Sliding Mode Control Design of Underactuated TORA

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Abstract

For the stabilization control problem of the underactuated translational oscillators with rotating actuator (TORA) system, a sliding mode control with pole assignment and exponential approach law is proposed. Compared with the existing control methods, the proposed algorithm is easier to obtain the parameters of the controller, and the stabilization time and oscillation attenuation amplitude are smaller than those of the control methods. Specifically, the first step is to simplify the dynamic equation of the system at its equilibrium point into a linear model, and design a sliding surface containing all state variables of the system; then, by pole assignment method, the system is assigned with several negative half plane poles to ensure the stability of the closed-loop system, and the values of the coefficients on the sliding surface can be determined; then, the control law is designed by using the exponential approach law, and the stability of the system is proved by Lyapunov function. Finally, the feasibility and effectiveness of the controller are verified by simulation experiments, and the test results show that the proposed method has better control performance.

Keywords

Translational oscillator with the rotating actuator; Pole assignment; Lyapunov function; Stability.

1. INTRODUCTION

As a nonlinear benchmark mechanical system, TORA was originally studied as a simplified model of a dual-spin spacecraft to investigate the resonance capture phenomenon [1] [2]. TORA system has two configuration states, translational position of the cart and rotating angle of the eccentric mass, to be controlled; however, only the rotating angle is actuated. Thus, TORA system is an underactuated system.

For the problem of controlling TORA, many scholars have been studied extensively. Scholars BUPP and TSIOTRAS have conducted special research on the control of TORA system [1] [3], and established the nonlinear benchmark of laboratory scale and the constraint of physical configuration of TORA system.

As the features and the lack of drive, generally there are two kinds of common control method designed for the TORA system. First approach is partial feedback linearization, in this method, the nonlinear terms are eliminated by global transformation of the coordinates of the system, the system is transformed into the nonlinear strict-feedback cascade canonical form, and then the nonlinear control law is designed by back-stepping, but complex structure is the disadvantage of this method, and large complexity is easy to appear in this design. The other one is to use the passive property of system to design control law, the control law is obtained by designing the corresponding Lyapunov function with energy function [4], which drives the total kinetic energy of the system towards zero to achieve the stability of the system. This kind of

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control has a good control effect, but the shortcoming is that the parameters of the controller need to tried.

The existing control methods of TORA system are all nonlinear controllers designed for the high nonlinearity of TORA system. There are problems such as complex process and difficult selection of controller parameters. For high-order complex nonlinear systems, a simple controller cannot meet many requirements, but the complex controller has many problems, such as too many parameters, resulting in the control effect is not good enough and the parameter is difficult to determine. In order to solve the complex control process and the controller parameters need to be tried problems, a simple design is proposed in this paper.

By referring to the method of sliding mode control based on pole placement in literature [5], the stability control design of high-order underactuated system can be realized. For the 2DTORA system, the sliding mode surface of all relevant state variables is designed in literature [6], by pole placement method, the system is assigned with several negative half plane poles to ensure the stability of the closed-loop system, and the values of the coefficients on the sliding surface can be determined, then the control law is designed by exponential approach law, and the control effect is good. Compared with literature [7]-[9], literature [6] uses pole placement method to effectively calculate the specific values of various of the controller, which makes up for the shortage of multiple parameters needing to be tried. In this paper, for the TORA system, a control law combining pole placement method and exponential approach law is designed. Finally, through numerical simulation, the stabilization control of TORA is effectively realized under the constraint conditions proposed in literature [1].

