# Admission Control and Resource Allocation based on Cognitive Radio Network Underlay Spectrum Sharing

Zilong Tang
Zhengzhou University, Zhengzhou, 450001, China

## Abstract

**The wireless networks today are based on fixed channel allocation. However, the spectrum utilization of fixed channel allocation is quite low, between 15% and 85%. As a result, much spectrum resource is laid unused. A resource allocation framework is presented for spectrum underlay in cognitive radio networks. QOS means the quality of service (QoS) auxiliary restricted users. Specififically, interference from secondary users to primary users is constrained to be below a tolerable limit. In this paper, we transform the real problems into mathematic optimization problem by building a model.**

## Keywords

**Primary user; Restricted user; Spectrum underlay, QOS.**

## 1. INTRODUCTION

Achieveing high-speed transmission and provisioning of quality of service (QoS) for emerging data-oriented wireless applications through intelligent and flflexible radio resource management are the key challenges for future-generation wireless networks [1-3]. QOS will be given high data rates and can be provided multimedia applications from the next-generation celluar wireless networks [4].

On the other hand, each admitted secondary user to access the spectrum band should have a minimum QOS, in terms of its BER. When the modulation scheme is chosen, the BER of each user is directly related to its SINR. Therefore, the requirement of BER is transferred to the requirement of minimum SINR. This paper mainly discuss how to find the largest secondary user set and how to maximize the total network utilities based on the chosen secondary user set.

### 1.1. Cognitive Radio Technology

Spectrum resource is getting in shortage from the rapid development of wireless services . Cognitive Radio technology as a new way to solve the increasing demand of wireless communication can solve the problem of the limited wireless spectrum resource also. The foundation of CR is that the secondary users can access the authorized spectrum in an opportunistic way by sensing spectrum holes and thus greatly improve the spectrum efficiency.

Built on the platform of Software Radio, Cognitive Radio can sense the environment, and are able to reconfigure itself by learning and adapting to the communication surroundings. Although Dynamic Spectrum Access is a very important application of Cognitive Radio, Cognitive Radio is a wider term. By Cognitive Radio, many aspects of wireless communication can be improved.

Cognitive Radio is to find the best available spectrum band by perception and restructure is the ultimate objective. Since most of the usable spectrum resource has already been assigned. Our most challenge today to spectrum sharing strategies[4]. Cognitive Radio can utilize the unused spectrum at some time and place, called the spectrum holes or white space. When the primary users, the users who are authorized to use this spectrum band, come back to use the spectrum again, secondary users can change the transmission parameters of transfer to another spectrum hole in order not to cause too much interference to the primary users.

In brief, two tactics constitute the Cognitive Radio. Firstly,the primary users won't be getting into interference under the secondary users accessing the spectrum band . Once the primary user come back to user the spectrum, the secondary must stop using this band. The other strategy is that the secondary users can access the spectrum in a fair and opportunistic way even if the primary user occupies the spectrum band, if only the total interference caused by secondary users is constrained. These two kinds of spectrum sharing strategy can be realized by Cognitive Radio technology. Cognitive Radio can sense and capture the outside working environment and sensitively change its transmission parameters to adapt to the new surroundings. Therefore, it can realize high efficiency transmission at any time and any place while the unauthorized users will not causing too much interference to the primary users [5].

Since the Cognitive Radio allows several secondary users to access the spectrum band of one primary user. The coexistence of several secondary users will certainly bring competition for more resource of each user. The selfish feature of each secondary user will cause a mess without a certain mechanism. Thus a reasonable spectrum accessing strategy is a must [5].

## 1.2. Spectrum Sharing Strategy

### 1.2.1 Spectrum Overlay

When the authorized primary user doesn't occupy the spectrum band, the unauthorized secondary users can detect this spectrum holes and then access it to do transmissions. When the primary user come back to transmit data again, the secondary users have to shift to other spectrum bands in order not to create interference to the primary user.

The Dynamic Spectrum Access based on spectrum Overlay can utilize the spectrum holes efficiently from the time and space angle, and thus improve the spectrum efficiency. From the time domain, the cognitive node can be divided into two groups: slow spectrum opportunity and fast spectrum opportunity[5]. Slow spectrum opportunity means in some circumstances the primary users will not utilize the spectrum band for a long time, and thus the secondary users can utilize it for long. For example, in some areas, the TV broadcast spectrum band can be rent for secondary users. While the fast spectrum opportunity means the spectrum holes will appear and disappear quickly due to the fast changing features of primary users' services. The research of fast spectrum opportunity has just taken off and is more common than the slow spectrum opportunity. Thus, research of fast spectrum opportunity is more meaningful for today's wireless communication systems.

### 1.2.2 Spectrum Underlay

Even if there is primary user occupying the spectrum band, the secondary users can still access the spectrum to transmit data, if only the interference caused by them is below a certain threshold. The reason to do this is that it can greatly improve the spectrum efficiency.

When the secondary users access the spectrum band owned by the primary users, the interference caused by them should be within constraints in order to ensure the QOS of primary user. In this article, we pay more attention to multi-secondary users. Obviously, when the transmission power of a SU is large, the SINR will be large and thus achieving high throughput. However, this will cause high interference to other secondary users as well as the primary user. As a result, the number of admitted secondary users will be decreased. In order to eliminate this kind of selfish actions, we devise a centralized power allocation and admission control algorithm.

## 2. SYSTEM MODEL & PROBLEM ANALYSIS

### 2.1. System Description

In this paper, our system is based on the Spectrum Underlay sharing strategy, that is, the secondary and primary users can access the spectrum band at the same time. In order to guarantee the QOS of primary user, the interference caused by all the secondary users should be below a certain designed threshold. As for each admitted secondary user, the QOS of it should be ensured, that is, the BER (bit error rate) should below a certain value. On the other hand, the system fairness quality should also be considered. Some secondary users with disadvantaged channel condition should not be neglected for long simply because we want to maximize the system throughput. With all the constraints, our first goal is to maximize the number of admitted secondary users. Based on the chosen secondary user set, our next goal is to maximize the total system utility.

Thus, the problem can be described as follows:

Constraints:1: the allover interference of primary user caused by secondary users should be lower than an exactly threshold.2: the QOS of each admitted secondary user should be guaranteed.3: some other constraints like upper limit of transmission power of secondary users, transmission rate bound.

Objectives:1: maximize the number of admitted secondary users.2: based on 1, maximize the integral system utility.3:a certain degree of fairness should be ensured.

### 2.2. System Model

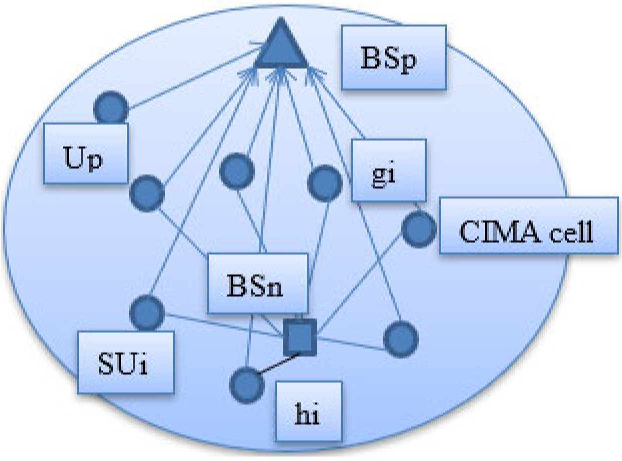

**Figure 1.** System Model of the3G cellular cognitive radio network

As the figure.1 shows, the model on the basis of 3G cellular network. In this model, there is only one primary user (PU), and several secondary users ( $SU_s$ ), the PU is the owner of the spectrum band and is authorized to use the band to communicate with the 3G base station( $BS_p$ ).The secondary user communicate with secondary base station ( $BS_s$ ) through CDMA protocols [6].

