

Structure Preserving Energy Function of Power System Considering Generator Subtransient

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Abstract

The direct method of transient stability based on energy is a practical method for power system stability analysis. It can be used as an off-line planning tool and on-line security assessment. Based on the six order generator model, the structure preserving energy function (SPEF) is established. Compared with the existing six order model, this six order model is more suitable for domestic application. Therefore, the SPEF based on this model can be used to determine the transient stability of power system more generally in China.

Keywords

Transient stability; Power system; Structure preserving energy function.

1. INTRODUCTION

At present, there are mainly time domain simulation method [1] and direct method [2] to evaluate power system stability, the latter is transient energy method. In time domain simulation method, a series of differential equations and algebraic equations are solved simultaneously to obtain the numerical solution under disturbance, and the curve of the state and algebraic variables of the system with time is obtained. According to the swing curve of the generator, whether the generator can keep synchronous under the disturbance is observed to judge the transient stability. Although the time-domain simulation method is very intuitive, the time-domain simulation is slow and complicated, and cannot give the characteristics of stability. On the contrary, the direct method has fast calculation speed, can take into account nonlinearity, and can give stability margins. With the application of computers and the development of power systems, the direct method is widely used. In the direct method, the state variable is used to represent the transient energy function of the system. It describes the transient energy during and after the fault. The transient energy is excited by the fault and forms in the fault stage. Transient energy includes kinetic energy and potential energy. The potential energy includes the potential energy of the speed governor system, the magnetic field energy storage of each component, and the dissipation of the network. When a fault occurs, the potential and kinetic energy of the system begin to increase significantly. The fault is cleared, the kinetic energy and potential energy are mutually converted. If the system can absorb the remaining kinetic energy, the system is stable.

In the early stage of direct method, the classical model is used to establish energy function, which has some limitations: canceling the load nodes of the network; network shrinkage makes the network lose the topology structure, so it can't study the transient energy change of each component; the transfer conductances in the reduced admittance matrix of the network are neglected; the generator model is too simplified and ignore the flux-decaying of exciter. It can only reflect the motion between different generators, that is, the stability of the first swing.

In order to overcome the above shortcomings, the direct method of SPEF [3-6] is proposed. Firstly, Bergen and Hill proposed the structure preserving model. The stability of the transmission network is analyzed by retaining the load nodes, while the structure of the transmission network is preserved. The active load is modeled depending on the frequency, but independent of the voltage, and assumes a linear relationship with the frequency. In reference [7], the loads are modelled as arbitrary functions of respectively bus voltages. Firstly, the earliest SPEF is based on the classical generator model, and then the third-order and fourth-order generator models are constructed. The energy functions of other components are also derived, such as HVDC [8], facts [9] and so on. The more detailed generator modeling near the fault side can improve the accuracy. The power system structure preserving model is actually a differential algebraic system, which is obtained by a series of changes of differential and algebraic equations. The energy function can be obtained by the first integral method [10]. Subsequently, the energy function based on Park equation is proposed. In [11], Younis proposed a sixth-order structural energy retention function that takes into account the saturated generator. However, this model is different from the generator model commonly used in the domestic power system analysis software BPA, and is not suitable for BPA. Reference [12] proposed a method to derive the analytical energy function of a power system with a certain form. This method is used to establish the analytical SPEF of the sixth-order generator model. This method is also suitable for simple governor models. When various conditions and AVR are considered, the analytical energy function obtained cannot satisfy the conditions of the energy function, and an alternative method is used to approximate the influence of AVR. Considering the dynamic characteristics of the governor and AVR, this paper establishes a six order generator SPEF which is commonly used in China.

In a word, the energy function based on the structure preserving model takes into account the load frequency and voltage characteristics, and considers the DC transmission system, etc. every time a new model is added, it is necessary to reconstruct the Lyapunov function to determine the transient energy of each component. The structure preserving model can adapt to various disturbances, such as three-phase fault, asymmetric fault, load shedding, reclosing, etc.

