

Adaptive Control of Quadrotor Aircraft Based on Error Saturation Function

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Abstract

The existing quadrotor aircraft control system has the problem of model uncertainties, low control precision and weak robustness. The whole control system adopts double closed loop structure, the inner loop is used to control the attitude, the outer loop is used to stabilize the position. In this paper, in order to reduce the influence on quadrotor flight from different external disturbances, a novel nonlinear robust controller is designed in the quadrotor system. The error saturation function is added to the controller. The simulation results show that the error saturation function is used to effectively suppress the influence of external disturbance.

Keywords

Quadrotor; Adaptive; Error saturation function; Trajectory tracking.

1. INTRODUCTION

Quadrotor aircraft has become an increasingly popular research platform due to its high maneuverability and the ability to change the size of the fuselage according to work requirements [1, 2]. Trajectory tracking is a basic function of quadrotor aircraft for performing various tasks, such as natural disaster surveillance or military surveillance. At present, the research on the control of quadrotor aircraft is a hot topic in the field of control theory. The predecessors have done a lot of research results in aircraft control, such as backstepping control [3, 4], adaptive control [5], fuzzy control [6, 7], neural network control [8], etc. Backstepping is one of the most common control methods. On the basis of the traditional backstepping method, the integral term of tracking error is added to make up for the steady-state error [9]. It is used to control the stable flight of the aircraft, but its performance is slightly poor when the aircraft is disturbed by the outside world.

In this paper, an error saturation function is proposed to suppress the disturbance of the disturbance team system. The combination of error saturation function and integral reverse step method is applied to the trajectory tracking of the aircraft disturbed by the external environment, which can not only reduce the steady-state error, but also improve the anti-interference performance of the aircraft itself, and greatly enhance the robustness of the aircraft.

2. QUADROTOR AIRCRAFT STATE MODEL

The quadrotor aircraft can be regarded as an underactuated rigid body with six degrees of freedom and four inputs. Its thrust is generated by four propellers, which can be controlled by command to make the UAV maintain a certain attitude and follow the required flight path [10]. The classic quadrotor aircraft dynamics model is shown below

$$\left\{ \begin{array}{l} \dot{x} = v_x \\ \dot{v}_x = (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \frac{b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)}{m} \\ \dot{y} = v_y \\ \dot{v}_y = (\cos \phi \sin \theta \cos \psi - \sin \phi \sin \psi) \frac{b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)}{m} \\ \dot{z} = v_z \\ \dot{v}_z = (\cos \phi \cos \theta) \frac{b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)}{m} - g \\ \dot{\phi} = v_\phi \\ \dot{v}_\phi = \dot{\theta} \dot{\psi} \left(\frac{I_y - I_z}{I_x} \right) + \frac{J_r}{I_x} \dot{\theta} (\omega_4 + \omega_2 - \omega_1 - \omega_3) + \frac{bl}{I_x} (\omega_4^2 - \omega_2^2) \\ \dot{\theta} = v_\theta \\ \dot{v}_\theta = \dot{\phi} \dot{\psi} \left(\frac{I_z - I_x}{I_y} \right) - \frac{J_r}{I_y} \dot{\phi} (\omega_4 + \omega_2 - \omega_1 - \omega_3) + \frac{bl}{I_y} (\omega_3^2 - \omega_1^2) \\ \dot{\psi} = v_\psi \\ \dot{v}_\psi = \dot{\phi} \dot{\theta} \left(\frac{I_x - I_y}{I_z} \right) + \frac{d}{I_z} (\omega_4^2 + \omega_2^2 - \omega_1^2 - \omega_3^2) \end{array} \right.$$