As for the N $SU_s$ , the channel gain between $SU_i$ and $BS_s$ is hi (i = 1, 2, ...,N),between the $SU_i$ and $BS_p$ the channel gain is $g_i$ (i = 1, 2, ...,N). For the QOS of primary user be ensured, the allover interference of primary user caused by secondary users should be lower than an exactly threshold. In this paper, we don't consider the interference caused by secondary base station. From the above, we get the total interference to the $BS_p$ is $\sum_{i=1}^{N} p_i g_i$ .Therefore we have

$$\sum_{i=1}^{N} P_i <= P_{th}$$

The SINR each SUi can get is:

$$\varepsilon_i = \frac{B}{r_i} \frac{h_i p_i}{\sum_{j \neq i}^{N} h_j p_j + \sigma^2}$$

Where, B stands for the spectrum bandwidth, $R_i$ stands for the transmission rate of $SU_i$. Thus B/ $R_i$ signals the PG (processing gain). $\sigma^2$ is the white noise of the system. We assume that the white noise for every link is in a same. $\sum_{j \neq i}^{N} h_j p_j$ is the interference caused by other secondary nodes to link of $SU_i$ . As for each $SU_i$ , the QOS (in terms of BER) should be guaranteed. When the modulation programme is determined for the system, the relationship of the BER between the SINR is affirm. Thus the QOS requirement can be transferred to the requirement of SINR.

Assume, for each $SU_i$, the minimum SINR is $\varepsilon_i^{\,q}$, that is

$$\varepsilon_i >= \varepsilon_i^{\,q}, i = 1, 2 ... N;$$

For each $SU_i$, the transmission power and transmission rate is within constraints

$$p_i < P_{\max};$$
$$R_{\min} <= R_i <= R_{\max};$$

Here, we assume, for each $SU_i$, the value of $P_{\max}$, $R_{\min}$, $R_{\max}$ are the same.

For $\varepsilon_i >= \varepsilon_i^{\,q}$ we have

$$\frac{B}{R_i} \frac{h_i p_i}{\sum_{j \neq i}^{N} h_j p_j + \sigma^2} >= \varepsilon_i^{\,q}$$

$$p_i >= \frac{\varepsilon_i^{\,q} R_i}{B h_i} (\sum_{j \neq i}^{N} h_j p_j + \sigma^2) >= \frac{\varepsilon_i^{\,q} R_{\min}}{B h_i} (\sum_{j \neq i}^{N} h_j p_j + \sigma^2)$$

Assume,: $P = [p_1, p_2 ... p_N], U = \frac{\sigma^2 R_{\min}}{B} [\frac{\varepsilon_1^{\,q}}{h_1}, \frac{\varepsilon_2^{\,q}}{h_2} ... \frac{\varepsilon_N^{\,q}}{h_N}]$

$$F_{ij} = \begin{cases} \frac{R_{\min} \varepsilon_i^{\,q} h_j}{B h_i} & i \neq j \\ 0 & , \; i = j \end{cases}$$

Then we have $(I - F) P^T >= U^T$

If $(I - F)$ is invertible and the spectrum radius is definitely less than 1 ( $\rho(F) < 1$ ), then we have $P^T >= (I - F)^{-1} U^T > 0$;

And thus we have one solution for this problem

$$P^* = U[(I - F)^{-1}]^T$$

Which is show in figure 2?

When $(\rho(F) >= 1)$, the solution of the problem doesn't exist. We can use the situation of two secondary users to explain the conclusion.

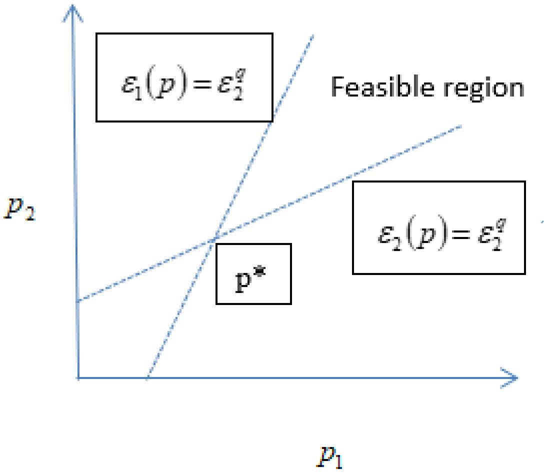

**Figure 2.** Graphic of p*geometry for N=2 case

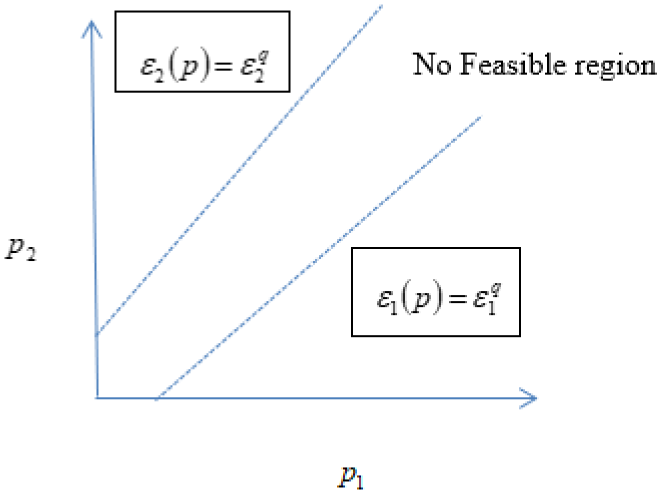

**Figure 3.** Graphic of the case N=2, and these is no feasible region

As the figure 3 illustrates, when the two lines has no intersection, the solution to the problem doesn't exist no matter how large P is. In this case, the QOS of each SU can't be satisfied if we admit all of them. Therefore, only a certain number of SU can have to chance to access the spectrum while others will be declined. Our first goal is to find the maximum secondary user set, in which case, we think the maximum interest of the system is achieved [7]. Based on the chosen set, our second goal is to maximize the total system utility by allocation different powers to each admitted $SU_i$.

# 3. OPTIMIZATION & SOLUTION

## 3.1. Optimal Algorithm

Define binary variable

$x_i \in \{0,1\}, i = 1, 2 \ldots N;$ Where $x_i = \begin{cases} 1, \text{SUi is admintted} \\ 0, \text{SUi is not admitted} \end{cases}$

Then the problem can be modeled as:

Maximize: $(\sum_{i=1}^{N} x_i)$

Subject to

$$\varepsilon_i = \frac{B}{R_i} \frac{x_i h_i p_i}{\sum_{j \neq i}^{N} x_j h_j p_j + \sigma^2} >= x_i \varepsilon_i^q \quad , \quad \text{i=1,2,...N} \tag{1}$$

$$\sum_{i=1}^{N} x_i p_i g_i <= P_{th} \tag{2}$$

$$p_i (x_i - 1) = 0, \text{ i=1, 2...N} \tag{3}$$

$$R_{\min} <= R_i <= R_{\max}, \text{ i=1, 2...N;} \tag{4}$$

$$0 <= p_i <= P_{\max}, \text{ i=1, 2...N;} \tag{5}$$

$$x_i \in \{0,1\}, \text{ i=1, 2...N;} \tag{6}$$

Where

Constraint (1) states that the QOS of each admitted $SU_i$ should be satisfied.

Constraint (2) states that the interference threshold of primary user should be satisfied.

Constraint (3) states that when $SU_i$ is not admitted, $x_i$ = 0, then the transmission power of it ( $p_i$ ) should be 0.

Constraint (4) graphic the constraints of the transmission power.

Constraint (5) graphic the constraints of the transmission rate.

