World Scientific Research Journal Volume 6 Issue 12, 2020
ISSN: 2472-3703 DOI: 10.6911/WSRJ.202012_6(12).0005

Recent Progress in Description of the Properties, States, and Interactions of Hexaquark States

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Abstract

Ever since Jaffe's prediction of deeply bounded H dibaryon, Hexaquark State has been intensively investigated due to its capability to evaluate the validity of distinct quark models. However, a unified quark model able to describe hexaquark state is not available either due to the lack of modifications (e.g. relativization) or is mathematically complicated. In this literature review, to evaluate the characteristics of available quark models and analysis methods, we discuss and compare dominant quark models and methods of calculation involved in the description of interaction within hexaquark state particles. The comparison of these models and methods indicate that the most promising model is chiral SU(3) quark model while the one with the greatest potentiality is the soliton model. In addition, comprehensively, the most promising calculation method is the lattice QCD simulation.

Keywords

QCD; Hexaquark State; Chiral SU(3) Quark Model; Lattice QCD simulation.

1. INTRODUCTION

Inspired by the introduction of the colour degree of freedom, Quantum Chromodynamics (QCD), the fundamental theory of the strong interaction, is a generalization of Quantum Electrodynamics to non-abelian groups. QCD is the only theory satisfying the invariant Lagrangian under SU(3) gauge group transformation. However, due to the complexity of non-perturbative QCD effect, it is inconclusive that the calculation proceeded directly through QCD theory, stimulating the proposal of different QCD-inspired quark models and analysis procedures.

Based on one of these QCD-inspired models, MIT bag model, Jaffe predicted an exotic particle constitutes of six quarks: deeply bound H dibaryon, a scalar one-flavour singlet [1]. Since then, the mainstream shifted to investigations of hexaquark state. In these investigations, researchers proceed QCD-inspired analysis procedures under QCD-inspired models to obtain states and interactions of these hexaquark states. While only one dibaryon, deuteron, was discovered in the experiment, various systems (e.g. H, NN, NΩ, ΩΩ, etc.) are all under the scope of investigations.

Although most hexaquark states have not been experimentally discovered, researchers continue to explore their possible identity in nature. One prevailing theory suggests that they might exist in the universe as dark matter and is gaining mounting attention. These hexaquark states also enable researchers to test the validity and applicability of established quark models.

This literature review is organized as follows. The history of QCD and hexaquark state is presented in the Section 2, including theories regarding strong force, quark and QCD and the prediction and investigation regarding quark models. In the Section 3, the original quark model and QCD-inspired quark models are discussed and compared. We discuss and compare previous and ongoing research regarding applications of QCD-inspired analysis procedures in the Section 4. Finally, we briefly summarize the literature review in the Section 5.

2. DEVELOPMENT OF THEORIES REGARDING HEXAQUARK STATES

2.1. Mesons and Strong Force

While repulsive electric force exists between protons in atomic nuclei, they are still stable and do not experience spontaneous decay. Due to this fact, D. Gross suggested that a stronger attractive force with short effective range acts between protons, keeping the nuclei from splitting apart. H. Yukawa proposed that protons and neutrons are attracted and aggregated by quantized fields created by mesons whose mass is between protons and electrons. Since then, researchers including Kemmer, Heisenberg, Nishijima, Gell-Mann, etc. investigated the description of strong force based on Yukawa's theory both theoretically and experimentally.

Theoretically, it was found that the strong force is independent of charge quantity, resulting in the unified "nucleon" representation of protons and neutrons in strong interactions. Such representation is introduced to both nucleons and mesons based on the isospin in Quantum Mechanics. The description of these meson exchange interactions was provided through the application of the lowest order of perturbative calculation supported by experiments. However, the introduction of Yukawa's theory makes the perturbative calculation meaningless, stimulating the development of S matrix theory and Axiomatic field theory [2].

Experimentally, the meson predicted by Yukawa's theory was discovered and named as "pion". Later, the development of high-energy collision experiment generated two primary results. On the one hand, these experiments assisted researchers to categorized fundamental particles into two groups, hadrons and leptons, according to the types of interaction they participate. On the other hand, the results of deep inelastic scattering experiments suggested that the form function is independent of any energy scale [3], indicating the existence of the enormous amount of point-like structures in hadron and a phenomenon known as "asymptotic freedom" shown by these point-like structures.

The development of symmetric theory contributed to the investigations concerning the internal structure and classification of hadrons. To investigate the internal structure of hadrons, Gell-Mann and Nishijima introduced hyperon based on symmetric theory, discovering that hadrons can be categorized based on $SU(3)$ groups [4]. Gell-Mann and Zweig independently proposed "quark" with three distinct flavour degree of freedom as fundamental entities inside hadrons [5].

In his theory, Gell-Mann found a statistical anomaly concerning the violation of Fermi-Dirac statistics. A new degree of freedom, colour, was introduced to eliminate such anomaly by O. Greenberg. Based on the colour degree of freedom and asymptotic freedom, Gell-Mann, Fritzsch, Adler, and etc. established Quantum Chromodynamics.

QCD is the fundamental theory describing the strong interaction based on fundamental entities quarks and gluons with two essential basic features: asymptotic freedom in high-energy scattering and colour confinement in low-energy hadrons. These two features signify that i). in high energy scenario, the behaviours of quarks are similar to that of free particles; and ii). In low energy scenario, all hadrons are colour singlet, and no single quark can be extracted. Besides, QCD is also the gauge non-abelian SU(3) field theory.