2. DYNAMICAL MODELING

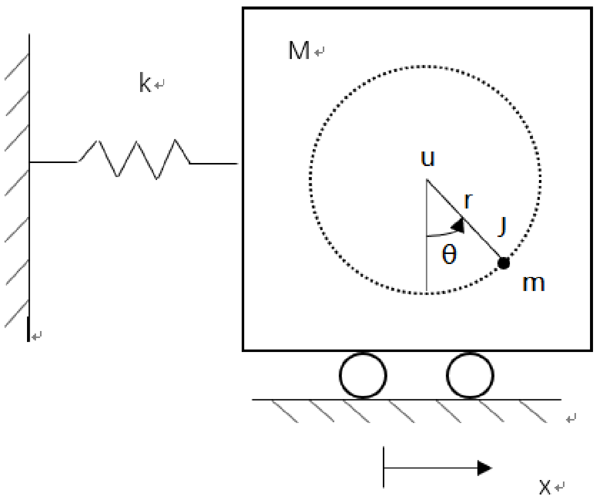

Figure 1. TORA system configuration

The system shown in Figure. 1 represents a translational oscillator with an eccentric rotational proof mass actuator. The oscillator consists of an unbalanced proof mass with mass m attached to a rotary bar of inertia J which is riding on a cart constrained to move horizontally. The moving cart of mass M is attached to a wall by a spring with spring constant k. Let r donate the distance between the rotary bar of the bar. For simplicity, we assume that the motion is confined to the horizontal plane so that there are no gravitational effects. In Figure. 1, u denotes the control torque applied to the proof mass. Let $x, \dot{x}$ denote the translational position and velocity of the cart, and let θ $\dot{\theta}$ denote the angular position and velocity of the rotational proof mass.

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According to the method in literature [4], through the Euler-Lagrange dynamics equation, the dynamic equation of TORA system can be express as

$$(M+m)\ddot{x}+mrcos\theta\ddot{\theta}-mrsin\dot{\theta}^2+kx=0 \tag{1}$$

$$mrcos\theta\ddot{x}+(mr^2+J)\ddot{\theta}+mgrsin\theta=u \tag{2}$$

Define state variable $\boldsymbol{x}=\left[x\ \dot{x}\ \theta\ \dot{\theta}\right]^T=[x_1\ x_2\ x_3\ x_4]^T$, and write the system as follows

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=f_1(x)+b_1(x)u\\ \dot{x}_3=x_4\\ \dot{x}_4=f_2(x)+b_2(x)u\end{cases} \tag{3}$$

Where

$$f_1(x)=\frac{-m^2r^2gsinx_3cosx_3+(mr^2+J)(kx-mr\sin x_3\,{x_4}^2)}{(mrcosx_3)^2-(mr^2+J)(M+m)}$$

$$f_2(x)=\frac{(M+m)mgrsinx_3+mrcosx_3(mr\sin\theta\,{x_3}^2-kx_1)}{(mrcosx_3)^2-(mr^2+J)(M+m)}$$

$$b_1(x)=\frac{mrcos\theta x_3}{(mrcosx_3)^2-(mr^2+J)(M+m)}$$

$$b_2(x)=\frac{-(M+m)}{(mrcosx_3)^2-(mr^2+J)(M+m)}$$

Under $\boldsymbol{u}=\boldsymbol{0}$ and $\boldsymbol{x_3}\in(-\boldsymbol{\pi},\boldsymbol{\pi}]$ conditions, there are two equilibrium points of system, one of them is an unstable equilibrium point $(\boldsymbol{x_1\ x_2\ x_3\ x_4})^{\boldsymbol{T}}=(\boldsymbol{0\ 0\ \pi\ 0})^{\boldsymbol{T}}$, the other is a stable equilibrium point $(\boldsymbol{x_1\ x_2\ x_3\ x_4})_{\boldsymbol{p}}^{\boldsymbol{T}}=(\boldsymbol{0\ 0\ 0\ 0})^{\boldsymbol{T}}$. The control goal of this paper is to stabilize the system at its stable equilibrium point.

3. CONTROL DESIGN

3.1. Model Linearization

Before using the pole placement sliding mode control method, the TORA system dynamics equation described in Equation (3) must be approximately linearized near its stable equilibrium point to transform the system into a linearized model:

Table 1. TORA parameters

Parameter	Value	Units
M	1.3608	kg
m	0.096	kg
r	0.0592	m
J	0.0002175	kg • m^2
K	186.3	N/m
g	9.81	N • m^2

$$\dot{x}=Ax+Bu \tag{4}$$

Where

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$$A=\left(\frac{\partial}{\partial x}\right)[x_2,f_1(x),x_4,f_2(x)]_{x=x_p}^T=\begin{bmatrix}0 & 1 & 0 & 0\\ -133.22 & 0 & 0.41 & 0\\ 0 & 0 & 0 & 1\\ 1366.72 & 0 & -104.84 & 0\end{bmatrix}$$

$$\boldsymbol{B}=[0,b_1(x),0,b_2(x)]_{x=x_p}^T=[0,-7.34,0,1880.50]^T$$

3.2. Sliding Surface Design

According to the method proposed in the literature [5], we can define the sliding surface as:

$$\dot{\boldsymbol{e}}=\dot{\boldsymbol{x}}=Ax+Bu. \tag{5}$$

$$s=c_1x_1+c_2x_2+c_3x_3+c_4x_4=Ce \tag{6}$$

Where $c_1 \sim c_4$ are positive constants and $\boldsymbol{C}=[c_1\ c_2\ c_3\ c_4]$.