In order to optimize problem, that is a classical combinatorial logical question. The optimization solution can be achieved through recursion and iteration. However, the computation complexity is quite large O (2^N). In order to reduce the computation complexity, we can devise a suboptimal algorithm of heuristic method to find the secondary user set, which will be discussed in the next section

### 3.2. Suboptimal Algorithm

In order to admit the most number of secondary users, the SINR of each $SU_i$ is first set to be $\varepsilon_i^q$,transmission rate, Because the larger $R_{\min}$ and $\varepsilon_i^q$, the larger the transmission power the node will have to pay. In this case, it will create larger interference to other nodes and the primary user, which will make the constraints difficult to meet and therefore, the number of admitted secondary users will decrease. Therefore, we first set the transmission rate and SINR to be $R_{\min}$ and $\varepsilon_i^q$ respectively. Assume $p_i(t+\Delta t), p_i(t)$ are the corresponding transmission power of node i after two consecutive updates. Then we get the listed below tentative algorithm.

$$p_i(t+\Delta t) = \frac{\varepsilon_i^q}{\varepsilon_i(t)} \tag{7}$$

In this case, P will converge to $P^* = U[(I-F)^{-1}]^T$ . If $SU_i$ is not admitted, then the above formula will not converge or will exceed Pmax. Therefore, the threshold Pmax is going to be used in the iteration to determine the convergence of P. when $p_i(t)$ iterates above Pmax, then we concludes that node I can't be admitted and it will be eliminate from the admitted user set.

Packing Algorithm

[P, FLAG] = PACK(SU)

//Here FLAG stands for whether the power Allocation is successful.

//SU stands for the set to be allocated the power

Step 1: choose a small initial $P^0 = [p_1^{\ 0}, p_2^{\ 0}, \ldots p_m^{\ 0}]$, and threshold $\lambda$;

Step 2: calculate $\varepsilon_i$ for each $SU_i$ in the set SU;

Step 3: doing iterations, $p_i^{n+1} = (\varepsilon_i^q / \varepsilon_i) p_i^n$ ,

If P exceeds Pmax, then the solution will not exist. Mark FLAG = 0. Breach through the iterations.

Else continue

Step 4: repeat the above, until for all $p_i$, we have $| p_i^{n+1} - p_i^n |$e= $\lambda$, set FLAG = 1

It can be proved [12] that if the solution exist, the above iteration will converge to $P^* = U[(I-F)^{-1}]^T$ . The corresponding SINR is $\varepsilon = [\varepsilon_1^{\ q}, \varepsilon_2^{\ q} \ldots \varepsilon_m^{\ q}]$ , power allocation is $P^q = [p_1^{\ q}, p_2^{\ q} \ldots p_m^{\ q}]$ and Ri = Rmin for all I;

Admission Control Algorithm

Through the above packing algorithm, a feasible power allocation solution can be found if the solution exists. In this part, we will discuss how to find the final secondary user set from the N requesting $SU_i$

[SET, P] = SystemPack(O, S)

//Here SET means the final chosen set result, SETi = 1 stands for admission and SETI = //0 stands for no admission

//P records the dependent power distribution result.

//O is the order for admission, which will be further discussed in later section

//S is the requesting secondary user set

Step (1): According to the order O, get the next $SU_i$, together with the admitted user set. Find whether FLAG is equal to 1 using the above PACK algorithm.

If FLAG = 1, then go to step (2)

Else delete this requesting node, continue step 1

Step (2): if FLAG = 1 and corresponding $P^q = [p_1^{\ q}, p_2^{\ q} \ldots p_m^{\ q}]$ satisfy $\sum_{i=1}^{m} g_i p_i^q <= P_{th}$, then this user is admitted into the admission set. Repeat step 1 again.

Else mark this user can't be admitted, continue step 1

Step (3): we get the final admission set $K=[SU_1, SU_2 ... SU_m]$, and the corresponding allocation power $P^q=[p_1{}^q, p_2{}^q ... p_m{}^q]$.

## 3.3. Second Goal-maximize the System Utility

After we get the secondary user set, our next goal is to maximize the system utility.

Define the utility of every admitted user is a function of its transmission rate R

$u_i = f(R_i)$

Here we simply define

$f(R_i) = R_i$

And we have $u_i = R_i$

Our goal is to maximize the total utility $\sum_{i=1}^{m} R_i$.

$$R_i = \frac{B}{\varepsilon_i} \frac{h_i p_i}{\sum_{j \neq i}^{N} h_j p_j + \sigma^2} <= \frac{B}{\varepsilon_i{}^q} \frac{h_i p_i}{\sum_{j \neq i}^{N} h_j p_j + \sigma^2}$$

Set each $\varepsilon_i = \varepsilon_i{}^q$

And we have $R_i = \frac{B}{\varepsilon_i{}^q} \frac{h_i p_i}{\sum_{j \neq i}^{N} h_j p_j + \sigma^2}$

Assume the admission set is $\{SU_i\}, i = 1, 2, ..m$

The model turns into the listed below majorization question.

Maximize $\sum_{i=1}^{m} R_i$

Subject to

$$R_i = \frac{B}{\varepsilon_i{}^q} \frac{h_i p_i}{\sum_{j \neq i}^{N} h_j p_j + \sigma^2} \in [R_{\min}, R_{,\max}] \text{, i=1,2,...m;} \tag{8}$$

$$\sum_{i=1}^{m} p_i g_i <= P_{th} \tag{9}$$

$$p_i \in [0, P_{\max}] \text{, i=1,2,...m;} \tag{10}$$

3.3.1 Optimal Algorithm

Assume matrix of F, M m*m . U, T (1*m) vector.

Define:

$$F_{ij} = \begin{cases} -Bh_i & i = j \\ R_{\min} \varepsilon_i{}^q h_j & i \neq j \end{cases}, \qquad U_i = -R_{\min} \varepsilon_i{}^q \sigma^2$$

$$M_{ij} = \begin{cases} Bh_i & i = j \\ -R_{\max}\varepsilon_i{}^q h_j & i \neq j \end{cases}, \qquad T_i = R_{\max}\varepsilon_i{}^q \sigma^2$$

Then the above constraints (8) can be turned into

$$FP <= U$$

$$MP <= T$$

Assume $G = [g_1, g_2 \ldots g_m]$, then the above constraints (8), (9), (10) can be turned into

$$\begin{bmatrix} F \\ M \\ G \end{bmatrix} P <= \begin{bmatrix} U \\ T \\ P_{th} \end{bmatrix}$$

$$p_i \in [0, P_{\max}]$$

The results are as follows, the upper model turns into a linear limit non-linear objective programming problem. We can use MATLAB to find the optimization result.

$$\text{Minimize } -\sum_{i=1}^{m} R_i \tag{11}$$

$$\text{Subject to} \quad \begin{bmatrix} F \\ M \\ G \end{bmatrix} P <= \begin{bmatrix} U \\ T \\ P_{th} \end{bmatrix} \tag{12}$$

$$p_i \in [0, P_{\max}] \tag{13}$$

### 3.3.2 Suboptimal Algorithm to for system utility

Assume the secondary user set we get through the first step is $K = [SU_1, SU_2 \ldots SU_m]$

The respective allocated power is $P^q = [p_1{}^q, p_2{}^q \ldots p_m{}^q]$

Then we must have: $\sum_{i=1}^{m} g_i p_i^q <= P_{th}$

Define variable

$$t = \frac{P_{th}}{\sum_{i=1}^{m} g_i p_i^q} >= 1, \qquad \alpha = \frac{P_{\max}}{\max(P)} >= 1$$

And define:

$$\beta = \min(t, \alpha) >= 1;$$

At most times: $P_{\max} >> \max(P)$, so $\frac{P_{\max}}{\max(P)} >> 1$

That is $\beta = \min(t, \alpha) = t = \frac{P_{th}}{\sum_{i=1}^{m} g_i p_i^q} >= 1;$