Based on QCD, a set of extended analytical procedures was developed to resolve difficulties of non-perturbative QCD effects. In this literature review, we discuss the lattice QCD simulation, large Nc limit calculation, and QCD sum rule. Also, a variety of quark models were proposed to classify and describe hadrons, including Potential model, MIT bag model, topological and nontopological soliton model, and chiral-quark model. These models are discussed in detail in section 3. With the MIT bag model, Jaffe predicted the existence of a hexaquark state: H particle. Starting from his prediction, researchers employed different models and predicted more hexaquark states.

2.2. QCD and Quark Models

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2.3. Hexaquark State and Dark Matter

Hexaquark state particles can be categorized into two groups: dibaryons and triple-quark-antiquark-pairs. Once the hexaquark state is formed, it remains stable bound state (or quasi bound state) or resonant state. Theoretically, various hexaquark states, such as H, NN, NΩ, ΩΩ, ΔΔ, Triply-charmed-diquark-hexaquark state, have been investigated through analytical procedures including lattice QCD simulation (regular/HAL QCD), large Nc limit, QCD sum rule. Experimentally, the products of weak decay of these hexaquark states have been detected. However, the confirmation of these states has yet been done.

The investigations regarding Hexaquark state usually aim at confirming the following quantities: potential, binding energy, mass, coupling, and scattering phase shift. These quantities enable researchers to determine the type of hexaquark states(dibaryon/triple-quark-antiquark-pairs), their state, and physical mechanisms serving as the primary contributor of interactions. Additionally, speculations regarding the form in which hexaquark states exist in the realistic universe are also investigated. Two of these theories stands out: Dark matter candidates and matter inside neutron stars.

3. QUARK MODELS

Quark model is an effective approach of categorizing and describing distinct hadrons. Before the proposal of the first quark model, all interactions and matters are investigated on the level of hadrons and leptons. The complexity of non-perturbative QCD effect imposes a barrier, preventing researchers from obtaining reliable, self-consistent, and precise calculation result and comprehension of the dynamical mechanism behind these numbers. Although investigations on this level are sufficient to account for baryon-baryon interactions, it cannot be used to investigate multiquark systems because the properties of multiquark states are determined by interactions on the level of quarks, and without these properties, problems involving interactions can neither be solved. However, the experiments and theories confirm the existence of multiquark states. Therefore, to describe these systems, Gell-Mann, Isgur, Glozman, and etc. developed various quark models.

Besides, it is difficult to find a unified model for all hadrons and multiquark states for the same reason. In the meantime, another approach, lattice QCD, involves complex calculations and simulations that rely on computers' capabilities. Such difficulty stimulates the development of various QCD-based quark models.

In this section, the development and main content and features of dominant QCD-based quark models and the original quark model will be introduced, discussed, and compared.

3.1. Non-relativistic Quark Model

Based on the assumption that the eightfold description of hadrons is correct, M. Gell-Mann and Zweig (1964) proposed the first quark model that can be adopted to explore the algebraic properties of the four fundamental interactions between fundamental entities of baryons and mesons independently and separately [6, 7]. Among these fundamental interactions, the gravitational, weak, and electromagnetic interactions are described by the dispersion theory while the strong interaction is described by scattering theory.

Gell-Mann assumed that these entities are octet, describing the underlying symmetric group as $SU(8)$ instead of traditional $SU(3)$. Based on this description, two forms of triplet representation were given: i). a unitary triplet of charge. ii). a unitary triplet of three degrees of freedom(spin, charge, and baryon number). These triplets are referred to as "quark". Since then, the latter entered the mainstream scientific community.

The mathematical model of quark under the representation of $SU(3) \times SU(3)$ group was established based on the field theory. The model allows him to abstract the relations from the field theory and to assign them to individual particles. Based on these relations, scattering theory and dispersion theory treatment can be employed to treat fundamental interactions between fundamental particles.

However, in this paper, quark is merely a mathematical concept instead of physical entities. Even Gell-Mann himself questioned the possibility of its existence.

3.2. Potential Models

Potential models introduce the phenomenological potential to describe the dynamical mechanisms of quark-quark interactions based on the global symmetry in the preliminary classification of hadrons. However, on the level of quarks, while some researchers including Isgur, Karl, Godfrey, and etc. believed that the primary contribution comes from a perturbative dynamical mechanism "one gluon exchange"(OGE), others, such as Glozman, believed that the primary contribution is a type of non-perturbative mechanism related to the exchange of mesons. Such a difference results in the development of different models. The most developed and successful potential model is the Constituent Quark Model (CQM) while the dominant model involving mesons exchange is the Goldstone Boson Exchange Model.

3.2.1. Non-relativistic CQM

To explain the pattern in size and angle of non-strange, negative parity baryon splitting in hyperfine interaction, N. Isgur and G. Karl(1977) established the prototype of CQM based on the assumption that all confined constituent quarks proceed non-relativistic motion [8]. In their model, the phenomenological potential is spin-independent and can be represented as the sum of one gluon exchange potential V_{ij}^{conf} and confining potential V_{ij}^{OGE}. The total effective Hamiltonian of their model is

$$H = \sum_{i}\left(m_i + \frac{p_i^2}{2m_i}\right) + \sum_{i<j}\left(V^{ij} + H_{hyp}^{ij}\right)$$

This expression comprises a spin-independent potential and Hamiltonian of hyperfine interaction, as shown:

$$H_{hyp}^{12} = \boldsymbol{A}\left\{\left(\frac{8\pi}{3}\right)\boldsymbol{S_1}\cdot\boldsymbol{S_2}\delta^3(\boldsymbol{\rho}) + \rho^{-3}(3\boldsymbol{S_1}\cdot\hat{\rho}\boldsymbol{S_2}\cdot\hat{\rho} - \boldsymbol{S_1}\cdot\boldsymbol{S_2})\right\}$$

The first term of hyperfine interaction Hamiltonian is the Fermi Contact, and the second term is the Tensor Part. The first term is operative only when both quarks have zero orbital angular momentum. The second term works only when the orbital angular momentum of both quarks is nonzero. These conditions clearly indicate two paths of physical process that only works on the particles with particular orbital angular momentum.

