World Scientific Research Journal Volume 7 Issue 4, 2021
ISSN: 2472-3703 DOI: 10.6911/WSRJ.202104_7(4).0034

Pricing of Vulnerable Power Exchange Options with Stochastic Default Barriers

Mingyun Sun

College of Economics, Jinan University, Guangzhou 510632, P. R. China

Abstract

This paper studies the pricing of vulnerable power exchange options with stochastic default barriers. Specifically, we assume that the two underlying assets and the option writer's assets are driven by three related geometric Brownian motions. Assuming that the counterparty default barrier is random, that is, driven by geometric Brownian motion, and is related to the two underlying assets and the option writer's assets, we use the method of measurement transformation to derive the vulnerable power exchange option with stochastic default barriers Pricing formula.

Keywords

Stochastic default barriers; Measurement transformation; Vulnerable power exchange options.

1. INTRODUCTION

Over-the-counter (OTC) contract holders are susceptible to the risk of counterparty default, that is, a counterparty fails to make payments in accordance with the terms of the contract. With the tremendous development of the over-the-counter derivatives market in the past 20 years and after the 2007 financial crisis, participants in over-the-counter financial derivatives transactions are increasingly concerned about the risk of counterparty default. Therefore, when pricing credit-sensitive OTC contracts (such as credit default swaps (CDS), forward contracts and options), the issue of credit exposure must be taken seriously.

For option pricing with credit risk, this pioneering work was done by Johnson and Stulz [1] (1987). They assumed that the option is the only liability of the counterparty, and considered the correlation between the underlying asset and the counterparty's assets, and named the option with credit risk as vulnerable option. Subsequently, Jarrow and Turnbull [2] (1995) discussed the pricing of vulnerable options under the assumption that the underlying asset and the counterparty's default strength are independent. Subsequently, many researchers studied the pricing of vulnerable options. Many researchers have studied the pricing of vulnerable options. Klein [3] (1996) obtained the vulnerable option pricing formula under the assumption that not only the correlation between the underlying asset of the option and the credit risk of the counterparty is allowed, but also the option author has other fixed liabilities. Klein [4] (2001) obtained the vulnerable option pricing formula under the assumption that the default barriers not only includes fixed liabilities but also brings income. Chang [5] (2006) discussed the pricing formula of American vulnerable options based on the two-point Geske and Johnson method. ShuIng [6] (2011) constructed a general binomial pyramid algorithm for vulnerable option pricing based on Rubinstein's method under the premise of random default obstacles. Yang [7] (2014) considered the vulnerable option pricing problem in which the underlying asset follows the stochastic volatility model. Jeon [8] (2019) used Mellin transformation to solve the partial differential equations for the pricing of vulnerable options with early credit risk.

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202104_7(4).0034

The power exchange option is the promotion of the power option proposed by Tompkins [9] (2000) and the Fischer-Margrabe exchange option proposed by Fischer and Margrabe [10] (1978). Blenman [11] (2005) studied power exchange options and solved the price problem of European power exchange options under the assumption that the underlying assets are controlled by geometric Brownian motion. Wang [12] (2016) extended Blenman's framework to exchange options by including related jump risks. Wang [13] (2017) proposed a vulnerable power exchange option pricing model that simultaneously considers counterparty default risk and jump risk. However, we find that the pricing of vulnerable options is affected by the underlying asset value model, volatility model, and default barriers. Therefore, this article discusses the pricing of vulnerable power exchange options with random default barriers, and fills the gaps in the research on vulnerable power exchange options.

The remaining chapters of this article are organized as follows. Section 2 outlines the proposed model, including two underlying assets, option writers, and a value-driven model of stochastic default barriers. Section 3 selects appropriate equivalent probability measures to derive power exchange options with counterparty risk Pricing formula. Section 4 gives conclusions and prospects.

2. CONSTRUCTION MODEL

In this section, we will discuss the pricing of exchange options with counterparties. The value of the underlying assets and the total assets of the option author should be driven by stochastic processes. We start with a limited time frame $\mathrm{T} > 0$. Assuming that the filtered probability space$(\Omega, \mathcal{F}, \mathrm{P})$ simulates the uncertainty in the economy, E represents the expectation of the risk neutral measure P. Refer to the power exchange option researched by Blenman [11] (2005). The power exchange option is a European option that exchanges the power value $S_1^{\beta_1}$ of one asset for the power value $S_2^{\beta_2}$ of another asset. It can be understood as the promotion of power options and Fischer-Margrabe exchange options. Here β_1 and β_2 are normal numbers. When the option holder chooses to exercise the option, if and only if the option brings him or her a positive return. This shows that the return of the T-power exchange option at expiry has the following form

$$\left[S_1^{\beta_1}(T) - S_2^{\beta_2}(T)\right]^+$$

Hypothesis 1: Under the risk-neutral measure P, assume that the price dynamics of the two types of basic assets $S_i(t)(i = 1,2)$ are driven by the following formula:

$$S_i(t) = S_i(0)exp\left\{\left(r - k_i - \frac{1}{2}\sigma_i^2\right)t + \sigma_i W_i(t)\right\}$$

Where r is the constant risk-free interest rate, and $\sigma_i(i = 1,2)$ represents the volatility of the corresponding asset.