Where, I_x, I_y, I_z is the inertia matrix, which is an important parameter of external force and torque in aerodynamics of aircraft. b is the drag coefficient; d is the lift coefficient; m is the quality of UAV; g is gravitational acceleration; l is the length of the arm; J_r is rotor inertia; ϕ is the roll angle, θ is the pitch angle, ψ is the yaw angle; x, y is the coordinate of the center of mass of the UAV on the fixed plane, z is the height; $\omega_i (i = 1, 2, 3, 4)$ is the four rotational angular velocities of the quadrotor UAV, and the relation with the corresponding four input items U_i can be expressed as

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ b(\omega_4^2 - \omega_2^2) \\ b(\omega_3^2 - \omega_1^2) \\ d(\omega_4^2 + \omega_2^2 - \omega_1^2 - \omega_3^2) \end{bmatrix}$$

The quadrotor UAV would be affected by some unpredictable external disturbance in actual application. Let $X = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$, The equation of state of the system is $\dot{X} = [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5, \dot{x}_6, \dot{x}_7, \dot{x}_8, \dot{x}_9, \dot{x}_{10}, \dot{x}_{11}, \dot{x}_{12}]^T$, With the external disturbance being taken into consideration, the nonlinear dynamic Equation should be rewritten as

$$\left\{ \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{u_x U_1}{m} + \delta_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{u_y U_1}{m} + \delta_4 \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= \frac{\cos x_7 \cos x_9 U_1}{m} - g + \delta_6 \\ \dot{x}_7 &= x_8 \\ \dot{x}_8 &= c_1 x_{10} x_{12} + c_2 \omega_d x_{10} + a_1 U_2 + \delta_8 \\ \dot{x}_9 &= x_{10} \\ \dot{x}_{10} &= c_3 x_8 x_{12} + c_4 \omega_d x_8 + a_2 U_3 + \delta_{10} \\ \dot{x}_{11} &= x_{12} \\ \dot{x}_{12} &= c_5 x_8 x_{10} + a_3 U_4 + \delta_{12} \end{aligned} \right. \quad \text{and,} \quad \left\{ \begin{aligned} c_1 &= \frac{I_y - I_z}{I_x} \\ c_2 &= \frac{J_r}{I_x} \\ c_3 &= \frac{I_z - I_x}{I_y} \\ c_4 &= -\frac{J_r}{I_y} \\ c_5 &= \frac{I_x - I_y}{I_z} \\ \omega_d &= \omega_4 + \omega_2 \\ &\quad -\omega_1 - \omega_3 \end{aligned} \right. , \quad \left\{ \begin{aligned} a_1 &= \frac{l}{I_x} \\ a_2 &= \frac{l}{I_y} \\ a_3 &= \frac{1}{I_z} \\ u_x &= \cos \phi \sin \theta \cos \psi \\ &\quad + \sin \phi \sin \psi \\ u_y &= \cos \phi \sin \theta \cos \psi \\ &\quad - \sin \phi \sin \psi \end{aligned} \right.$$

Where $\delta_2, \delta_4, \delta_6, \delta_8, \delta_{10}, \delta_{12}$ is defined as the external disturbance term, and the bound of the disturbance is $|\delta_i| \leq \varrho (i = 2,4,6,8,10,12)$, while ϱ is a given positive constant.

3. SATURATION INTEGRAL BACKSTEPPING CONTROL

The desired trajectory is $X_d = [x_{1d}, x_{3d}, x_{5d}, x_{7d}, x_{9d}, x_{11d}]^T = [x_d, y_d, z_d, \phi_d, \theta_d, \psi_d]^T$. In this paper, we give the control method and the proof process of the altitude direction, and the other directions are similar to the altitude direction. Take the altitude direction control input U_1 for example, the design method is given step-by-step as follows:

Step 1. The equation of state in the altitude direction

$$\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{\cos x_7 \cos x_9 U_1}{m} - g + \delta_6 \end{cases} \tag{1}$$