Set the final power allocation as $p_i^* = \beta p_i^q$, then we have:

$$R_i^* = \frac{B}{\varepsilon_i^q}\frac{h_i p_i^*}{\sum_{j\neq i}^{N} h_j p_j^* + \sigma^2} = \frac{B}{\varepsilon_i^q}\frac{\beta h_i p_i^q}{\beta\sum_{j\neq i}^{N} h_j p_j^q + \sigma^2} >= \frac{B}{\varepsilon_i^q}\frac{\beta h_i p_i^q}{\beta\sum_{j\neq i}^{N} h_j p_j^q + \beta\sigma^2} = R_{\min}$$

;

Since $R_i <= R_{\max}$

So set the final allocated rate is $R_i = \min\{R_i^*, R_{\max}\}$ ; then:

$$\varepsilon_i = \frac{B}{R_i}\frac{h_i p_i}{\sum_{j\neq i}^{N} h_j p_j + \sigma^2} = \frac{B}{\min\{R_i^*, R_{\max}\}}\frac{h_i p_i}{\sum_{j\neq i}^{N} h_j p_j + \sigma^2}$$

$$>= \frac{B}{R_i^*}\frac{h_i p_i}{\sum_{j\neq i}^{N} h_j p_j + \sigma^2} = \varepsilon_i^q$$

Therefore, the final SINR of each admitted secondary user is also satisfied. In the later section, we will prove our result through MATLAB simulation.

The System Model in Suboptimal Algorithm.

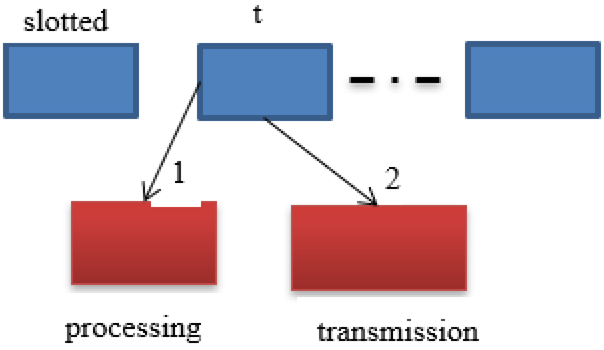

**Figure 4.** The System Model in Suboptimal Algorithm

As figure 4 shows, the system time can be filled in. In each time slot, the secondary users first tell the secondary base station about its QOS, channel parameters g h and so on. Then, the BS will calculate who will be admitted and who don't according to the designed priority and assign each admitted SU power as according to the previous algorithm. Then each $SU_i$ will change its parameter Di. Next, the admitted $SU_i$ use the left time in the time slot to do transmissions. then,the next round begins. The algorithm can be set to:

Step 1: Bp receives the information of $\varepsilon_i^q$ , h, g from each requesting $SU_i$ ;

Step 2: According to the $\varepsilon_i^q$ and historical Di, calculate the priority of $SU_i$ $y_i = (D_i + 1)/\varepsilon_i^q$ , and then sort them.

Step 3: According to the order, by the previous suboptimal selection algorithm, a admission set can be achieved $K = [SU_1, SU_2 ... SU_m]$, for each $SU_i$ , $SU_i \in K$ ,set $D_i = 0$, for others, set $D_i = \min\{D_i + 1, M\}$

Step 4: The admitted $SU_i$ do transmission according to the allocated $p_i^*$ and transmission

Step 5: A new round.

# 4. FAIRNESS QUALITY ANALYSIS

## 4.1. One Time Allocation Fairness Quality Analysis

Define F1

$F_1 = \frac{(\sum_{i=1}^{m} R_i)^2}{m \cdot (\sum_{i=1}^{m} R_i^2)}$, where i=1,2...m; represents the admitted users.

After the first step we know the admission set.

User set: $K = [SU_1, SU_2 ... SU_m]$

Power allocation: $P^q = [p_1{}^q, p_2{}^q ... p_m{}^q]$

Rate allocation: $R = [R_1, R_2 ... R_m] = [R_{\min}, R_{\min} ... R_{\min}]$

Here F1 = 1.

If we use the suboptimal algorithm to find the system utility,

$\beta = \min(t, \alpha) >= 1$

$$R_i = \frac{B}{\varepsilon_i{}^q} \frac{h_i p_i{}^q}{\sum_{k \neq i}^{m} h_k p_k{}^q + \sigma^2} = R_{\min}$$

$$R_j = \frac{B}{\varepsilon_j{}^q} \frac{h_j p_j{}^q}{\sum_{k \neq j}^{m} h_k p_k{}^q + \sigma^2} = R_{\min}$$

$$R_i / R_j = \frac{\varepsilon_j{}^q}{\varepsilon_i{}^q} \cdot \frac{h_i p_i{}^q}{h_j p_j{}^q} \cdot \frac{\sum_{k \neq j}^{m} h_k p_k{}^q + \sigma^2}{\sum_{k \neq i}^{m} h_k p_k{}^q + \sigma^2} = 1$$

And $R_i^* / R_j^* = \frac{\varepsilon_j{}^q}{\varepsilon_i{}^q} \cdot \frac{\beta h_i p_i{}^q}{\beta h_j p_j{}^q} \cdot \frac{\beta \sum_{k \neq j}^{m} h_k p_k{}^q + \sigma^2}{\beta \sum_{k \neq i}^{m} h_k p_k{}^q + \sigma^2}$

If $\sigma^2 << \sum_{k \neq j}^{m} h_k p_k{}^q$, then

$$R_i^* / R_j^* = \frac{\varepsilon_j{}^q}{\varepsilon_i{}^q} \cdot \frac{\beta h_i p_i{}^q}{\beta h_j p_j{}^q} \cdot \frac{\beta \sum_{k \neq j}^{m} h_k p_k{}^q + \sigma^2}{\beta \sum_{k \neq i}^{m} h_k p_k{}^q + \sigma^2} \approx \frac{\varepsilon_j{}^q}{\varepsilon_i{}^q} \cdot \frac{\beta h_i p_i{}^q}{\beta h_j p_j{}^q} \cdot \frac{\beta (\sum_{k \neq j}^{m} h_k p_k{}^q + \sigma^2)}{\beta (\sum_{k \neq i}^{m} h_k p_k{}^q + \sigma^2)} = 1$$

Then $F_1 \approx 1$, as a result, the final result using the suboptimal algorithm to find the maximum system utility will achieve a good fairness quality.

## 4.2. Long Term Fairness Quality

Define $K_i(L) = kr_i / L$

Here L stands for the L time slots, $kr_i$ means the number of admission of $SU_i$ in the L time slots. That is, $K_i(L)$ stands for the possibility of admission of $SU_i$ in L time slots.

Define long term fairness index F2, considering all the user set {$SU_i$, i=1,2,...N}

$$F_2(L) = \frac{(\sum_{i=1}^{N} K_i(L))^2}{N \cdot (\sum_{i=1}^{N} K_i^2(L))}$$

As for the suboptimal selection algorithm, since we use the priority method to find the order. Those nodes that are in disadvantaged condition will have more chance to be admitted if it has been declined for long. Thus, the lasting fairness will be guaranteed. In the later section, we will prove our conclusion by MATLAB simulation.

### 4.3. Stationary Fairness Quality

In order to find the stationary fairness quality after many rounds, we define

$$F(B)_T = Average_T(F_2(L)) = \sum_{L=1}^{T} F_2(L)/T$$

Here we get the average F2 in T time slots. F is a function of B and we want to research how F and B are related.

## 5. SYSTEM SIMULATION

### 5.1. Simulation Configuration

In this part, the different results by two different algorithms-optimal and suboptimal algorithms under two different goals will be shown and their respective advantages and disadvantages will be illustrated. For our system, we assume the system parameters as $h_i = k * ds_i^{-4}$, $g_i = k * dp_i^{-4}$, here, $ds_i$, $dp_i$ stands for the distance between $SU_i$ and secondary base station, primary base station respectively. We assume k = 21.8. Here, the parameters $ds_i$, $dp_i$ will be generated by uniform random generator in [100, 1000] and [500, 4000] respectively. The SINR requirement for each $SU_i$ is also generated by uniform random generator in [0.05, 0.5]. We set B = 10000, interference Pth = 0.1, upper limit of transmission power for each $SU_i$ Pmax = 100, transmission rate bound Rmin = 4500, Rmax = 100000, system white noise Sigma = 10^-8[8].