After assigning the negative parity baryons with the lowest mass to a seventyplet, Isgur and Karl evaluated the effective contribution of both terms to the strength in the hyperfine interaction. Their result is in accordance with experiments.

The non-relativistic CQM is able to provide the resonance spectroscopy of most hadrons under low-energy condition. The calculations of hadron spectroscopy and heavy quarks are agreed with the experimental value.

3.2.2. Relativistic CQM

In [6] and [7], S. Godfrey (1985), N. Isgur (1985-1986) and S. Capstick (1986) proposed relativistic CQM for mesons (1985) and baryons (1986) to extend the applicability of non-relativistic CQM from heavy quarks to light quarks. They separately introduced the relativistic modification for baryons and mesons. Their relativized CQM improves researchers' comprehension regarding the physical mechanism of colour confinement.

In the relativization of meson CQM under soft QCD, they introduced two fundamental assumptions: i). rest-frame Schrödinger-type equation describes the quark-antiquark wave function and dominates the total Fock-space wave function. ii). Potential is a variant of the sum of V_{ij}^{OGE} and linear V_{ij}^{conf}. Based on the frame Schrödinger-type equation, the Hamiltonian of this model is written as

$$H = H_0 + V = E$$

The first term takes the non-relativistic limit:

$$H_0 \to \sum_{i=1}^{2}\left[m_i + \frac{p^2}{2m_i}\right]$$

The second term includes the Hamiltonian of spin-independent linear confinement, colour hyperfine interaction, and spin-orbit interaction with colourful magnetic piece and Thomas-precession term:

$$V_{ij}(\boldsymbol{p},\boldsymbol{r}) \to +H_{ij}^{conf} + H_{ij}^{hyp} + H_{ij}^{so(cm)} + H_{ij}^{so(tp)}$$

Based on the potential description, Isgur and Godfrey investigated the source and effect of short-ranged annihilation with Feynman graph and simulate the expression of flavour coupling constant and the meson spectroscopy.

One year later, Isgur and Capstick extended their previous discussion to baryons in [7], forming a complete picture of hadron relativistic CQM. With a similar procedure under qqq instead of $q\bar{q}$, the solution of rest-frame Schrödinger-type equation provided them with a similar form of the general expression for Hamiltonian, as above.

However, the explicit, detailed expression is different. Due to the system change, the first term is now

$$H_0 = \sum_{i=1}^{3}[p_i^2 + m_i^2]^{\frac{1}{2}}$$

In addition, the potential term is now a relativized triquark momentum-dependent potential with V_{string}, V_{Coul}, V_{hyp}, $V_{so(cm)}$, and $V_{so(tp)}$. The confining potential is generated by the string potential, a spin-independent term, and spin-orbit interaction potential under Thomas precession, a spin-dependent term. The Coulombic potential, hyperfine interaction potential, and the colour-magnetic piece of spin-orbit interaction potential are all spin-dependent term and construct the one gluon exchange potential.

Working collectively, the relativistic CQM is formed. Based on this model, Isgur and Capstick compared the experimental results and theoretical predictions of baryons constituting of different quarks.

3.2.3. Goldstone Boson Exchange Model

In their research regarding the spectroscopy of Δ nucleon and Λ hyperon, L. Ya. Glozman and D. O. Riska(1996) proposes that the interaction between quarks is primarily contributed by a non-perturbative QCD effect: exchange of the $SU(3)_F$ octet of pseudoscalar mesons [9]. Their belief was drastically different from their contemporaries who believed that the short-range interaction is primarily contributed by the perturbative effect: one gluon exchange.

They believed that this mesons exchange mechanism indirectly reveals the existence of a hidden chiral symmetry, indicating that the exchange of these pseudoscalar mesons is a type of chiral field-quark interaction. These mesons are bosons and were named as “Goldstone Boson”.

Their model indicates that the internal structure of hadrons includes three constituent quarks with flavours. Within the confining scale of QCD, the interactions between quarks include a harmonic interaction and a chiral interaction mediated by the Goldstone Boson.

This model was developed based on the cold lattice QCD. In their model, the most critical term of Hamiltonian is the quark-chiral field coupling. This interaction is spin-flavour dependent.

$$H_X \sim -\sum_{i<j} \frac{V(\boldsymbol{r}_{ij})}{m_i m_j} \lambda_i^F \cdot \lambda_j^F \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

The term λ_i^F is the Gell-Mann matrix of the flavour space. Under the limit of chiral invariance, this expression suggests that the baryon multiplet structure belongs to group $SU(3)_F \times SU(2)_S \times U(6)_{Conf}$, indicating that baryons with the same radial structure and permutational spin-flavour symmetry but different spin or flavour symmetry possesses different mass.