Introduce the altitude tracking error as

$$e_5 = x_{5d} - x_5 \tag{2}$$

A new error integral variable is defined as

$$p_5 = \int_0^t e_5(\tau) d\tau \tag{3}$$

The first Lyapunov function is chosen as

$$V_5(e_5, p_5) = \frac{1}{2} e_5^2 + \frac{1}{2} \lambda_5 p_5^2, (\lambda_5 > 0) \tag{4}$$

The derivative of V_5 with respect to time is

$$\dot{V}_5(e_5, p_5) = e_5 \dot{e}_5 + \lambda_5 p_5 \dot{p}_5 = e_5 (\dot{x}_{5d} - \dot{x}_6 + \lambda_5 p_5) \tag{5}$$

In order to make the system stable, a virtual control quantity α_6 is introduced to instead of the state variable x_6

$$\alpha_6 = \dot{x}_{5d} + \lambda_5 p_5 + \beta_5 e_5, (\beta_5 > 0) \quad (6)$$

Substituting x_6 by Equation (6), then the Equation (5) can be rewritten as

$$\dot{V}_5(e_5, p_5) = -\beta_5 e_5^2 \leq 0 \quad (7)$$

Step 2. The deviation of α_6 from the desired value x_6 can be defined as the second-order tracking error e_6 , the expression is shown below

$$e_6 = \alpha_6 - x_6 = \dot{x}_{5d} + \lambda_5 p_5 + \beta_5 e_5 - x_6 \quad (8)$$

The derivative of e_6 can be represented as

$$\dot{e}_6 = \dot{\alpha}_6 - \dot{x}_6 = \ddot{x}_{5d} + \lambda_5 \dot{p}_5 + \beta_5 \dot{e}_5 - \dot{x}_6 \quad (9)$$

Where $\dot{p}_5 = e_5$, $\dot{e}_5 = e_6 - \lambda_5 p_5 - \beta_5 e_5$, $\dot{x}_6 = \frac{\cos x_7 \cos x_9 U_1}{m} - g$ in Equation (9).

Chosen the second positive definite Lyapunov function as

$$V_6(e_5, p_5, e_6) = \frac{1}{2} e_5^2 + \frac{1}{2} \lambda_5 p_5^2 + \frac{1}{2} e_6^2 \quad (10)$$

The derivative of V_6 with respect to time is

$$\dot{V}_6(e_5, p_5, e_6) = e_5 \dot{e}_5 + \lambda_5 p_5 \dot{p}_5 + e_6 \dot{e}_6 = -\beta_5 e_5^2 + e_6(e_5 + \dot{e}_6) \quad (11)$$

Step 3. In order to make the system stable, the following inequality should be hold

$$e_6(e_5 + \dot{e}_6) = -\beta_6 e_6^2, (\beta_6 > 0) \quad (12)$$

Substituting the differential term for the second-order tracking error e_6 in the Equation (11) to give the expression for the controller U_1

$$U_1 = \frac{m[(1+\lambda_5-\beta_5^2)e_5+(\beta_5+\beta_6)e_6-\beta_5\lambda_5 p_5+\ddot{x}_{5d}+g-\delta_6]}{\cos x_7 \cos x_9} \quad (13)$$

Finally, substituting the Equation (13) into (11), the derivative of V_6 with respect to time is

$$\dot{V}_6(e_5, p_5, e_6) = -\beta_5 e_5^2 - \beta_6 e_6^2 \leq 0 \quad (14)$$

According to Lyapunov stability theory, the nonlinear system of quadrotor UAV with Equation (1) is asymptotically stabilized using the controller (13).