### 5.2. Simulation Results

By changing the number of requesting nodes, we get the following admission set number by optimal and suboptimal algorithm.

From figure 5, we see that when requesting number n is small, the result we get by suboptimal algorithm and optimal algorithm are quite near. As n gradually increases, optimal algorithm will get much better admission set number. Here, we didn't give the result when n is very large because the computation complexity of optimal algorithm is quite large. For big one, obtaining the result will cost a long-long time.

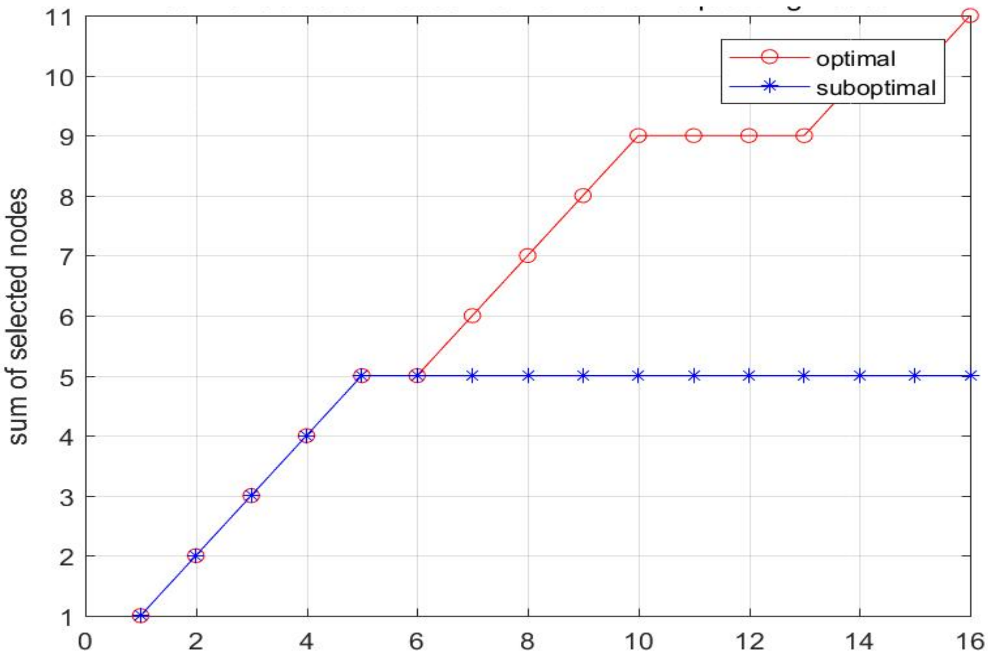

**Figure 5.** Sum of selected nodes VS number requese=ting nodes

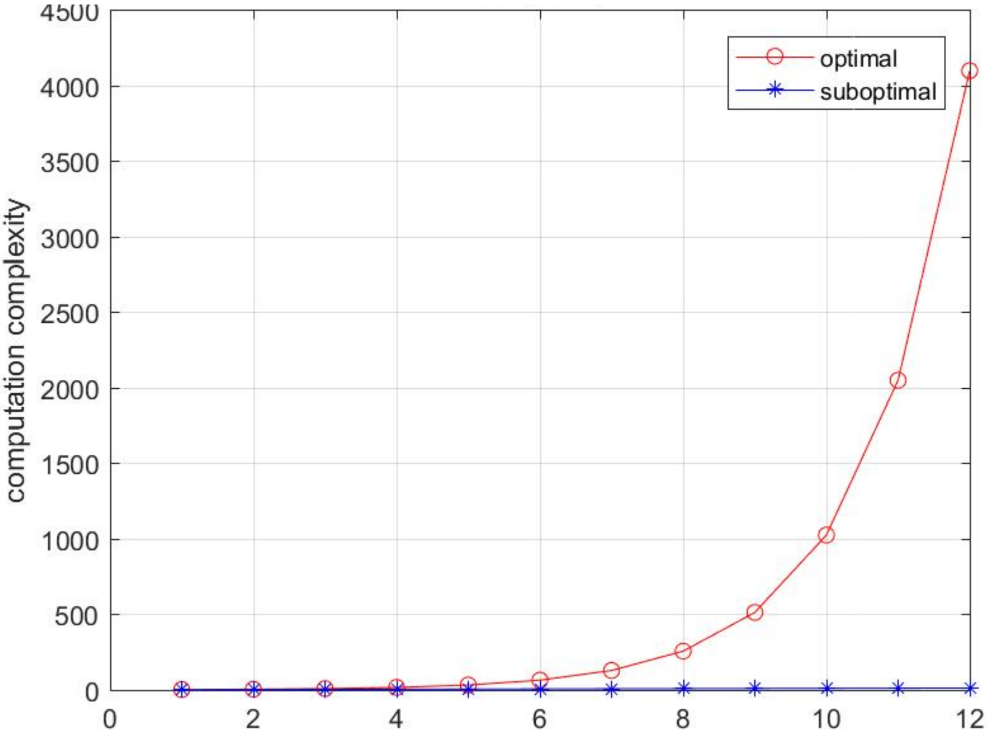

**Figure 6.** Complexity

As figure 6 shows the complexity of two algorithms, optimal and suboptimal algorithms, when n becomes very large, computation complexity for optimal increases very quickly and thus it is not suitable for large network. On the other hand, the suboptimal algorithm will only scan the requesting nodes one time and the computation complexity of it is small.