Based on this model, they investigated the baryon spectroscopy, tensor interaction and its correction with magnetic moments. Later, Glozman et al. generalize their model to scalar and vector meson [10]. Their calculation of baryon spectroscopy is even closer to experimental value than relativistic CQM.

3.3. MIT Bag Model

Another class of potential models is the bag model. In 1974, A. Chodos et al. developed a new bag model “MIT bag Model” under the quasiclassical frame and two-dimensional quantization frame [11]. They eliminated the dynamical degree of freedom by assigning a constant energy B to a space of strongly interacting particle. Space with such a property is referred to as a “bag”.

The prototype description of the energy of this model is under the classical frame:

$$W = \int_{t_2}^{t_2} dt \int_R d^3r \left[\frac{1}{2}\phi^2 - \frac{1}{2}(\boldsymbol{\nabla\phi})^2 - B\right]$$

In the expression, ϕ is the prototype of a hadronic field. This description is in lack of a kinetic term, indicating that it is an equation of constraint.

One description of local coupling among hadrons, the method is to integrate every single bag over the point under the condition that constant energy parameter B must be universal. The physical significance of the “bag” is a space of quarks and gluons with colours. In addition, they also introduced the relationship between charge, isospin, hyperon, and colour after introducing hyperon.

$$Q = I_3 + \frac{1}{2}Y + \frac{1}{3}C$$

In the expression, I_3 and Y belong to ordinary $SU(3)$, and C belongs to coloured $SU(3)$, indicating that the quarks are under the frame of Fermi-Dirac statistics.

Another description is based on the massless non-abelian coloured-gluon field confined in the bag. This model indicates that quarks possess fraction charge and reintroduces the relation between charge, isospin, and hyperon

$$Q = I_3 + \frac{1}{2}Y$$

Based on this model, Chodos et al investigate the energy and angular momentum of hadrons. In addition, Jaffe firstly predicted the existence of hexaquark state based on this model.

3.4. Soliton Model

R. Friedberg and T. D. Lee (1977) proposed the soliton interpretation of hadron based on renormalizable local quantum field theory with the interaction between scalar Hermitian field ϕ and Dirac field ψk [12]. Their work aimed at judging whether quarks are soliton solution. The model interprets interactions by decomposing Hamiltonian density into two terms: quasiclassical Hamiltonian and its quantum correction term:

$$\mathcal{H} = \frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + U(\phi) + \sum_{k=1}^{n} \psi^{k*}(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta m + g\beta\phi)\psi^k + counterterm$$

In the prototype Hamiltonian density equation, Π and ϕ satisfy the usual commutation relation while ψ^k and ψ^{k*} satisfy the usual anticommutation relation. The term $U(\phi)$ represents the renormalizability with a, b, c, m, and g as renormalizable constants. The Dirac field simulates the quark field while its superscripts indicate its colour.

By their definition of the soliton, Friedberg and Lee investigated solitons’ general properties and behaviours under boundary conditions. They verified the validity of soliton theory in non-relativistic fermion, including its existence, non-relativistic limit, and stability. They also investigated the weak coupling with fermion number N=1 and N=2 for relativistic fermion.

This model was further generalized to topological soliton model under which hexaquark states have been discussed. The description of hexaquark states under such a model will be introduced and evaluated in Section 4.

3.5. Chiral SU(3) Quark Models

When the mass of quarks is considered as zero, QCD possesses a rigorous chiral symmetry. In the case of light quarks, chiral symmetry still holds as an effective approximation. This symmetric nature is considered as one of the most effective symmetries in the strong interaction, giving rise to the description of quark under chiral field interaction.

As mentioned in Subsection 3.2, the dynamical mechanism of short-ranged interactions between quarks was unclear, inspiring researchers to develop a model comprising both OGE and Goldstone Boson exchange. As a result, to provide a unified description for N-N interaction, N-Y interaction, and Y-Y interaction, Z. Y. Zhang et al. established the chiral $SU(3)$ quark model and generalized it to vector meson exchange.

3.5.1. Chiral SU(3) Quark Model

Due to the effectiveness of the chiral-quark model in describing baryon-baryon interaction, Z. Y. Zhang et al.(1997) generalized chiral $SU(2)$ quark model to $SU(3)$ by extending the scalar

σ field and pseudoscalar π field to the $SU(3)_F$ space and introducing all nonets of scalar and pseudoscalar field. This generalized chiral-quark model is "chiral $SU(3)$ quark model" [13].

The chiral $SU(3)$ quark model includes both the short-ranged interaction and long-ranged interaction. For short-ranged interactions, it includes both the contribution of OGE and Goldstone boson exchange.

With the model, Z. Y. Zhang and his group obtained the expression of linear realization of quark-chiral $SU(3)$ coupling, Lagrangian of quark-chiral filed interaction, and chiral-quark interaction Hamiltonian:

$$\sum = \sum_{a=0}^{8} \sigma_a \lambda_a - i \sum_{a=0}^{8} \pi_a \lambda_a$$

$$\mathcal{L} = -g_{ch}\bar{\psi}\left(\sum_{a=0}^{8} \sigma_a \lambda_a + i \sum_{a=0}^{8} \pi_a \lambda_a \gamma_5\right)\psi$$

$$H_{ch} = g_{ch}F(q^2)\bar{\psi}\left(\sum_{a=0}^{8} \sigma_a \lambda_a + i \sum_{a=0}^{8} \pi_a \lambda_a \gamma_5\right)\psi$$

They also proved that the Lagrangian is invariant under $SU(3)_L \times SU(3)_R$ transformation. In the expression of chiral-quark interaction Hamiltonian, $F(q^2)$ is the form factor describing the structure of chiral field with the cut-off mass that indicates the scale of chiral symmetry breaking.