Step 4. In theory, the controller (13) can not be used because the external disturbance term δ_6 is unknown, but the influence of disturbance on the system trajectory can be represented by the error. The error sign function ρ_6 is used to reverse suppress the interference and realize the adaptive control of the system.

$$\rho_6 = k_6 \text{sign}\left(\frac{e_6}{\mu_6}\right), (k_6 > 0, \mu_6 > 0) \tag{15}$$

Since the sign function is not Lipschitz's at zero, chattering phenomenon may occur in practical application, resulting in poor stability of the system. In this paper, we consider using saturation function σ_6 to improve the performance of sign function ρ_6 near zero.

$$\sigma_6 = k_6 \text{sat}\left(\frac{e_6}{\mu_6}\right) \tag{16}$$

The saturation function can be written as

$$\text{sat}\left(\frac{e_6}{\mu_6}\right) = \begin{cases} 1, & \frac{e_6}{\mu_6} > 1 \\ -1, & \frac{e_6}{\mu_6} < -1 \\ \frac{e_6}{\mu_6}, & \left|\frac{e_6}{\mu_6}\right| \leq 1 \end{cases} \tag{17}$$

In the controller (13), we using the error saturation function σ_6 to substitute the external disturbance term δ_6 , the altitude controller (13) can be rewritten as

$$U_1 = \frac{m[(1 + \lambda_5 - \beta_5^2)e_5 + (\beta_5 + \beta_6)e_6 - \beta_5\lambda_5 p_5 + \ddot{x}_{5d} + g + \sigma_6]}{\cos x_7 \cos x_9} \tag{18}$$

Theorem 3.1: If the altitude tracking error is controlled by the controller (18), the solution to the nonlinear system of quadrotor aircraft in the altitude direction will be uniformly ultimately bounded.

Proof: substituting the controller (18) into (11), the derivative of V_6 can be rewritten as

$$\dot{V}_6(e_5, p_5, e_6) = -\beta_5 e_5^2 - \beta_6 e_6^2 - e_6(\sigma_6 + \delta_6) \tag{19}$$

Two inequalities below are used here,

$$\begin{cases} -e_6 \sigma_6 \leq 0 \\ -e_6 \delta_6 \leq \frac{1}{2\gamma_6} e_6^2 + \frac{\gamma_6}{2} \delta_6^2 \end{cases} \tag{20}$$

Where γ_6 is a positive constant, and substitute (20) into Equation (19) can be obtained as

$$\dot{V}_6(e_5, p_5, e_6) \leq -2\beta_5 \frac{e_5^2}{2} - 2\left(\beta_6 - \frac{1}{2\gamma_6}\right) \frac{e_6^2}{2} + \frac{\gamma_6}{2} \delta_6^2 \tag{21}$$

Let $\xi = \min\left\{2\beta_5, 2\left(\beta_6 - \frac{1}{2\gamma_6}\right)\right\}$, $\varepsilon = \frac{\gamma_6}{2} \delta_6^2 + \frac{1}{2} \xi \lambda_5 p_5^2$, where $\beta_6 > \frac{1}{2\gamma_6}$ and the bound of the parameter ε is η , the following inequality is obtained

$$\begin{cases} \xi = \min\left\{2\beta_5, 2\left(\beta_6 - \frac{1}{2\gamma_6}\right)\right\} > 0 \\ \varepsilon = \frac{\gamma_6}{2} \delta_6^2 + \frac{1}{2} \xi \lambda_5 p_5^2 \leq \eta \end{cases} \tag{22}$$

Substituting (22) into (21), \dot{V}_6 can be rewritten as

$$\dot{V}_6 \leq -\xi \left(\frac{e_5^2}{2} + \frac{e_6^2}{2} + \frac{1}{2} \lambda_5 p_5^2 \right) + \frac{1}{2} \xi \lambda_5 p_5^2 + \frac{\gamma_6}{2} \delta_6^2 \leq -\xi V_6 + \eta \tag{23}$$

Which implies [11]

$$V_6(t) \leq V_6(0)e^{-\xi t} + \frac{\eta}{\xi} \tag{24}$$

According to the uniformly ultimately bounded theorem of the nonvanishing perturbation [11,12], the altitude tracking errors e_5 and the second-order tracking error e_6 are uniformly ultimately bounded.