Then, before using the pole placement method, for convenience, equation (5) is transformed into controllable standard form. Suppose the transformation matrix is T, by transforming $\boldsymbol{e}=\boldsymbol{T}\overline{\boldsymbol{e}}$, equation (5) is transformed into the follow form

$$\dot{\boldsymbol{e}}=\overline{A}x+\overline{B}u, s=Ce=\overline{\boldsymbol{C}}\overline{\boldsymbol{e}} \tag{7}$$

Where $\overline{\boldsymbol{A}}=\boldsymbol{T}^{-1}\boldsymbol{A}\boldsymbol{T},\overline{\boldsymbol{B}}=\boldsymbol{T}^{-1}\boldsymbol{B},\overline{\boldsymbol{C}}=\boldsymbol{C}\boldsymbol{P}=[\overline{c_1}\ \overline{c_2}\ \overline{c_3}\ \overline{c_4}]$. When the state variables of the system move on the sliding surface, we can obtain $s=\overline{\boldsymbol{C}}\overline{\boldsymbol{e}}=0$. The control input of the system is obtained:

$$\boldsymbol{s}=\overline{\boldsymbol{C}}\overline{\boldsymbol{e}}=\overline{\boldsymbol{C}}(\overline{\boldsymbol{A}}\overline{\boldsymbol{e}}+\overline{\boldsymbol{B}}\boldsymbol{u})=\boldsymbol{0}\Rightarrow\boldsymbol{u}=-\left(\overline{\boldsymbol{C}\boldsymbol{B}}\right)^{-1}\overline{\boldsymbol{C}\boldsymbol{A}}\overline{\boldsymbol{e}} \tag{8}$$

By substituting equation (8) into (7), the closed-loop state equation of the system with sliding surface $s=0$ can be obtained:

$$\dot{\boldsymbol{e}}=\left(\boldsymbol{I}-\overline{\boldsymbol{B}}\left(\overline{\boldsymbol{C}\boldsymbol{B}}\right)^{-1}\overline{\boldsymbol{C}}\right)\overline{\boldsymbol{A}}\overline{\boldsymbol{e}} \tag{9}$$

Where

$$\left(\boldsymbol{I}-\overline{\boldsymbol{B}}\left(\overline{\boldsymbol{C}\boldsymbol{B}}\right)^{-1}\overline{\boldsymbol{C}}\right)\overline{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1}\\ \boldsymbol{0} & -\frac{\overline{\boldsymbol{c}_1}}{\overline{\boldsymbol{c}_4}} & -\frac{\overline{\boldsymbol{c}_2}}{\overline{\boldsymbol{c}_4}} & -\frac{\overline{\boldsymbol{c}_3}}{\overline{\boldsymbol{c}_4}}\end{bmatrix}$$

Then, we can judge the stability of the system by calculating the eigenvalues of $\left(I-\overline{B}\left(\overline{CB}\right)^{-1}\overline{C}\right)\overline{A}$. The characteristic polynomial of $\left(I-\overline{B}\left(\overline{CB}\right)^{-1}\overline{C}\right)\overline{A}$ is

$$\Delta=\left|sI-\left(I-\overline{B}\left(\overline{CB}\right)^{-1}\overline{C}\right)\overline{A}\right|=s^4+\frac{\overline{c_3}}{\overline{c_4}}s^3+\frac{\overline{c_2}}{\overline{c_4}}s^2+\frac{\overline{c_1}}{\overline{c_4}}s \tag{10}$$

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Therefore, as long as the polynomial (10) is Hurwitz stable, the close-loop state system (9) is stable. Obviously, one solution the equation (10) is $s = 0$, so we need to assign another three poles which have negative real part to promise the system can stable.

3.3. Control Design

According to the design idea of sliding mode control, the designed control input should make the sliding surface s close to zero in a limited time, and the control input of the system is obtained by Lyapunov method.