2. SYSTEM MODEL

Considering a multi machine system with n nodes and m generators, assuming that the transmission network is lossless and neglecting the resistance of the network, each generator is represented as a subtransient model with two damping windings on the q -axis, one damping winding and the field coil on the d -axis

2.1. Generator Model

$$\frac{d\delta_i}{dt} = \Delta\omega_i \quad (1)$$

$$\frac{M_i d\Delta\omega_i}{dt} = P_{mi} - P_{ei} - D\Delta\omega_i \quad (2)$$

$$T'_{doi} \dot{E}'_{qi} = E_{fi} - E'_{qi} - \frac{x_{di} - x'_{di}}{x'_{di} - x''_{di}} (E'_{qi} - E''_{qi}) \quad (3)$$

$$T'_{qoi} \dot{E}'_{di} = -E'_{di} - \frac{x_{qi} - x'_{qi}}{x'_{qi} - x''_{qi}} (E'_{di} - E''_{di}) \quad (4)$$

$$T''_{doi} \dot{E}''_{qi} = E'_{qi} - E''_{qi} - (x'_{di} - x''_{di}) i_{di} \quad (5)$$

$$T''_{qoi}E''_{di} = E'_{di} - E''_{di} + (x'_{qi} - x''_{qi})i_{qi} \tag{6}$$

$$V_{di} = E''_{di} + x''_{qi}i_{qi} \tag{7}$$

$$V_{qi} = E''_{qi} - x''_{di}i_{di} \tag{8}$$

$$V_i e^{j\theta_i} = (V_{di} + jV_{qi})e^{j(\delta_i - \frac{\pi}{2})} \tag{9}$$

From equation (9), we can get

$$V_{di} = V_i \sin(\delta_i - \theta_i) \tag{10}$$

$$V_{qi} = V_i \cos(\delta_i - \theta_i) \tag{11}$$

By introducing equations (10) and (11) into (7) and (8), respectively, we can obtain

$$i_{di} = \frac{E''_{qi} - V_i \cos(\delta_i - \theta_i)}{x''_{di}} \tag{12}$$

$$i_{qi} = \frac{V_i \sin(\delta_i - \theta_i) - E''_{di}}{x''_{qi}} \tag{13}$$

From equations (10) - (13), the following two equations can be derived:

$$\begin{aligned} P_{Gi} &= V_{di}i_{di} + V_{qi}i_{qi} \\ &= \frac{E''_{qi}V_i \sin(\delta_i - \theta_i)}{x''_{di}} - \frac{E''_{di}V_i \sin(\delta_i - \theta_i)}{x''_{qi}} \\ &\quad + \frac{E''_{di}V_i^2 \sin(2(\delta_i - \theta_i)) (x''_{di} - x''_{qi})}{2x''_{di}x''_{qi}} \end{aligned} \tag{14}$$

$$\begin{aligned} Q_{Gi} &= V_{qi}i_{di} - V_{di}i_{qi} \\ &= \frac{-V_i^2 (1 - \cos(2(\delta_i - \theta_i))) (x''_{di} - x''_{qi})}{2x''_{di}x''_{qi}} \\ &\quad + \frac{E''_{qi}V_i \cos(\delta_i - \theta_i)}{x''_{di}} - \frac{V_i^2}{x''_{di}} + \frac{E''_{di}V_i \cos(\delta_i - \theta_i)}{x''_{qi}} \end{aligned} \tag{15}$$

2.2. Power Flow Equations

The active and reactive power equations of bus K are as follows:

$$0 = -P_{ek} + \sum_{j=1}^n B_{kj}V_kV_j \sin(\theta_k - \theta_j) + P_{Lk} \tag{16}$$

$$0 = -Q_{ek} + \sum_{j=1}^n B_{kj}V_kV_j \cos(\theta_k - \theta_j) + Q_{Lk} \tag{17}$$

3. SYSTEM ENERGY FUNCTION

The energy flow into the generator [13] is formulated as follows:

$$\begin{aligned}
 W_{in} &= \int_{(V_0, \theta_0)}^{(V, \theta)} (P_{ei} d\theta_i + \frac{Q_{ei}}{V_i} dV_i) \\
 &= \int_{V_{g0i}}^{V_{gi}} (I_{di} dV_{qi} - I_{qi} dV_{di}) + P_{ei} d\delta_i \\
 &= \int_{V_{g0i}}^{V_{gi}} \mathbf{IM}(\mathbf{I}_{Gi}^* d\mathbf{V}_i)
 \end{aligned}
 \tag{18}$$