In addition to the chiral-quark interaction Hamiltonian, they also provide the expression of total Hamiltonian:

$$H = \sum_i T_i - T_G + \sum_{i<j} V_{ij}$$

Based on the expression of Hamiltonian, they also calculate the potentials of the $SU(3)$ chiral-field-induced quark-quark interactions. It includes scalar meson exchange potential and pseudoscalar meson exchange potential. As a type of extended potential model, chiral SU(3) quark model is capable of calculating the potential of hexaquark states, suggesting the binding energy and state in which these hexaquark state particles lie.

3.5.2. Extended Chiral SU(3) Quark Model

Later, L. R. Dai, Z. Y. Zhang and their group (2003) extended their model to include coupling between vector chiral meson field and quarks from scalar and pseudoscalar ones based on the investigation of nucleon-nucleon interactions [14].

The Lagrangian of this model is different from its original version because of the introduction of vector chiral field:

$$\mathcal{L}^{v} = -ig_{chv}\bar{\psi}\gamma_\mu \boldsymbol{\varphi}_\mu \cdot \boldsymbol{\tau}\psi - i\frac{f_{chv}}{2M_p}\bar{\psi}\sigma_{\mu v}\partial_v \boldsymbol{\varphi}_\mu \cdot \boldsymbol{\tau}\psi$$

$$H = \sum_i T_i - T_G + \sum_{i<j} V_{ij}$$

Where the potential term includes the confining potential, one gluon exchange potential, and chiral-quark interaction potential. In this model, the chiral-quark interaction potential includes scalar, pseudoscalar, and vector meson exchange potential. While the general expression of total Hamiltonian is the same in both models, the potential terms are different.

In this model, the most challenging term is the confining potential due to the complexity non-perturbative QCD effect in the low-energy region. The treatment usually is phenomenological and harmonic [15]. Based on this model, the scattering of the nucleon-nucleon(N-N) system is investigated. Their analysis indicates that in short-ranged baryon-baryon interactions, the contribution of Goldstone boson exchange outweighs that of one gluon exchange in the Hamiltonian. However, the contribution of OGE remains critical and is not negligible. In addition, the effect of mesons exchange is primarily dominated by vector mesons instead of scalar or pseudoscalar mesons.

3.6. Discussion and Comparison of Quark Models

These quark models are developed based on different assumptions, making them work on different aspects of hadrons and quarks. The similarities/differences, description of interactions, and effective range of application are discussed, compared, and summarized in Table 1. The hexaquark state-related information is also given in this subsection.

Table 1. Comparison of Quark Models

Name	Proposer(Year)	Quantity described	Dynamical Mechanism	Type of Dynamical Mechanism	Reference
Non-relativistc CQM	Isgur, Karl (1977)	H	OGE	Perturbative	[8]
Relativistic CQM	Godfrey, Isgur, Capstick (1986)	H	OGE	Perturbative	[6], [7]
Goldstone Boson Exchange Model	Glozman, Riska(1996)	H	Pseudoscalar Mesons Exchange	Non-perturbative	[9]
MIT Bag Model	Chodos. *et al.* (1974)	Energy	-	-	[11]
Nontopological Soliton Model	Friedberg, Lee (1977)	$\mathcal{H}$	Scalar Mesons Exchange	Perturbative	[12]
Chiral *SU(3)* Quark Model	Z. Y. Zhang *et al.* (1997)	H & $\mathcal{L}$	Scalar/Pseudoscalar Mesons Exchange & OGE	Perturbative & non-perturbative	[13]
Extended Chiral *SU(3)* Quark Model	L. R. Dai *et al.*(2003)	H & $\mathcal{L}$	Vector Mesons Exchange & OGE	Perturbative & non-perturbative	[14]

There are three primary contributions provided by Gell-Mann's original quark model. Primarily, the concept of quark was proposed and described by spin, baryon number, and charge degrees of freedom abstracted from the field theory. Secondly, the currents of fundamental interactions provided enabled researchers to investigate interactions between quarks. Finally, the statistical anomaly in Gell-Mann's work directly results in the introduction of the colour degree of freedom, stimulating the development of QCD. These contributions served as a foundation and a standard, guiding researchers to further develop standard theories describing quark-quark interactions and quark models.

After Gell-Mann and Zweig's quark assumption, whose existence as physical entities was later verified by experiments, QCD was founded and developed with different branches, stimulating the development of all other quark models discussed in subsections above.

The first generation of potential models, the CQM and Goldstone Boson exchange model, describes the quark-quark interactions with different dynamical mechanisms. The former attributed the interactions to the OGE while the latter attributed them to the mesons exchange. The fact that the simulation results for both of them is close to experimental values causes a physical dilemma: which is the primary contributor? The answer was provided by chiral-quark model discussed in the next paragraph. From the perspective of applications, CQM involves only the perturbative QCD effect while the Goldstone Boson exchange model involves the non-perturbative QCD effect. This feature indicates that the latter shows a strong dependence on computer technology. Besides, in Glozman and Riska's original works, the latter is equipped with much more interpretations of the physical features of interactions and particles, making them more useful in describing hadrons interactions in nature. For both models, these descriptions are based on Hamiltonian with potential and kinetic terms. However, the latter did not explicitly indicate whether their model includes relativistic modification in the kinetic term, making us being reserved about applying them to light quarks.