We propose the Lyapunov function candidate as $V = 0.5s^2$, according to Lyapunov theorem, the sliding surface s will converge to zero when $\dot{V} < 0$. The expression of $\dot{V}$ is

$$\dot{V} = s\dot{s} = s[c_1x_2 + c_2f_1 + c_3x_4 + c_4f_2 + (c_2b_1 + c_4b_2)u]$$

Where control input can be written as

$$\tau = u_{eq} + u_{sw} \tag{11}$$

Where u_{eq} is equivalent control and u_{sw} is switching control which using exponential reaching law

$$u_{eq} = -\frac{c_1x_2+c_2f_1+c_3x_4+c_4f_2}{c_2b_1+c_4b_2} \tag{12}$$

$$u_{sw} = -\frac{-\eta\cdot sign(s)+ks}{c_2b_1+c_4b_2} \tag{13}$$

Where η and k is positive constant. Substituting the expression of τ into Lyapunov function, we have

$$\dot{V} = -s(\eta \cdot sign(s) + ks) = -\eta|s| - k \cdot s^2 \leq 0 \tag{14}$$

So the system is stable in the sense of Lyapunov.

4. SIMULATION RESULTS

Referencing the parameters of TORA in [4], simulation parameter are chosen as in Table I; and according to experience, we choose three poles as $p_1 = -0.5 + 11i$, $p_2 = -0.5 - 11i$, $p_3 = -5$, and the gains of the controller (13) are chosen as $\eta = 1$, $k = 10$.

According to the assigned poles and prior knowledge, the parameters of sliding surface are determined as

$$s = 14429x_1 + 2404.8x_2 + 683.6x_3 + 9.54x_4 \tag{15}$$

And the control input is

$$\tau = -\frac{14429x_2+2404.8f_1+683.6x_4+9.54f_2-sign(s)+10s}{2404.8b_1+9.54b_2} \tag{16}$$

The simulation results are shown in the Figure 2. ~Figure 4. Under the initial condition is $[0.01, 0, \pi/3, 0]$, we can see that the cart' positions are stabilized to 0 and the rotor angle is move

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to the zero after 4.3 seconds. Then the robust test is carried out in the Figure 5., under the initial condition is $[0.01,0,\pi/3,0]$, sinusoidal interference with amplitude of 0.5 and frequency of 4π is added between 10-11 seconds, and random interference with amplitude of 0.1 is added between 20-30 seconds. Under the action of the controller designed in this paper, the result show that different types of external disturbances can be attenuated and eliminated quickly, and the system is guaranteed to run within the constraints. The simulation results show that this method is robust. At the same time, the carts position and the continuous control torque which is allowed higher for short periods can meet the constrains of $|x| \leq 0.25m$ and $|u| \leq 0.100N \cdot m$ [1].