Step 5. The designs of the other control inputs are similar to U_1 . Altogether, the control inputs U_1, U_2, U_3, U_4 and the virtual control inputs u_x and u_y for the quadrotor aircraft nonlinear system can be formulated as follows:

$$\begin{cases} U_1 = \frac{m[(1 + \lambda_5 - \beta_5^2)e_5 + (\beta_5 + \beta_6)e_6 - \beta_5 \lambda_5 p_5 + \ddot{x}_{5d} + g + \sigma_6]}{\cos x_7 \cos x_9} \\ U_2 = \frac{(1 + \lambda_7 - \beta_7^2)e_7 + (\beta_7 + \beta_8)e_8 - \beta_7 \lambda_7 p_7 + \ddot{x}_{7d} - c_1 x_{10} x_{12} - c_2 \omega_d x_{10} + \sigma_8}{a_1} \\ U_3 = \frac{(1 + \lambda_9 - \beta_9^2)e_9 + (\beta_9 + \beta_{10})e_{10} - \beta_9 \lambda_9 p_9 + \ddot{x}_{9d} - c_3 x_8 x_{12} - c_4 \omega_d x_8 + \sigma_{10}}{a_2} \\ U_4 = \frac{(1 + \lambda_{11} - \beta_{11}^2)e_{11} + (\beta_{11} + \beta_{12})e_{12} - \beta_{11} \lambda_{11} p_{11} + \ddot{x}_{11d} - c_5 x_8 x_{10} + \sigma_{12}}{a_3} \\ u_x = \frac{m}{U_1} [(1 + \lambda_1 - \beta_1^2)e_1 + (\beta_1 + \beta_2)e_2 - \beta_1 \lambda_1 p_1 + \ddot{x}_{1d} + \sigma_2] \\ u_y = \frac{m}{U_1} [(1 + \lambda_3 - \beta_3^2)e_3 + (\beta_3 + \beta_4)e_4 - \beta_3 \lambda_3 p_3 + \ddot{x}_{3d} + \sigma_4] \end{cases} \tag{25}$$

Remark 3.1: The design process of control method and uniformly ultimately boundedness proving process of the other controllers U_2, U_3, U_4 and virtual control inputs u_x, u_y are

similar to those of U_1 . By using virtual control inputs u_x and u_y which were derived from Equation (25), the desired value of roll angle ϕ_d and pitch angle θ_d can be obtained by the inverse solution module.

$$\begin{cases} u_x = \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ u_y = \cos \phi \sin \theta \cos \psi - \sin \phi \sin \psi \end{cases} \Rightarrow \begin{cases} \phi_d = x_{7d} = \arcsin(u_x \sin \psi - u_y \cos \psi) \\ \theta_d = x_{9d} = \arcsin\left(\frac{u_x - \sin \phi_d \sin \psi}{\cos \phi_d \cos \psi}\right) \end{cases}$$

4. SIMULATION RESULTS

In this section, in order to validate the designed controllers, simulation in trajectory tracking is carried out by MATLAB, with periodic disturbance taken into consideration. The difference of the general integral backstepping method (IB) [13,14] and the adaptive control method (AIB) in this paper are compared.

The model parameters of the quadrotor used in simulation are modified as: $m = 1.0\text{kg}$, $g = 9.80\text{m/s}^2$, $l = 0.2\text{m}$, $b = 3 \times 10^{-5}\text{Ns}^2$, $d = 6 \times 10^{-7} \text{ Nms}^2$, $I_x = I_y = 5 \times 10^{-3}\text{kgm}^2$, $I_z = 0.01\text{kgm}^2$, $J_r = 2 \times 10^{-5}\text{kgm}^2$. The control parameters are chosen as $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 5$, $\beta_5 = 3$, $\beta_6 = 2$, $\beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = \beta_{12} = 5$; $\lambda_1 = 2$, $\lambda_3 = 3$, $\lambda_5 = 3$, $\lambda_7 = \lambda_9 = \lambda_{11} = 2$; $k_2 = k_4 = k_6 = 6$, $k_8 = k_{10} = k_{12} = 2$; $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = 0.1$.