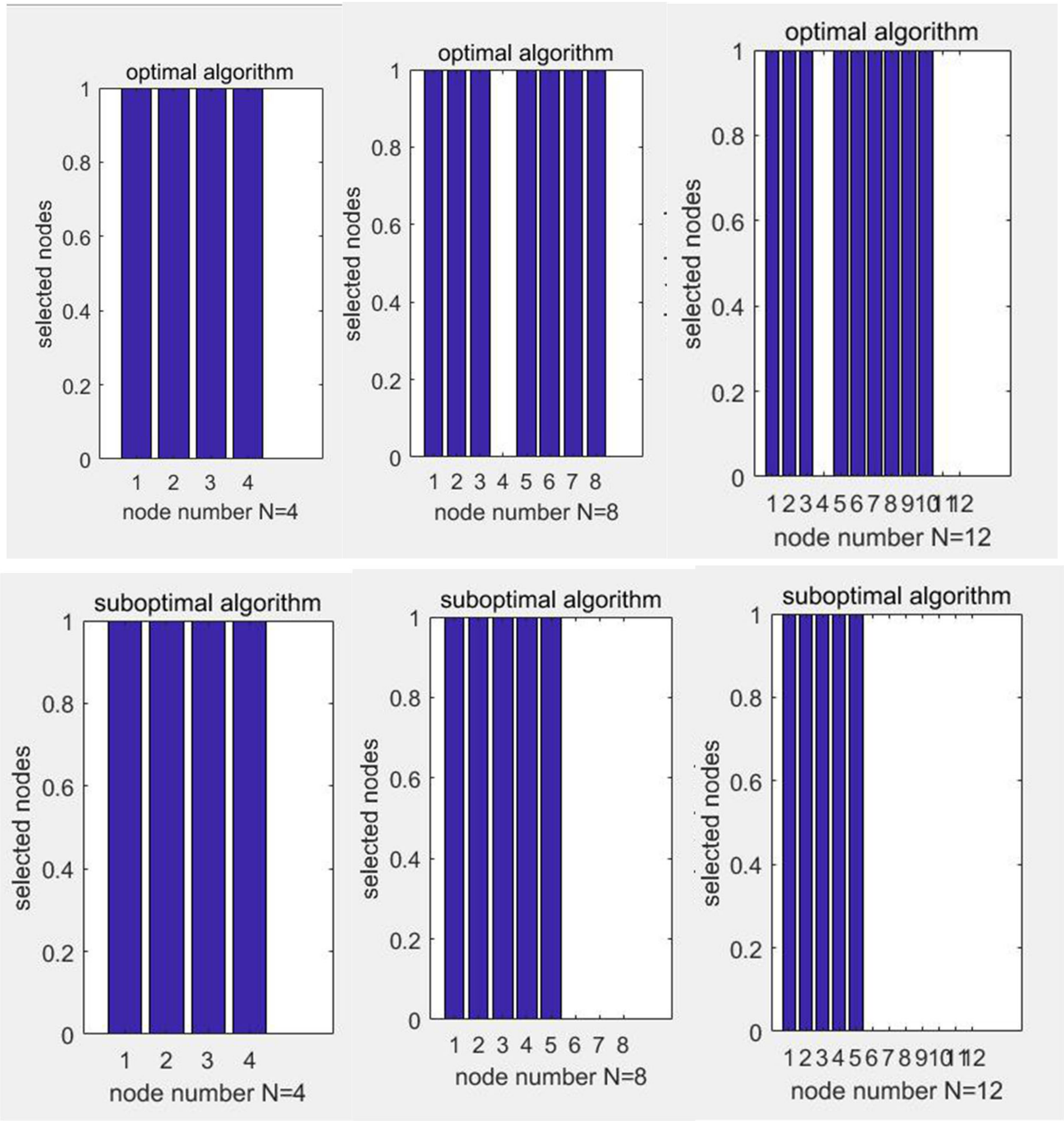

**Figure 7.** Different selection results by optimal and suboptimal algorithms

In figure 7, four different selection results by optimal and suboptimal algorithms were show, requesting node number N = 4, 8, 12, 16. Blue bar stands for the node is admitted while white vacancy stands for no admission. We see that when N is small, the result of suboptimal selection matches this of the optimal algorithm, which accords with the figure 5. As N rise, they become different.After we get the admission set by the first step, there are two algorithms to get the maximum system utility (sum of R), optimal algorithm and suboptimal algorithm. The result of optimal algorithm will be very good, but the computation complexity will be quite large while the computation complexity of suboptimal algorithm is quite small. In the following figure, we will compare the system utility achieved by the two different algorithms [9].

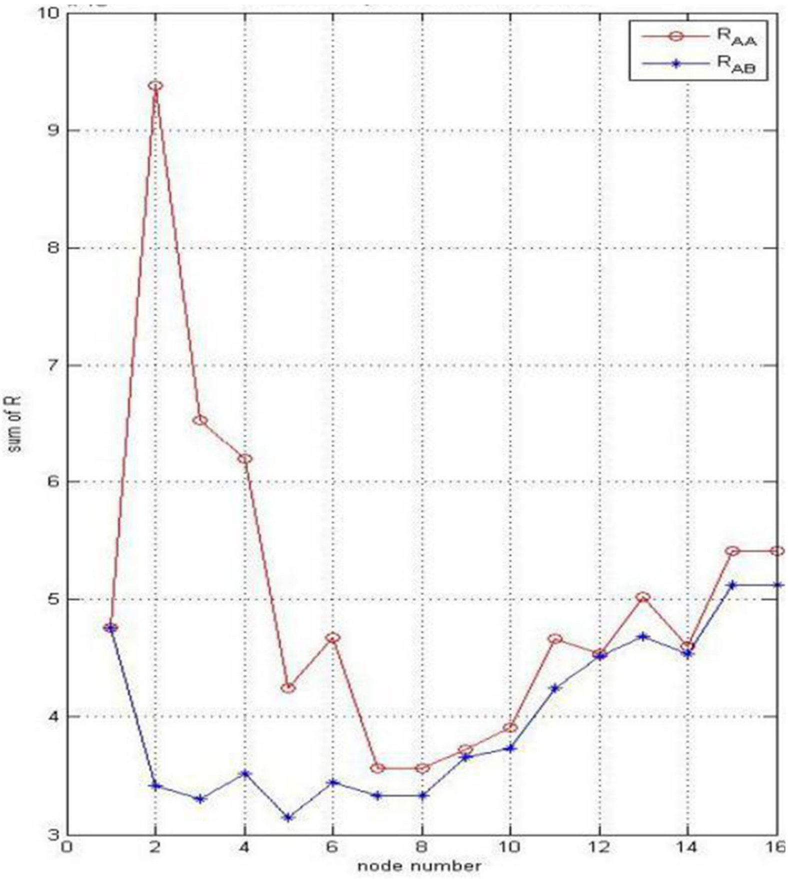

**Figure 8.** (a) Sums of the system transmision rates

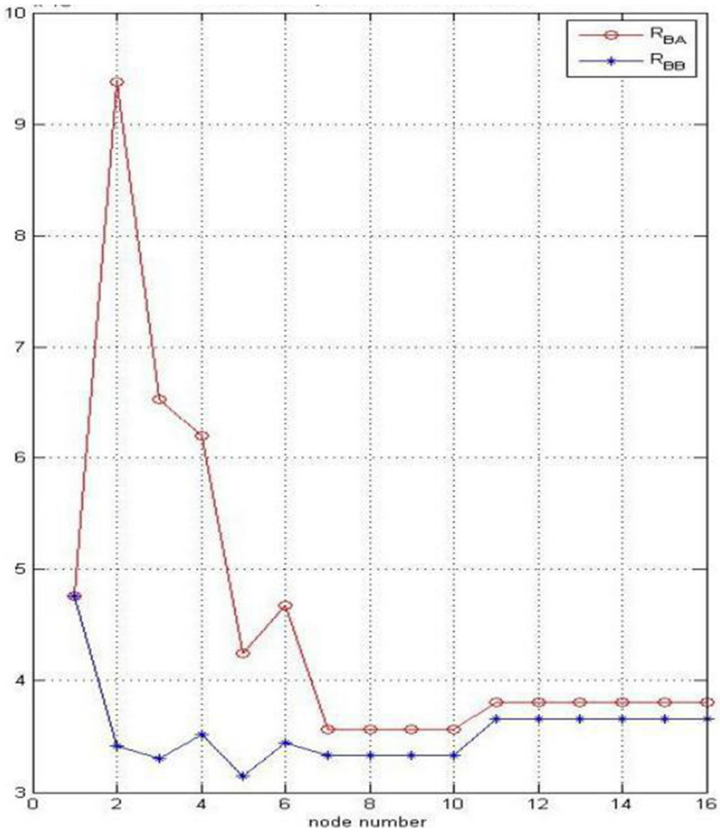

**Figure 8.** (b) Sums of the system transmision rates

In figure 8, RAA stands for the system utility achieved by optimal selection algorithm and optimal maximization algorithm; RAB stands for system utility achieved by optimal selection algorithm and suboptimal maximization algorithm; RBA stands for system utility achieved by suboptimal selection algorithm and optimal maximization algorithm; RBB stands for system utility achieved by suboptimal selection algorithm and suboptimal maximization algorithm.In this figure, we get the different system utility under the condition of different number of requesting nodes.

From the above figure, we see that when the number of requesting nodes increases, the system utility tends to decrease. This is because when the node number increases, the number of admitted users increases, and the interference between them and interference to the primary use will be large. Thus we see that the spectrum underlay sharing strategy is not suitable for large system inherently. On the other hand, we see that when N becomes large, the system utilities achieved by suboptimal and optimal maximization algorithm become quite near[10].