The physical dilemma mentioned stimulated the development of quark models with both dynamical mechanisms. Z. Y. Zhang et al. developed and generalized the chiral quark model based on SU(3) group representation in [13]. The effectiveness of the chiral quark model is a direct result of chiral symmetry of QCD under the assumption that a quark possesses zero mass. This assumption obviously indicates that the validity of their model cannot be extended to heavy quarks due to the chiral symmetry breaking. Although the symmetry breaking is physically weary, it is able to aid researchers to assign dynamical mass to quarks. The description of interactions in the chiral quark model is based on Hamiltonian and Lagrangian, which is an invariant under chiral transformation. The difference between chiral SU(3) quark model to extended chiral SU(3) quark model is the type of meson under exchange. The former involves the exchange of scalar and pseudoscalar meson while the latter involves the exchange of vector mesons. An essential contribution of their model resolves the dilemma: The contribution of Goldstone Boson exchange outweigh that of OGE.

Bag models belong to the potential models. In this literature review, the bag model discussed is the MIT bag model. The reason for choosing this specific bag model is that the first prediction of hexaquark state was made based on this model. The description of interactions in this model is based on energy and momentum, which is clearly different from the other model discussed previously. Thus, simulations made based on this model reveal different aspects of hadron interactions. Besides, this model also provided researchers with algebraic properties of quarks through the relation across different degrees of freedom. From the perspective of applications, it is relatively concise to proceed simulations for its elimination of dynamical degrees of freedom. However, the effective range of application of the original MIT bag model is relatively narrow because it was based on quasiclassical and two-dimensional quantized frame. Therefore, currently, it can neither serve as a unified model for all hadrons. However, it is likely that this problem will be resolved in further development.

Finally, the quark model with greatest potential development is the soliton model. Different from other quark models, this model describes interactions with Hamiltonian density (quasiclassical and quantum correction). This model attributes the dynamical mechanism to mesons exchange and employs the Hermitian field for mesons and the Dirac field for quark. In Friedberg and Lee's original work, they discussed the general properties and weak coupling interaction, which is different from all other contemporary works discussed in this literature review. From the perspective of applications, it is usually employed with the lattice QCD method. Also, this model is qualified to apply on relativistic fermions (light quarks). Currently, the topological soliton model has already been developed and applied. It is rational to assume that the investigation regarding strong coupling, electromagnetic decay, strong decay, and weak decay might greatly widen its effective range of application.

Currently, the chiral quark model and its extension is the most prevalent quark model. The model with the greatest potential is the soliton model.

4. CALCULATION AND ANALYSIS

The behaviours, general properties, and state of hexaquark states are determined by calculations and analysis. The techniques chosen for calculations and analysis are based on different features of different hexaquark state systems and the model based on which the hexaquark states are predicted. Currently, the most dominant calculation techniques include Lattice QCD, QCD Sum rule, Large Nc limits, Resonance Group Method, and RGM with Permutation Group Representation.

In this section, we discuss the features and types of systems in which the calculation techniques are applied.

4.1. Lattice QCD Method

Lattice QCD method can be regarded as an application of Lattice Gauge Theory. It was proposed based on the transformation from four-dimensional Minkowski spacetime to four-dimensional Euclidean spacetime. Currently, the most prevalent method concerning lattice QCD method is the lattice QCD simulation. It enables researchers to calculate the strong coupling constant, current quark mass with great precision. It also enables researchers to investigate the mass spectroscopy of hadrons.

A. Francis et al. (1994) investigated the binding energy and spectroscopy of bounded H dibaryon [16]. Their work employed the distillation method for time-slice-to-all propagator under lattice QCD simulation. They fitted the spectroscopy through the system of physical entities constructed by mass-degenerating flavour degree of freedom of Wilson quark and Wilson lattice element improved by O(a) group. In their methods, this particular construction has resulted from DD-HMC and MP-HMC algorithm. To find the energy scale, Francis et al calculated the correlated matrix under interpolating operator as the base (hexaquark operator/dibaryon operator) and solved the generalized eigenvalue problem. Based on these general procedures, A. Francis et al. stated that the energy in the nth level is the solution diagonalized matrix obtained from the diagonalization of non-Hermitian correlator matrix with hexaquark state operator as both the source operator and the sink operator in the [16].

HAL QCD collaboration (2019) investigated the state and interaction of the NΩ system [17]. Instead of using traditional QCD method suggested in the last paragraph, they employed the time-dependent HAL QCD method. Specifically, the HAL QCD collaboration extracted the potential function from time-dependence and correlation function after the construction of correlation function and R correlator. Afterwards, they set up the lattice condition for simulation. The calculation of scattering length and effective range was calculated based on the solution of the Schrödinger equation with a potential value from the simulation. Under such conditions, the simulation result indicated that this system is under the quasi bound state, as shown in Figure 1. The effective range and scattering length are similar to ΩΩ system.