The desired spiral trajectory of the quadrotor aircraft is shown below

$$\begin{cases} x_d = 2 \sin(2\pi t/5) \\ y_d = 2 \cos(2\pi t/5) \\ z_d = t/2 \\ \psi_d = 0.1 \end{cases}$$

The external disturbances in the six directions are respectively $\delta_2 = \delta_4 = \delta_6 = 3 \times \sin(t)$, $\delta_8 = \delta_{10} = \delta_{12} = \sin(t)$. Simulation result of trajectory tracking by using general integral backstepping method (IB) is shown in Figure 1, and the simulation result of trajectory tracking by using the adaptive control method (AIB) is shown in Figure 2. The simulation results of trajectory tracking show that adaptive control method (AIB) is better than the integral backstepping method (IB). The simulation result in Figure 1 shows that it is hard to track the trajectory by using general integral backstepping method (IB). However, the simulation result in Figure 2 shows that the quadrotor can track the desired trajectory very well. Therefore, the control method in this paper is better than the previous results [15].

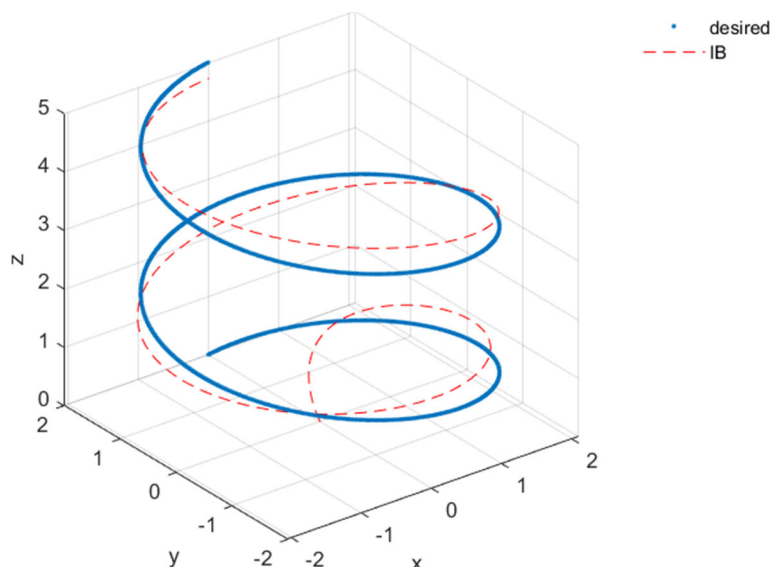


Figure 1. Integral Backstepping (IB)

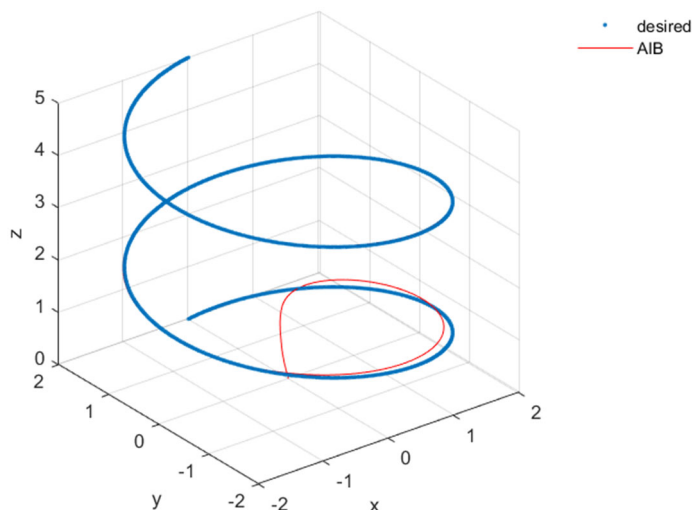


Figure 2. Adaptive Control (AIB)

The tracking errors in three directions (x, y, z) are shown in Figure 3. The horizontal axis is time, and the vertical axis is the error of three directions. It is easy to see that the tracking error x_e and z_e by using adaptive control method (AIB) has smaller tracking error than that by using integral backstepping method (IB).