After two different algorithms, we get the final transmission rate of each admitted $SU_i$. In this case, we can evaluate the final fairness quality

$$F_1 = \frac{(\sum_{i=1}^{m} R_i)^2}{m \cdot (\sum_{i=1}^{m} R_i^2)}$$, where i=1,2…m stands for the admitted $SU_i$

The result is as follows

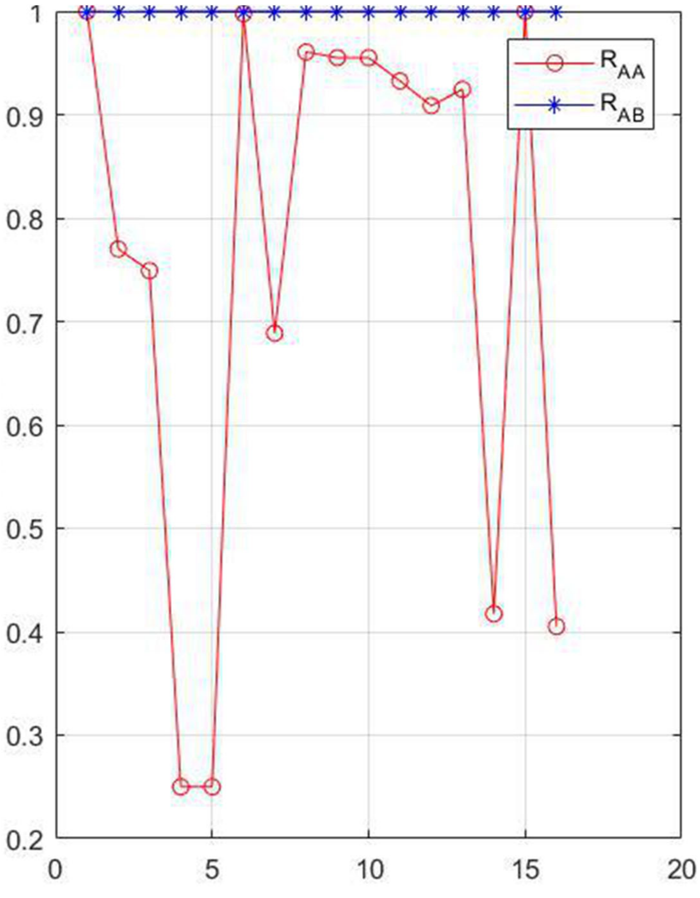

**Figure 9.** (a) Fairness index

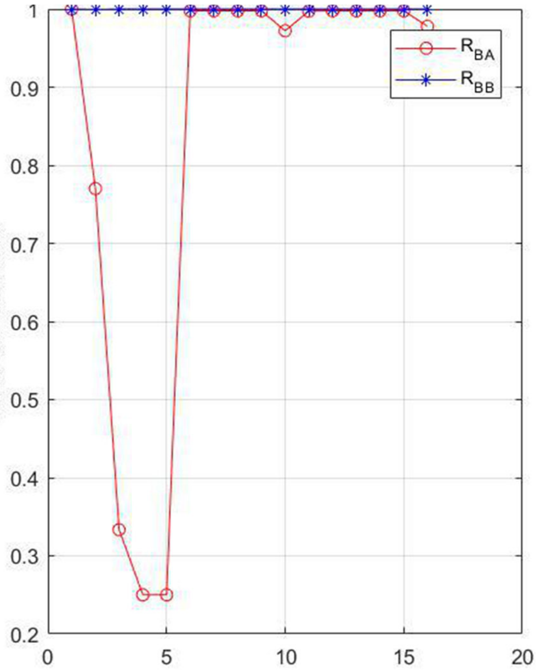

**Figure 9.** (b) Fairness index

As figure 9 shows, the fairness quality achieved by suboptimal maximization algorithm is better than that achieved by optimal maximization algorithm, which proved our conclusion in the previous section. As for the long term fairness quality, we define

$$F_2(L) = \frac{(\sum_{i=1}^{N} K_i(L))^2}{N \cdot (\sum_{i=1}^{N} K_i^{\,2}(L))}$$

$K_i(L)$ represents the admitted possibility in L time slots. The details are listed below.

**Table 1.** $K_i(L)$ stands for the admitted possibility in L time slots.

| Secondary user node serial number | G(Channel gain G) | H(Channel gain H) | Eq(SINR QOS) |
|---|---|---|---|
| 1 | 2.74E-12 | 1.52E-10 | 0.265306 |
| 2 | 1.75E-12 | 1.05E-10 | 0.077473 |
| 3 | 1.54E-13 | 7.30E-09 | 0.321216 |
| 4 | 3.71E-13 | 3.22E-11 | 0.170684 |
| 5 | 2.67E-12 | 3.07E-11 | 0.478313 |
| 6 | 4.73E-13 | 3.40E-10 | 0.349219 |
| 7 | 4.06E-12 | 9.36E-08 | 0.232574 |
| 8 | 1.12E-13 | 2.67E-11 | 0.065328 |
| 9 | 1.99E-11 | 8.54E-09 | 0.373131 |
| 10 | 8.60E-13 | 2.93E-10 | 0.173569 |
| 11 | 1.66E-13 | 2.52E-09 | 0.445203 |
| 12 | 1.60E-11 | 5.79E-11 | 0.061107 |
| 13 | 6.56E-11 | 7.39E-08 | 0.198893 |
| 14 | 4.38E-12 | 1.89E-08 | 0.378058 |

Choose the time slots varies from 1 to 100.

The figure 10 are derived when B = 6500, which shows the relationship between F2 and the time slot-N, we see that when N becomes large, F2 becomes stationary and converge a value near 0.9.

Figure 11 describes the relationship between F2 and time slot-N under the condition B = 10000. It is also shown that the final value becomes stationary at the value of 0.96.

In order to evaluate the fairness quality when N becomes large, we discuss the stationary fairness quality and investigate the relationship between F and B.

$$F(B)_T = Average_T(F_2(L)) = \sum_{L=1}^{T} F_2(L)/T$$

In this expression, we set T = 100.