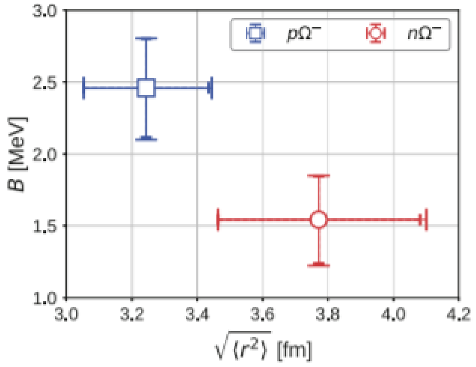

Figure 1. The binding energy B and the root mean square distance for nΩ-(red) and pΩ-(blue) systems [17]

HAL QCD collaboration (2020) proceeded another investigation of the potential, scattering phase shift, and binding energy of a special ΔΔ system, d*(2380) [18]. d*(2380) is a ΔΔ system with (spin-parity, isospin) =(3+, 0). The basic assumption of their analysis was that the coupling of the NN system, the product of the decay of d*(2380), through D-wave and G-wave is extremely weak. In their work, the solution of equal time Nambu-Bethe-Salpeter (NBS) wave function aided the HAL QCD collaboration to solve Schrödinger type equation, thus defining the potential function. The NBS wave function is related to the simplified version of the four-point function, which satisfies the time-dependent HAL QCD equation when coupling strength is lower than the inelastic threshold. Afterwards, the derivative expansion of the non-local term of potential function provided the leading order local potential term. After setting up the condition under this term, the HAL QCD collaboration proceeded the lattice QCD simulation. The potential of this system indicates that within the effective range, the system shows attractive behaviour. Besides, they obtained the scattering phase shift in the form of a kinetic energy function and the binding energy from fitted potential function and the solution of Schrödinger type equation in a lattice space with infinite volume. Both of these results indicated that the system is in the quasi bound state.

4.2. QCD Sum Rule

QCD sum rule is a semi-theoretical and semi-phenomenological non-perturbative method. The critical element of this method is that the QCD correlation function is connected to theory in quark level and phenomenology in hadron level by dispersion relation. In this method, the non-perturbative QCD effects are absorbed by a set of vacuum condensates that can be phenomenologically confirmed [19].

X. H. Chen et al.(2019) investigated the mass of ΩΩ dibaryon state as a molecular picture [20]. Their calculation based on the QCD sum rule with ten-dimensional vacuum condensates was employed. They provided the interpolating current of the system based on the Loffe current of Ω baryon. Based on this interpolating current, the molecular form of ΩΩ system was constructed according to the different spin-parity quantum number and the two-point correlation function, which is invariant under dispersion relation, was given. The coupling between the current and the scalar/tensor dibaryon state provided the full QCD sum rule for the calculation purpose.

Z. G. Wang (2020) investigated the mass and interaction of triply-charmed hexaquark states [21]. He constructed the hexaquark state current from the scalar and axial-vector form of the charm diquark operator. The general procedure is to calculate the mass and spectroscopy under model parameter from experimental data based on QCD sum rule of triply-charmed hexaquark state. This expression is an analytical expression of QCD spectroscopy obtained from dispersion relation. Specifically, he isolated the two-point correlation function, forming QCD part and hadron part. Afterwards, he adopted smearing method on the former and contracted the latter with Wick Theorem, obtaining a fifteen-dimensional quark-gluon operator, as shown in Figure 2. Based on this result, Wang expanded the product of operators to fifteen-dimensional vacuum condensates, receiving the QCD sum rule he used for calculation.

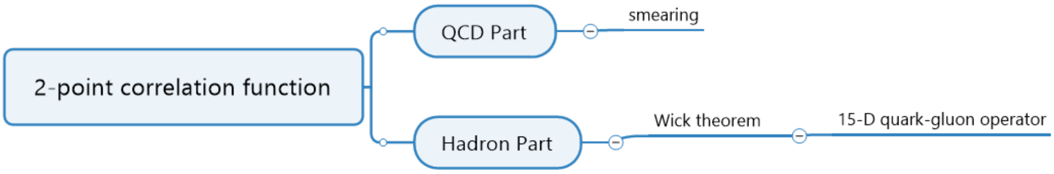

Figure 2. The treatment of the two-point correlation function under HAL QCD method.

4.3. Large N_C Limits Method

The large N_c limit method of QCD is the discussion of the leading order term of the large N_c expansion, which was developed based on the analysis of Feynman graph. It possesses a particular feature that any operator X that is invariant under gauge transformation can be a local operator, non-local operator similar to Wilson loop, or the determinant of a strange Dirac operator. It is a non-perturbative calculator method on quark-gluon level based on the colour number of hadrons. Its primary function is to investigate the properties of hadrons under the expansion of large Nc limit.

Chi-Keung Chow (1995) discussed the existence of multiquark states (tetra, penta, and hexa) [22]. His work included two interactions: the binding of heavy meson and chiral soliton, and Coulombic attraction between quarks. Utilizing a fake quark to substitute a heavy quark in the pentaquark state (the result of chiral soliton-heavy meson binding), he constructed the hexaquark state: H dibaryon and investigated its existence and state. The binding energy and elastic constant are given by the odd functional of the profile function, and the binding potential is given by the profile function. The result showed that binding potential takes harmonic form, indicating the Isgur-Wise form constant. In this system, the light degree of freedom is the same as the pentaquark state, and the Isgur-Wise form factor is similar to that of the tetraquark state. Physically, the form factor is regarded as the product of perturbative effect part and non-perturbative effect part where the former is caused by the colour Coulombic attraction and the latter is caused by the overlap of the initial and final light degree of freedom. Based on this analysis, the H dibaryon is stable when the sum of isospin and light degree of freedom is 1. The stability comes from its large colour Coulombic binding energy.

4.4. Resonance Group Method (RGM)

RGM is the standard method used to investigate the interaction between two systems. The primary procedure is adding the spin, flavour, and colour degree of freedom to the hexaquark state wave function to form the resonance group wave function of dibaryon system.