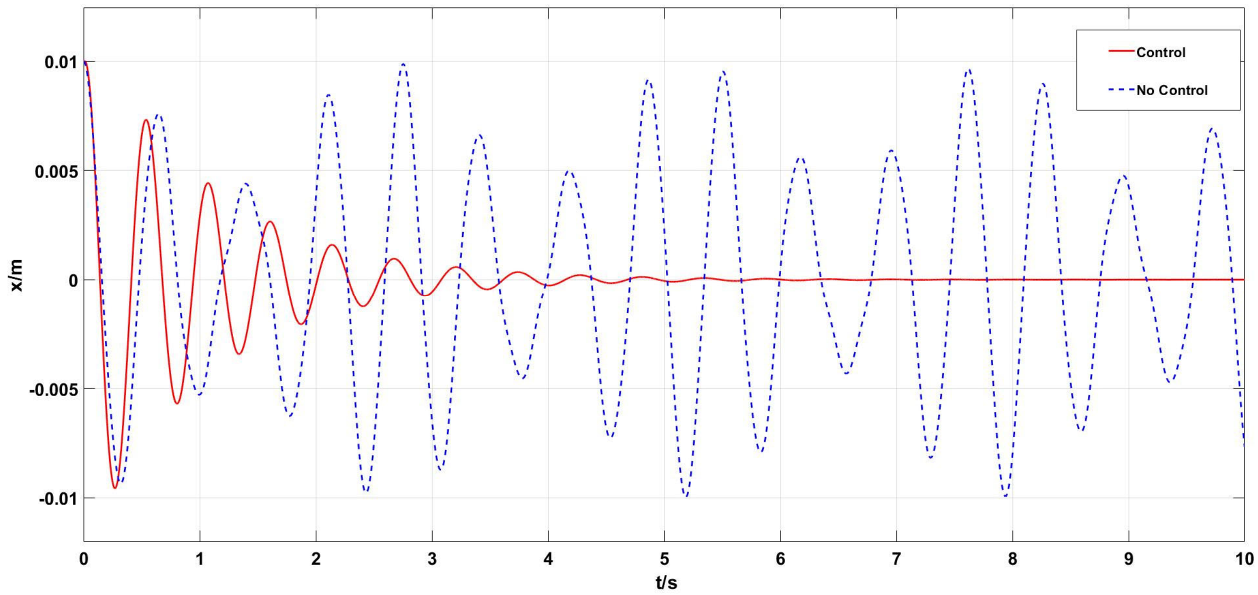

Figure 2. Displacement of the cart

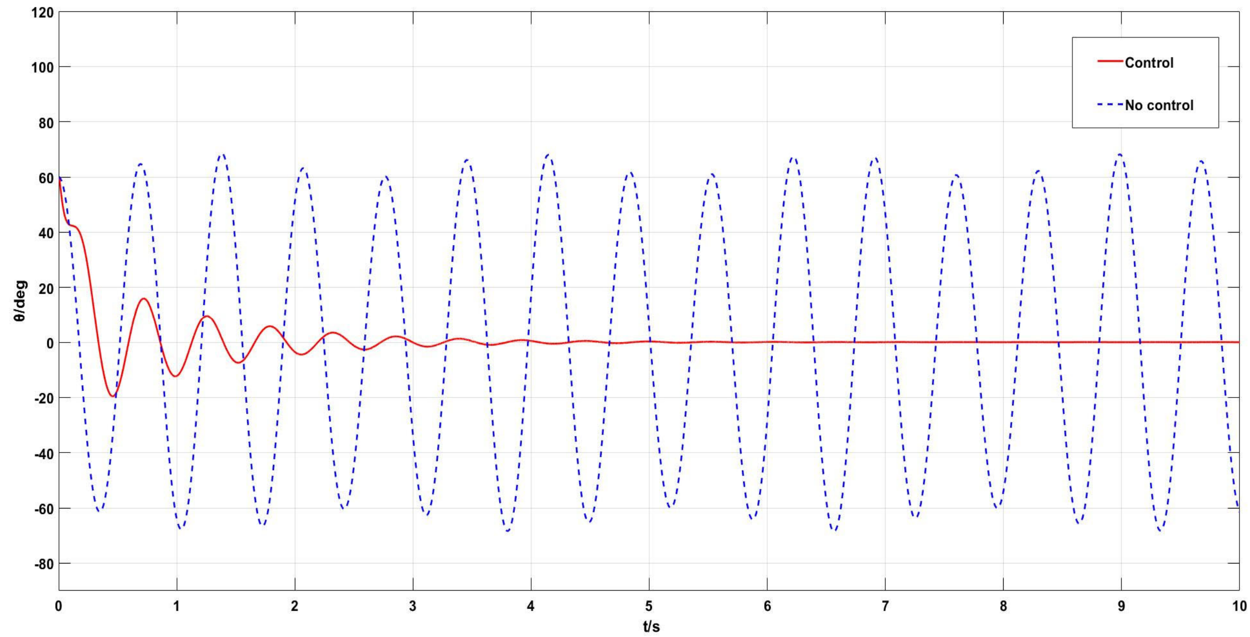

Figure 3. Displacement of the proof mass

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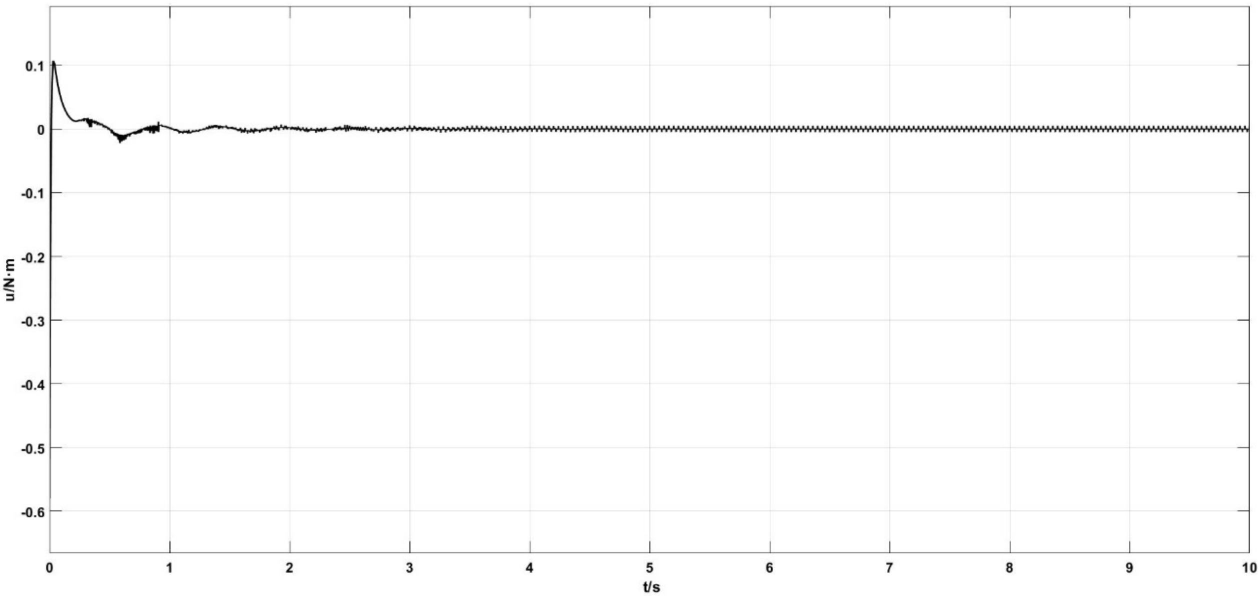

Figure 4. Torque

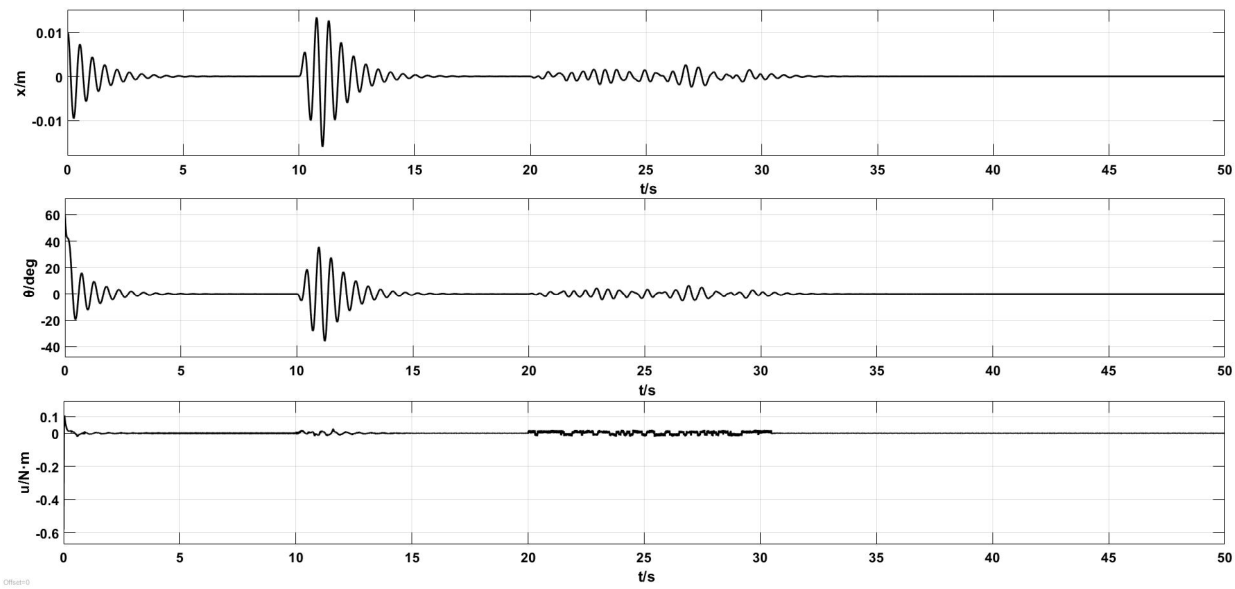

Figure 5. Robust test

5. CONCLUSION

In this paper, a sliding mode controller based on pole placement method is designed to solve the problem of the complex control process of TORA and difficulty in choosing the controller parameters. The experimental results show that this method can easily obtain the parameters, effectively realize the stabilization control of TORA, and has certain robust.

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