protocol can alleviate the relay node compresses the received signal followed by encoding the compressed signal. Recently, many papers study these relaying schemes and compare their rate performance [1]-[4].

Based on asymptotic high-SNR diversity-multiplexing tradeoff (DMT) proposed by Zheng and Tse [5], Narasimhan [6] presents new definitions of diversity gain and multiplexing gain at finite SNRs. This is the reason that some practical communication systems would rather operate in low to moderate SNR region. In this new framework, the multiplexing gain is defined as a ratio of the target data rate to the capacity of an AWGN channel at a certain finite SNR. It indicates the sensitivity to finite SNRs of the rate-adaptive strategy. Due to capacity-achieving codes applied in per packet, the packet error rate (PER) is equal to outage probability. Under a fixed multiplexing gain and finite SNR, the diversity gain is defined as a negative slope of the log-log of outage probability versus SNR. The diversity gain at a particular SNR can determine the additional SNR required to decrease a given amount of outage probability for a fixed multiplexing gain. In the finite-SNR DMT (f-DMT) framework, Bai and Chen analyze the outage performance and DMT for the parallel fading channels in realistic SNRs [7]. Outage probability and f-DMT performance of one-way relay channel is also obtained in [8]. Furthermore, Liu and Kim in [9] derive the outage and f-DMT performance for single-antenna TWRC DF protocol with

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non-uniform power allocation under equal time-sharing. And Yi and Kim in [10] make use of the outage probability and f-DMT to characterize the performance of single antenna. In this paper, we analyze outage probability and f-DMT for TWRC single-antenna DF protocol as well as multi-antenna AF and DF protocols. Moreover, we study the general case with arbitrary time-sharing for DF and varying rate allocation for both AF and DF. Further comparison of the outage and f-DMT property between AF and DF protocols is also provided.

2. SYSTEM MODEL AND FINITE-SNR DMT

2.1. System Model

Consider a TWRC shown in Fig. 1, where two source nodes S1 and S2 communicate with each other aided by the relay node R. It is supposed that two source nodes S1, S2 and R are equipped with a single antenna. We assume that every node has the same transmit power E and operates in half-duplex mode. Moreover, the channel state information (CSI) is perfectly known in each node. The links between S1, S2 and R are reciprocal, i.e., in the TWRC the channel coefficients satisfy $h_{1R} = h_{R1} = h$, $f_{2R} = f_{R2} = f$, where both h and f are zero-mean complex Gaussian random variables following $h \sim CN(0, \beta_h)$ and $f \sim CN(0, \beta_f)$, respectivly. As a result, the pre-defined X and Y are simplified as $X = |h|^2$ and $Y = |f|^2$, which follow Exponential distribution $p_x(X) = \frac{1}{\beta_h} e^{-\frac{x}{\beta_h}}$ and $p_y(Y) = \frac{1}{\beta_f} e^{-\frac{x}{\beta_f}}$.

In this paper, we study the two-phase TWRC with CF relay protocol. The transmission time of each round of information exchange is normalized to one. In specific, in the first phase, also called multiple-access (MAC) phase, S1 and S2 simultaneously transmit towards R during a time fraction of t, which is a time-sharing parameter. In the second phase, also named as broadcast (BC) phase, R transmits to both S1 and S2 is naturally ignored in the two-phase scheme. Let R denote the target sum-rate of TWRC and α be a rate allocation parameter. Hence, the target rates of S1 and S2 are defined as $R_1 = \alpha R$ and $R_2 = (1-\alpha)R$, respectively. The outage probability of TWRC is defined as the probability that the targe rate pair $R = [R_1, R_2]^{\mathrm{T}}$ lies outside the achievable rate region conditioned on h and f.

2.2. Finite-SNR DMT

While the traditional diversity and multiplexing gains refer to asymptotically high SNR, in this paper, we recall the definitions of multiplexing gain r and diversity gain d at finite SNRs [6] as:

$$\mathrm{r} = \frac{\mathrm{R}}{\log_2(1+\mathrm{g}\gamma)},\ \mathrm{d}(\mathrm{r},\gamma) = -\frac{\partial \ln \mathrm{P_{out}}(\mathrm{r},\gamma)}{\partial \ln \gamma}, \tag{1}$$

Where $\mathrm{P_{out}}(\mathrm{r},\gamma)$ represents the outage probability of MIMO channels as function of multipleing gain r and SNR γ. The multiplexing gain r is defined as the ratio of rate R to capacity of an AWGN channel. The array gain g is chosen such that $\mathrm{g} = 1$ due to fair comparison of diversity and outage performance across different antenna numbers of both two source nodes and the relay node antenna at low to medium SNRs. Furthermore, for a given r, the diversity gain is defined as the negative slope of the log-log plot of outage probability versus SNR. In the following derivations, since the noise is assumed to have unit variance, we simply replace the SNR γ in (1) with the transmit power E. Then the diversity gain d can be re-expressed as the function of both multiplexing gain r and transmit power E:

$$\mathrm{d}(\mathrm{r},\mathrm{E}) = -\frac{\partial \ln \mathrm{P_{out}}(\mathrm{r},\mathrm{E})}{\partial \ln \mathrm{E}} = -\frac{\mathrm{E}}{\mathrm{P_{out}}(\mathrm{r},\mathrm{E})}\frac{\partial \mathrm{P_{out}}(\mathrm{r},\mathrm{E})}{\partial \mathrm{E}}. \tag{2}$$