Where $\mathbf{V}_i = V_i e^{j\theta_i}$

By introducing equation (2) into (18), we can get:

$$\begin{aligned}
 W_{in} &= \int_{(V_0, \theta_0)}^{(V, \theta)} (P_{ei} d\theta_i + \frac{Q_{ei}}{V_i}) \\
 &= \int_{V_{g0i}}^{V_{gi}} (I_{di} dV_{qi} - I_{qi} dV_{di}) - \int_{t_0}^t D_i \Delta\omega_i^2 dt \\
 &\quad + \int_{\delta_0}^{\delta_i} P_{mi} d\delta_i - \frac{1}{2} M_i (\Delta\omega_i^2 - \Delta\omega_{i0}^2)
 \end{aligned}
 \tag{19}$$

For multi bus network, there is KCl equation:

$$\mathbf{Y}_{Bus} \mathbf{V}_{Bus} - \mathbf{I}_G + \mathbf{I}_L = \mathbf{0}
 \tag{20}$$

Through (20), we can get the following results:

$$\mathbf{W} = \int_{V_{Bus0}}^{V_{Bus}} (\mathbf{Y}_{Bus} \mathbf{V}_{Bus} - \mathbf{I}_G + \mathbf{I}_L) d\mathbf{V}_{Bus} \equiv \mathbf{0}
 \tag{21}$$

Ignoring the resistance of the transmission network, i.e. the transmission network is lossless, the imaginary part of equation (21) and the joint equation (18) can be obtained

$$\begin{aligned}
 W_{imag} &= -\frac{1}{2} \left[\sum_{i=1}^n \sum_{j=1}^n B_{ij} V_i V_j \cos \theta_{ij} \right]_{(V_0, \theta_0)}^{(V, \theta)} \\
 &\quad - \sum_{i=1}^m \int_{\delta_0}^{\delta_i} P_{mi} d\delta_i + \sum_{i=1}^m \frac{1}{2} M_i (\Delta\omega_i^2 - \Delta\omega_{i0}^2) \\
 &\quad - \sum_{i=1}^m \int_{V_{g0i}}^{V_{gi}} (I_{di} dV_{qi} - I_{qi} dV_{di}) + \sum_{i=1}^n \int_{\theta_0}^{\theta} P_{Li} d\theta_i \\
 &\quad + \sum_{i=1}^n \int_{V_0}^{V_i} \frac{Q_{Li}}{V_i} dV_i + \sum_{i=1}^m \int_{t_0}^t D_i \Delta\omega_i^2 dt \equiv 0
 \end{aligned}
 \tag{22}$$

Combining equations (7) and (8), we can be obtained expressions as follows:

$$\begin{aligned} & \int_{V_{g0i}}^{V_{gi}} (I_{di}dV_{qi} - I_{qi}dV_{di}) \\ &= \int_{E_{qio}}^{E_{qi}} I_{di}dE''_{qi} - \frac{1}{2}x''_{di}(I_{di}^2 - I_{dio}^2) \\ & \quad - \int_{E_{dio}}^{E_{di}} I_{qi}dE''_{di} - \frac{1}{2}x''_{qi}(I_{qi}^2 - I_{qio}^2) \end{aligned} \tag{23}$$

Simultaneous equations (22), (23), we can get

$$\begin{aligned} & -\frac{1}{2} \left[\sum_{i=1}^n \sum_{j=1}^n B_{ij}V_iV_j \cos \theta_{ij} \right] \Bigg|_{(V_0, \theta_0)}^{(V, \theta)} - \sum_{i=1}^m \int_{\delta_0}^{\delta_i} P_{mi}d\delta_i \\ & + \sum_{i=1}^m \frac{1}{2}M_i\Delta\omega_i^2 - \sum_{i=1}^m \left(\int_{E_{qio}}^{E_{qi}} I_{di}dE''_{qi} - \int_{E_{dio}}^{E_{di}} I_{qi}dE''_{di} \right) \\ & + \sum_{i=1}^m \left(\frac{1}{2}x''_{di}I_{di}^2 + \frac{1}{2}x''_{qi}I_{qi}^2 \right) + \int_{t_0}^t D_i\Delta\omega_i^2 dt \\ & + \sum_{i=1}^n \left(\int_{\theta_0}^{\theta} P_{Li}d\theta_i + \int_{V_0}^{V_i} \frac{Q_{Li}}{V_i} dV_i \right) \equiv C \end{aligned} \tag{24}$$

Where C is a constant.