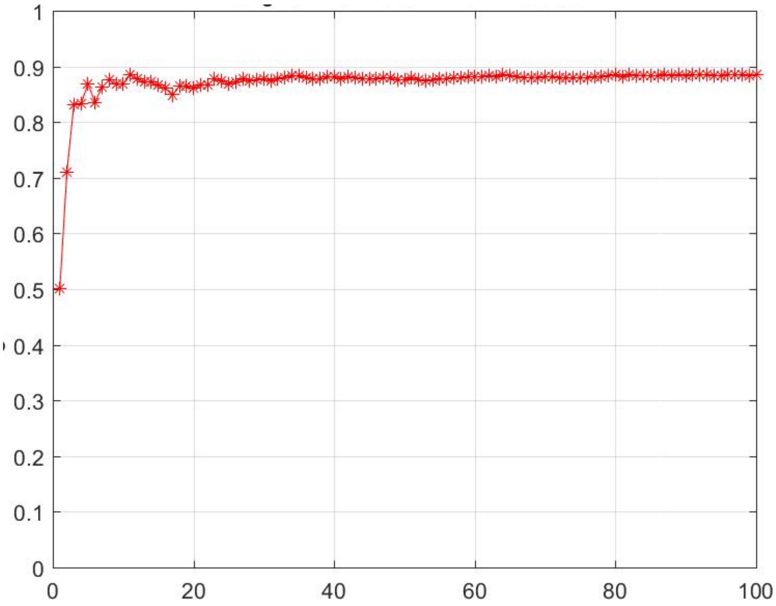

**Figure 10.** Lasting fairness when B=6500

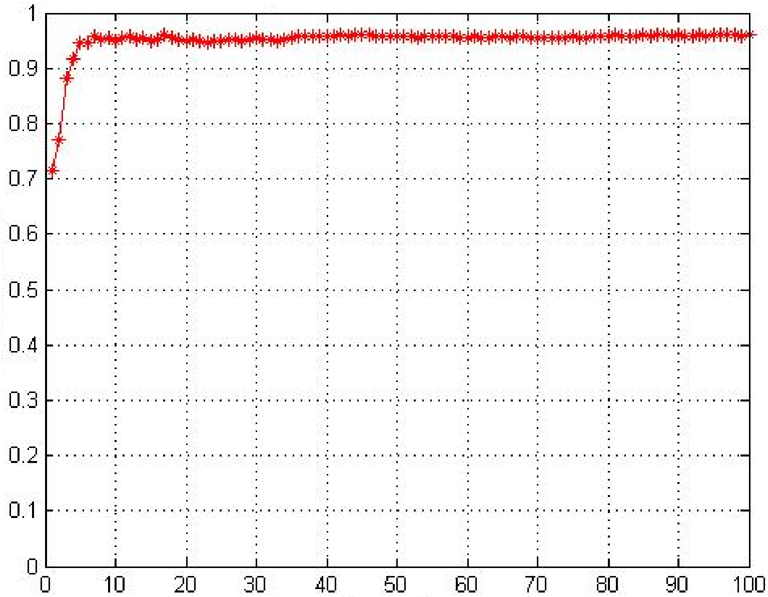

**Figure 11.** Lasting fairnees when B=10000

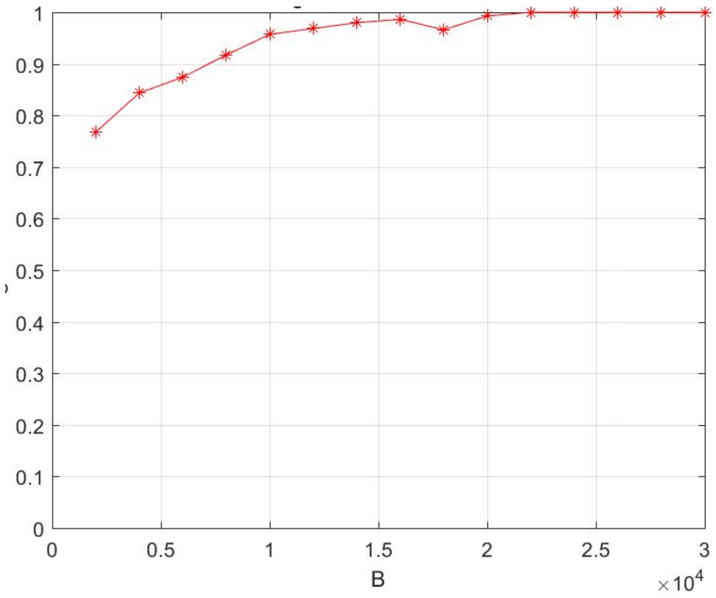

**Figure 12.** Lasting fairness VSB

As figure.11and figure.12 shows, when the system bandwidth increases, the long term fairness quality will gradually become better and finally converges to 1. This is because when B becomes larger, the system will have more capacity to admit secondary users, and thus the possibility of a secondary user being admitted will increase and approach 1. Thus the Fairness Quality will finally turn 1[11].

As for optimal selection algorithm, since it will not consider the historical condition, if the system parameters don't change, then the final selection result keep the same. As a result, this will create some phenomenon of "hunger" as those who are in unfavorable condition will be declined for admission for long. Therefore, suboptimal selection algorithm is comparatively better when computation complexity, adaptive ability, long term fairness quality are considered.

## 6. CONCLUSION

In this paper, an admission control and resource allocation model based on CRN spectrum underlay was discussed. In this model, the secondary users can visit the spectrum band without bother the major. However, the interference threshold of the primary user must be satisfied in order to ensure its QOS. As for each admitted $SU_i$, the QOS in terms of SINR must be guaranteed. Besides, each $SU_i$ has its own limitations such as upper limit of transmission power, transmission rate bound due to its basic physical realities. Our target is to first maximize the number of admitted secondary users, second maximize the total system utility based on the admission result. In this paper, we solve this problem by building a mathematical model. As for the first objective, it is turned into a Mixed Integer Linear Programming (MILP). Two algorithms were proposed to solve it, optimal and suboptimal. The optimal selection algorithm can find the optimal result while the computation complexity is very large and it can not ensure long term fairness quality of the system. The suboptimal algorithm uses heuristic methods and has low computation complexity. And the admission order ensures long term fairness.

## 7. PROSPECTIVE

In this paper, we discuss the situation of one primary user and several secondary users share the spectrum band by spectrum underlay strategy. In order to simplify the model, the interference from the base station was not considered. In advanced discussions, it can be contained into the model. On the other hand, a more complex model can be further researched, a system including many primary users, secondary users, several primary and secondary base stations. Of course, the model will become quite complex and difficult to solve. One short-coming of this paper is that no proof is provided to show that the suboptimal algorithm is the best one. Although we get a suboptimal algorithm and show that the simulation result approaches that of the optimal one while keeping low computation complexity and ensuring Fairness Quality, maybe there are some other more effective suboptimal ones. In our future research project, we may try to do more work in this aspect. Finally, since most resource allocation issues are quite similar. More effective methods can be introduced from other places into network allocation issues.

## REFERENCES

[1] Akyildiz, I., Lee, W., Vuran, M. and Mohanty, S., 2006. NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey. Computer Networks, 50(13), pp.2127-2159.

[2] Le, L. and Hossain, E., 2008. Resource allocation for spectrum underlay in cognitive radio networks. IEEE Transactions on Wireless Communications, 7(12), pp.5306-5315.

[3] Le, L., Hossain, E. and Alfa, T., 2006. Delay Statistics and Throughput Performance for Multi-rate Wireless Networks Under Multiuser Diversity. IEEE Transactions on Wireless Communications, 5(10), pp.3234-3243.

[4] Le, L. and Hossain, E., 2007. Multihop Cellular Networks: Potential Gains, Research Challenges, and a Resource Allocation Framework. IEEE Communications Magazine, 45(9), pp.66-73.

[5] Qing Zhao and Sadler, B., 2007. A Survey of Dynamic Spectrum Access. IEEE Signal Processing Magazine, 24(3), pp.79-89.

[6] Le, L. and Hossain, E., 2008. Cross-layer optimization frameworks for multihop wireless networks using cooperative diversity. IEEE Transactions on Wireless Communications, 7(7), pp.2592-2602.

[7] Andersin, M., Rosberg, Z. and Zander, J., 1996. Gradual removals in cellular PCS with constrained power control and noise. Wireless Networks, 2(1), pp.27-43.

[8] T. Banerjee, C. Ghosh, and D. P. Agrawal, “Wireless sensor based dynamic channel selection in cellular communication by cognitive radio approach,” in Proc. 1st International Conference on Cognitive Radio Oriented Wireless Networks and Communications, June 2006.

[9] Peng, C., Zheng, H. and Zhao, B., 2006. Utilization and fairness in spectrum assignment for opportunistic spectrum access. Mobile Networks and Applications, 11(4), pp.555-576.

[10] Y. Xing, C. N. Mathur, M. A. Haleem, R. Chandramouli,and K.P.Subbalakshmi,“Dynamic spectrum access with QoS and interference temperature constraints,” IEEE Trans. Mobile Comp., vol. 6, no. 4, pp. 423-433,Apr. 2007.

[11] Jafar, S. and Goldsmith, A., 2003. Adaptive multirate CDMA for uplink throughput maximization. IEEE Transactions on Wireless Communications, 2(2), pp.218-228.

[12] Meshkati, F., Poor, H., Schwartz, S. and Mandayam, N., 2005. An Energy-Efficient Approach to Power Control and Receiver Design in Wireless Data Networks. IEEE Transactions on Communications, 53(11), pp.1885-1894.

[13] Qing Zhao and Sadler, B., 2007. A Survey of Dynamic Spectrum Access. IEEE Signal Processing Magazine, 24(3), pp.79-89.