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3. OUTAGE PROBABILITY AND FINITE-SNR DMT

In this section, we consider single-antenna two-phase CF protocol. In the MAC phase, S1 and S2 simultaneously transmit towards R, and the received signal at R is $y_r = h\sqrt{E}x_1 + f\sqrt{E}x_2 + n_r$, where n_r is the additive white Gaussian noise with $CN(0,1)$. Then R compresses the received signal y_r into a compressed version $\hat{y}_r$. The compression method applied by the relay node is source coding with side information, i.e., Wyner-Ziv coding, since each source node can take its transmitting signal as self side information. In practice, $\hat{y}_r$ can be thought of as the quantized version of y_r. Herein, we consider the quantized signal only depends on the present signal and are independent of the previously received messages. Therefor, y_r can be modeled as:

$$\hat{y}_r = y_r + q \tag{3}$$

Where q, the termed as quantization noise, is an additive white Gaussian noise with $CN(0, N_q)$ and has and one-to-one mapping index $\mathrm{w_r}$ which is encoded to the codeword $\boldsymbol{x_r}$ denoted as the transmitting signal of relay node. Hence, in the BC phase, the received signals at two source nodes read

$$y_1 = h\sqrt{E}x_r + n_1, \tag{4}$$

$$y_2 = h\sqrt{E}x_r + n_2. \tag{5}$$

The decoding at the two source nodes consists of two step: 1) the first step: after decoding x_r with the help of self side-information, recovering the index estimation $\hat{\mathrm{w}}_\mathrm{r}$, hence, obtaining $\boldsymbol{\hat{y}_r}$, 2) the second step, called as decompressing step: obtaining the objective message from $\hat{y}_r$, also using the self side-information.

As a result, the achievable region is the closure of the set of all points (R_1, R_2)

$$\begin{aligned}
&\mathrm{R_1} \le \mathrm{I_1} = \mathrm{t\,I(x_1; \boldsymbol{\hat{y}_r} | x_2) = tlog_2}\left(1 + \frac{|\mathrm{h}|^2\mathrm{E}}{1+\mathrm{N_q}}\right),\\
&\mathrm{R_2} \le \mathrm{I_2} = \mathrm{t\,I(x_2; \hat{y}_r | x_1) = tlog_2}\left(1 + \frac{|\mathrm{f}|^2\mathrm{E}}{1+\mathrm{N_q}}\right),\\
&\mathrm{s.t.}\quad \mathrm{t\,I(y_r; \boldsymbol{\hat{y}_r} | x_1) = tlog_2}\left(1 + \frac{|\mathrm{f}|^2\mathrm{E}+1}{\mathrm{N_q}}\right) \le (1-\mathrm{t})\mathrm{I(x_r; y_1)} = (1-\mathrm{t})\mathrm{log_2}(1 + |\mathrm{h}|^2\mathrm{E}),\\
&\mathrm{t\,I(y_r; \boldsymbol{\hat{y}_r} | x_2) = tlog_2}\left(1 + \frac{|\mathrm{h}|^2\mathrm{E}+1}{\mathrm{N_q}}\right) \le (1-\mathrm{t})\mathrm{I(x_r; y_2)} = (1-\mathrm{t})\mathrm{log_2}(1 + |\mathrm{f}|^2\mathrm{E}).
\end{aligned} \tag{6}$$

To satisfy the two constraints in (6),

$$\mathrm{N_q} = \max\left\{\frac{|\mathrm{f}|^2\mathrm{E}+1}{(|\mathrm{h}|^2\mathrm{E})^{\frac{1-\mathrm{t}}{\mathrm{t}}}}, \frac{|\mathrm{h}|^2\mathrm{E}+1}{(|\mathrm{f}|^2\mathrm{E})^{\frac{1-\mathrm{t}}{\mathrm{t}}}}\right\} \tag{7}$$