Putting equations (3)-(6) into (23) we can get

$$\begin{aligned} & \int_{V_{g0i}}^{V_{gi}} (I_{di}dV_{qi} - I_{qi}dV_{di}) \\ &= \frac{x''_{qi}I_{qio}^2}{2} - \frac{E'_{qi} - E''_{qi}}{x'_{di} - x''_{di}} \int_{(E'_{qio}, E''_{qio})}^{(E'_{qi}, E''_{qi})} d(E'_{qi} - E''_{qi}) \\ & \quad - \frac{E'_{di}}{x_{qi} - x'_{qi}} \int_{E'_{dio}}^{E'_{di}} dE'_{di} + \frac{E_f}{x_{di} - x'_{di}} \int_{E'_{qio}}^{E'_{qi}} dE'_{qi} \\ & \quad - \frac{T''_{qoi}}{x_{qi'} - x''_{qi}} \int_{t_0}^t (\dot{E}''_{di})^2 dt - \frac{E'_{qi}}{x_{di} - x'_{di}} \int_{E'_{qio}}^{E'_{qi}} dE'_{qi} \\ & \quad - \frac{T'_{doi}}{x_{di} - x'_{di}} \int_{t_0}^t (\dot{E}'_{qi})^2 dt - \frac{1}{2}x''_{di}I_{di}^2 - \frac{1}{2}x''_{qi}I_{qi}^2 \\ & \quad - \frac{T''_{doi}}{x'_{di} - x''_{di}} \int_{t_0}^t (\dot{E}''_{qi})^2 dt - \frac{T'_{qoi}}{x_{qi} - x'_{qi}} \int_{t_0}^t (\dot{E}'_{di})^2 dt \end{aligned}$$

$$-\frac{E'_{di} - E''_{di}}{x'_{qi} - x''_{qi}} \int_{(E'_{dio}, E''_{dio})}^{(E'_{di}, E''_{di})} d(E'_{di} - E''_{di}) + \frac{1}{2} x''_{di} I_{dio}^2 \tag{25}$$

Substituting equation (25) into equation (24), the following equation can be obtained:

$$\begin{aligned} & -\frac{1}{2} \left[\sum_{i=1}^n \sum_{j=1}^n B_{ij} V_i V_j \cos \theta_{ij} \right] \Big|_{(V_0, \theta_0)}^{(V, \theta)} - \sum_{i=1}^m \int_{\delta_0}^{\delta_i} P_{mi} d\delta_i \\ & - \sum_{i=1}^m \left(\frac{E_f}{x_{di} - x'_{di}} \int_{E'_{qio}}^{E'_{qi}} dE'_{qi} + \frac{(E'_{qi} - E''_{qi})^2}{2(x'_{di} - x''_{di})} \right) \\ & + \sum_{i=1}^m \left(\frac{x''_{di} I_{di}^2}{2} + \frac{x''_{qi} I_{qi}^2}{2} + \frac{T''_{doi}}{x'_{di} - x''_{di}} \int_{t_0}^t (\dot{E}''_{qi})^2 dt \right) \\ & + \sum_{i=1}^m \left(\frac{T''_{qoi}}{x'_{qi} - x''_{qi}} \int_{t_0}^t (\dot{E}''_{di})^2 dt + \frac{(E'_{di} - E''_{di})^2}{2(x'_{qi} - x''_{qi})} \right) \\ & + \sum_{i=1}^n \int_{\theta_0}^{\theta} P_{Li} d\theta_i + \sum_{i=1}^m \frac{T'_{qoi}}{x_{qi} - x'_{qi}} \int_{t_0}^t (\dot{E}'_{di})^2 dt \\ & + \sum_{i=1}^m \frac{T'_{doi}}{x_{di} - x'_{di}} \int_{t_0}^t (\dot{E}'_{qi})^2 dt + \sum_{i=1}^m \frac{E'_{di}{}^2}{2(x_{qi} - x'_{qi})} \\ & + \sum_{i=1}^n \int_{V_0}^{V_i} \frac{Q_{Li}}{V_i} dV_i + \sum_{i=1}^m \int_{t_0}^t D_i \Delta \omega_i^2 dt \\ & + \sum_{i=1}^m \frac{1}{2} M_i \Delta \omega_i^2 + \sum_{i=1}^m \frac{E'_{qi}{}^2}{2(x_{di} - x'_{di})} \equiv C_1 \tag{26} \end{aligned}$$

Where C_1 is a constant.