In [15], J. Zhao (2005) investigated the scattering, annihilation of NΩ system and calculated the matrices elements of the system under the extended chiral SU(3) quark model. Based on the SU(3)F symmetry breaking, Zhao constructed the eightfold and tenfold single baryon wave function. Afterwards, she defined the Jacobi coordinate transformation, combining the single baryon wave function of N and Ω baryon to form hexaquark state wave function. As mentioned, she obtained the resonance group wave function of NΩ system through the flavour symmetry breaking. With the RGM wave function, the calculation of scattering and bound state can proceed with different expansion method (Generating Coordinate Method & sub-wave expansion). She also proceeded a simulation to investigate the dynamical mechanism in the system (chiral field-quark interaction).

4.5. Discussion and Comparison of Calculation and Analysis Procedures

All calculation and analysis procedures discussed above were developed based on QCD theory. Although different methods provide researchers with different physical quantities, some of these quantities are able to generate the same conclusion (e.g. potential, binding energy, and scattering phase shift can all indicate the state of hexaquark state particles). In this subsection, the methods and results of these previous works are compared, and these comparisons are summarized in the Table 2.

The investigation of hexaquark states, as mentioned, involves the calculation of mass, energy, scattering phase shift, scattering length, and coupling strength to determine the state and general properties, fit the spectroscopy, and describe interactions. From the perspective of completeness, both works based on RGM only generate the results of state and strength of interactions and is in lack of general properties and spectroscopy. Thus it is incomplete In [22], Chow's work using large Nc limit method generate the state and part of the general properties of hexaquark states. Though it is more general than the RGM method, it is still incomplete. Besides, the QCD sum rule provides researchers with a reliable prediction of spectroscopy, strength and mechanism of interactions, and state of hexaquark state. The effective range of QCD sum rule and large Nc limit is similar. However, lattice QCD methods provided researchers with binding energy, potential, energy, scattering phase shift, scattering length, and mass. In other words, investigations with the lattice QCD method generate all three required aspects of hexaquark states.

Table 2. Comparison of Calculation and Analysis Procedures

Methods Names	Category	Author(year)	System	features	applicability	Reference
Regular Lattice QCD Simulation	Lattice QCD	Francis et al. (1994)	H	State, General Properties, Interaction, Spectroscopy	Choice of propagator	[16]
HAL-QCD	Lattice QCD	HAL QCD Collaboration (2019, 2020)	NΩ/d*(2380)	State, General Properties, Interaction, Spectroscopy	-	[17], [18]
QCD-Sum Rule	QCD Vacuum Condensate	Chen et al. (2019); Wang (2020)	ΩΩ/Triply-charmed-hexaquark state	State, Interaction, Spectroscopy	Specific interpolate current choice	[20], [21]
Large Nc Limit	Large Nc Expansion	Chow (1995)	H	State, General Properties (mass only)	Only systems with fake heavy quark	[22]
RGM regular	Two-body Interactions	Zhao (2005)	NΩ	State, Interaction (strength only)	Short range interactions (strength, mechanism)	[15]
RGM under PGR	Two-body Interactions	Su (2019)	H	State, Interaction (strength only)	Short range interactions (strength, mechanism)	[23]

From the perspective of applicability, the RGM method is viable only when the interactions involve only one dynamical mechanism within one effective range(short/intermediate/long). Though it is not mathematically complicated, it cannot be applied to investigate the general properties and the fitting of spectroscopy. The large Nc limit method should be applied with caution for its construction mechanism of hexaquark state. In Chow's work, the hexaquark was constructed by the transformation of mesons, which has not been found in the experiment. Besides, the results of the QCD sum rule are consistent with the method using other methods (e.g. lattice QCD method). However, in some cases, the choice of interpolating current is overly specific for general application because of the symmetry breaking. Finally, the lattice QCD method, though is complete in descriptions, is lack of consistency when different operators are applied. In [16], A. Francis mentioned that the results of effective energy under the point-to-all propagator and the time-slice-to-all propagator are different, demonstrating inconsistency for the same quantity of the same system. However, its statistical precision is promising, and the choice of specific method (point-to-all propagator/time-slice-to-all propagator) can help to resolve this issue.

5. SUMMARY

In this literature review, we introduce the development and classification of descriptions of hexaquark state particles and list the physical quantities under the scope of investigations in the Section 2. Based on these quantities, original quark model, CQM, Goldstone Boson exchange model, MIT bag model, soliton model, chiral SU(3) quark model, and extended chiral SU(3) quark model are discussed and compared in detail, including the development, validity, and primary equation for description in the Section 3. These models indicate the dynamical and construction mechanism for the hexaquark states. Afterwards, in the Section 4, we reexamine some investigations of hexaquark state applying different analytical techniques and compare each of them.

The result of this comparison indicates that currently, the most effective model is the extended chiral SU(3) quark model as shown in the Table 1. This is primarily because of the approximate chiral symmetry of interactions on the level of quarks. Also, the soliton model shows great potentials: this model includes the chiral field-quark interaction and is capable of predicting the construction hexaquark states. If its validity in determining the dynamical mechanism in electromagnetic, strong, and weak decay and strong coupling is confirmed, then its effectiveness might even surpass the chiral-quark model.

The analytical techniques, in this literature review, are also compared. From both angles of the completeness of description and the universality, the lattice QCD simulation (regular and HAL QCD) is most promising. However, it still has some limitations (e.g. the dependence on operator choice, dependence on computer technology). Moreover, some techniques are very limited and are only applicable and useful for the specific choices of system and quantities calculated. Therefore, these QCD-inspired theories need further developments.

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