The outage events in regard to two traffic flows are then given by

$$\mathrm{O_1} = \{\mathrm{tI(x_1; \hat{y}_r | x_2) < R_1}\}, \tag{8}$$

$$\mathrm{O_2} = \{\mathrm{tI(x_2; \hat{y}_r | x_1) < R_2}\}, \tag{9}$$

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Thus the outage probability of TWRC single-antenna CF protocol is as follows

$$P_O = P(O) = P(O_1 \cup O_2) \tag{10}$$

For varying parameters, such as time-sharing t and rate allocation α, the formulation on outage probability is difficult to obtain a closed-form, due to be divided into many situations to consider integration. Another reason is that the derivation is the same with different t and α. So this section considers a special case with $t = 1/2$, $\alpha = 1/2$, i.e. equal time-sharing and fair rate allocation. And

$$\begin{aligned} P_O &= P(I_1 < R_1, \text{or } I_1 < R_1) \\ &= P\left(\log_2\left(1 + \frac{|h|^2 E}{1+N_q}\right) < R, or \log_2\left(1 + \frac{|f|^2 E}{1+N_q}\right) < R\right) \\ &= P\left(\frac{|h|^2}{1+N_q} < \frac{2^R - 1}{E}, \text{or } \frac{|f|^2}{1+N_q} < \frac{2^R-1}{E}\right) \end{aligned} \tag{11}$$

Here, assume that $a = \frac{2^R-1}{E}$, $X = |h|^2$,$Y = |f|^2$. And the probability is equal to

$$P_O = P\left(\frac{X}{1+\frac{XE+1}{YE}} < a, or \frac{Y}{1+\frac{XE+1}{YE}} < a, \quad X \geq Y\right) + P\left(\frac{X}{1+\frac{YE+1}{XE}} < a, or \frac{Y}{1+\frac{YE+1}{XE}} < a, \quad X < Y\right). \tag{12}$$

The above probability can be simplifed with conditional $X \geq Y$ and $X < Y$. Then,

$$P_O = P(\frac{Y^2 E}{YE+XE+1} < a, \quad X \geq Y) + P\left(\frac{X^2 E}{XE+YE+1} < a,, \quad X < Y\right) \quad . \tag{13}$$

Due to $1/E \ll XE + YE$, the probability is as

$$P_O = P(\frac{Y^2}{Y+X} < a, \quad X \geq Y) + P\left(\frac{X^2}{Y+X} < a,, \quad X < Y\right) \ . \tag{14}$$

After some mathematical calculation and integral operation, we can get the lower bound of outage probability as follows

$$P_O = 1 - e^{\left(-\frac{1}{\beta_h} - \frac{1}{\beta_f}\right)2a} \quad . \tag{15}$$

Note that the above outage probability is only the similar solution, in order to get the closed-form of finit-SNR DMT. And substitute (15) into (2), we can get the DMT expression

$$d(r, E) == -\frac{E}{P_O}\frac{\partial P_O}{\partial E} = 2\frac{E}{P_O} e^{\left(-\frac{1}{\beta_h} - \frac{1}{\beta_f}\right)} \left(-\frac{1}{\beta_h} - \frac{1}{\beta_f}\right) \frac{da}{dE} \quad . \tag{16}$$

4. NUMERICAL RESULTS

This section presents some numerical results of outage probability to validate the better performance of the two-phase CF TWRC comparison with AF and DF. We assume that all the three terminals are in a straight line and the relay is at the middle point between S1 and S2. The distance between S1 and S2 is fixed at unit 1 and let D and 1-D represents the distances from S1 to R and from S2 to R, respectively. So the path loss factor is assumed as $\beta_h = D^{-4}$ and $\beta_f = (1 - D)^{-4}$.

As shown in Fig. 1, the outage performance of AF is worst among these three schemes from middle to high SNR region. With high multiplexing gain r, the outage performance of CF betters than that of DF at high-SNRs. In the low-SNR regime, the outage performance of CF is the worst.

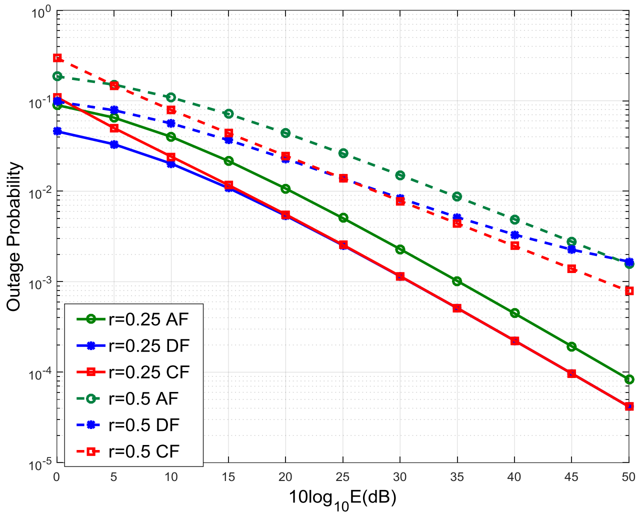

Fig 1. Comparion on outage probability of AF, DF and CF.

5. CONCLUSION

This paper studies the performances of outage probability and finite-SNR for TWRC CF relaying scheme. The derivation process applies some mathematical methods, such as variable substitution, integral operation and approximation to get the closed-form. Finally, the simulation results show that CF has advantages of outage performance in high-SNR regime and under some fixed multiplexing condition.

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