Observing equation (26), according to the law of conservation of energy, the energy function of the system can be obtained:

$$\begin{aligned} E = & \sum_{i=1}^m \frac{1}{2} M_i \Delta \omega_i^2 - \sum_{i=1}^m \int_{\delta_0}^{\delta_i} P_{mi} d\delta_i - \sum_{i=1}^m \frac{E_f}{x_{di} - x'_{di}} \int_{E'_{qio}}^{E'_{qi}} dE'_{qi} \\ & + \sum_{i=1}^n \int_{\theta_0}^{\theta} P_{Li} d\theta_i + \sum_{i=1}^n \int_{V_0}^{V_i} \frac{Q_{Li}}{V_i} dV_i + \sum_{i=1}^m \frac{(E'_{qi} - E''_{qi})^2}{2(x'_{di} - x''_{di})} \\ & + \sum_{i=1}^m \left(\frac{x''_{di} I_{di}^2}{2} + \frac{x''_{qi} I_{qi}^2}{2} \right) + \frac{(E'_{di} - E''_{di})^2}{2(x'_{qi} - x''_{qi})} + \frac{E'_{di}{}^2}{2(x_{qi} - x'_{qi})} + \frac{E'_{qi}{}^2}{2(x_{di} - x'_{di})} \end{aligned}$$

$$-\frac{1}{2} \left[\sum_{i=1}^n \sum_{j=1}^n B_{ij} V_i V_j \cos \theta_{ij} \right] \Bigg|_{(V_0, \theta_0)}^{(v, \theta)} \tag{27}$$

In equation (27), the first term is kinetic energy, the second term is the potential energy of the governor, the third term is the potential energy of the excitation system, the fourth and fifth terms are the potential energy of the load, and the last term is the potential energy of the network. The term is the potential energy of the generator. It is easy to see:

$$E = C_1 - \sum_{i=1}^m \int_{t_0}^t D_i \Delta \omega_i^2 dt + \sum_{i=1}^m \frac{T'_{doi}}{x_{di} - x'_{di}} (\dot{E}'_{qi})^2 dt - \sum_{i=1}^m \int_{t_0}^t \frac{T'_{qoi}}{x_{qi} - x'_{qi}} (\dot{E}'_{di})^2 + \frac{T''_{doi}}{x'_{di} - x''_{di}} (\dot{E}''_{qi})^2 dt - \sum_{i=1}^m \left(\frac{T'_{qoi}}{x_{qi} - x'_{qi}} \int_{t_0}^t (\dot{E}'_{di})^2 dt \right) \tag{28}$$

It is necessary to prove that the energy function satisfies the condition of the energy function to ensure that there is an attractive region around the stable equilibrium point. The derivative of the energy function to time is less than or equal to 0, which is at the stable equilibrium point.

$$\dot{E} = - \sum_{i=1}^m (D_i \Delta \omega_i^2 + \frac{T'_{doi}}{x_{di} - x'_{di}} (\dot{E}'_{qi})^2 + \frac{T''_{doi}}{x'_{di} - x''_{di}} (\dot{E}''_{qi})^2) - \sum_{i=1}^m \left(\frac{T'_{qoi}}{x_{qi} - x'_{qi}} (\dot{E}'_{di})^2 + \sum_{i=1}^m \left(\frac{T'_{qoi}}{x_{qi} - x'_{qi}} (\dot{E}'_{di})^2 \right) \right) \leq 0 \tag{29}$$

4. CONCLUSION

In this paper, the direct method and time-domain simulation method are briefly introduced. In order to preserve the topology of the system and facilitate the analysis of the transient energy of different components, based on the six order generator model, the SPEF is derived, and it is proved that the derived energy function satisfies the condition of energy function. The stability of the system can be judged by analyzing the potential energy and kinetic energy of the energy function. The six order generator model is consistent with that of BPA, which is commonly used in China, so it can be applied to this software.

Nowadays, low frequency oscillation [14] seriously threatens the stability of power system. In the next step, the transient energy based on structure preserving model is used to analyze the energy dissipation of the system, so as to suppress the low-frequency oscillation.

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