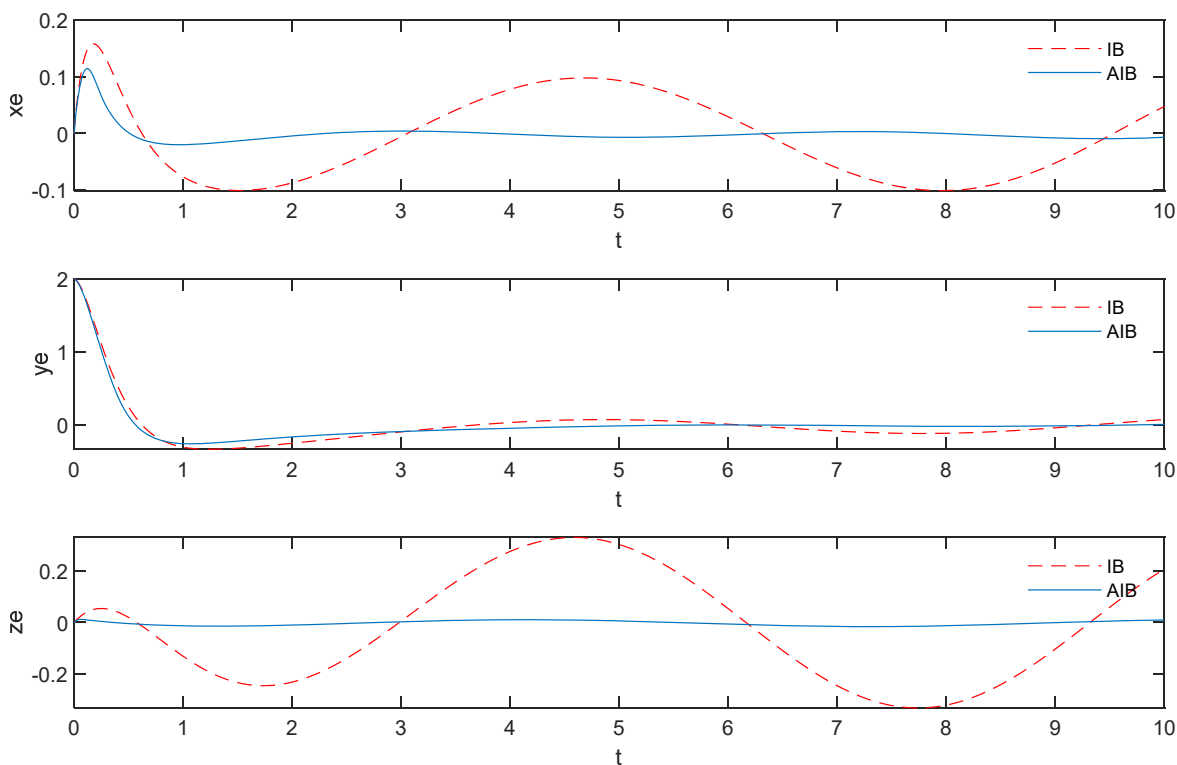


Figure 3. Tracking Errors

5. CONCLUSIONS

In order to reduce the effect from external disturbance on quadrotor aircraft which is a unstable nonlinear system in actual application, an adaptive controller is designed in this paper. The simulation results (MATLAB) show that this method has good tracking effect, short convergence time, and can effectively reduce the effect under external disturbance conditions.

In the follow-up work, the control algorithm in this paper will be tested in real flight to further test the optimization performance of the algorithm in the real flight process.

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