Hypothesis 2: Under the risk-neutral measure P, the asset value of the option writer is driven by the following formula:

$$V(t) = V(0)exp\left\{\left(r - k_3 - \frac{1}{2}\sigma_3^2\right)t + \sigma_3 W_3(t)\right\}$$

Where, σ_3 represents the instantaneous volatility of the option writer's assets.

Hypothesis 3: Under the risk-neutral measure P, the value of the option writer's corporate liabilities is driven by the following formula:

$$D(t) = D(0)exp\left\{\left(r - k_4 - \frac{1}{2}\sigma_4^2\right)t + \sigma_4 W_4(t)\right\}$$

Hypothesis 4: Assumption $\left(W_1(t), W_2(t), W_3(t), W_4(t)\right)$ is a four-dimensional Brownian motion with the following dependencies:

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$\mathrm{E}\left[dW_i(t)dW_j(t)\right] = \rho_{ij}dt, (i,j = 1,2,3,4)$,where $\rho_{ij} = 1$, $\rho_{ij} = \rho_{ji}$ and $|\rho_{ii}| = 1$.

Hypothesis 5: This article discusses situations in which option holders may suffer losses when their counterparties default. We assume that the default situation is: as long as the option writer's asset value $V(T)$ is less than the company's liability value $D(T)$ at the option delivery time T, a default event will be triggered. $D(t)$ at T is roughly equal to the amount of outstanding claims $D(t)$, but may be less than $D(t)$, because even if $V(t)$ is less than $D(t)$, the counterparty is still operating. In the event of credit loss, the recovery of the nominal claim is written down as $\frac{(1-\omega)V_T}{D_T}$, which includes the cost of financial distress or bankruptcy. The ratio $\frac{V_T}{D_T}$ represents the value of assets that the option author can allocate to all creditors, expressed as a proportion of the total debt. Therefore, the profit function of the vulnerable power exchange option at the delivery time T is

$$\left[S_1^{\beta_1}(T) - S_2^{\beta_2}(T)\right]^+ \left\{I_{\{V(T)\geq D(T)\}} + \frac{(1-\omega)V_T}{D_T} I_{\{V(T)<D(T)\}}\right\}$$

Where, $I_{\{\cdot\}}$ represents indicative function.

3. PRICING OF VULNERABLE POWER EXCHANGE OPTIONS

In this section, we use the method of measurement transformation to give the pricing formula of vulnerable power exchange options. According to the no-arbitrage principle, under the risk-neutral measure Q, the initial price of a vulnerable power exchange option with counterparty default risk is given as

$$P(0) = e^{-rt}E_P\left[\left[S_1^{\beta_1}(T) - S_2^{\beta_2}(T)\right]^+ \left\{I_{\{V(T)\geq D(T)\}} + \frac{(1-\omega)V_T}{D_T} I_{\{V(T)<D(T)\}}\right\}\right]$$

$$= e^{-rt}E_P\left[\left[S_1^{\beta_1}(T) - S_2^{\beta_2}(T)\right] I_{\left\{V(T)\geq D(T), S_1^{\beta_1}(T)\geq S_2^{\beta_2}(T)\right\}}\right]$$

$$+ e^{-rt}E_P\left[\frac{(1-\omega)V_T}{D_T}\left[S_1^{\beta_1}(T) - S_2^{\beta_2}(T)\right] I_{\left\{V(T)<D(T), S_1^{\beta_1}(T)\geq S_2^{\beta_2}(T)\right\}}\right]$$

According to the previous research literature, our goal is to obtain a pricing formula with a cumulative normal density function. Through careful observation of the expression $P(0)$, we find that since $P(0)$ involves a four-dimensional complex normal density integral, if you use mathematical knowledge to directly calculate it, it seems that $P(0)$ is difficult to get an elegant formula. Therefore, we use the technique of measure transformation to simplify the calculation of $P(0)$. Next we introduced the measurement changes. To this end, we define an equivalent probability measure Q of the probability measure P, where τ is the Radon-Nikodym derivative of the measure transformation from P to Q, as follows:

$$\tau = \frac{d\mathrm{Q}}{d\mathrm{P}} = \frac{S_2^{\beta_2}(T)}{E_P\left[S_2^{\beta_2}(T)\right]} = exp\left\{-\frac{1}{2}\sigma_2^2\beta_2^2 T + \beta_2\sigma_2 W_2(t)\right\}$$

Under the probability measure Q defined in Radon-Nikodym derivative $\frac{d\mathrm{Q}}{d\mathrm{P}}$, according to Girsanov's theorem, we have the following results:

$$\widetilde{W}_1(t) = W_1(t) - \rho_{12}\beta_2\sigma_2\mathrm{t}$$

$$\widetilde{W}_2(t) = W_2(t) - \beta_2\sigma_2\mathrm{t}$$

$$\widetilde{W}_3(t) = W_3(t) - \rho_{23}\beta_2\sigma_2\mathrm{t}$$

$$\widetilde{W}_4(t) = W_4(t) - \rho_{24}\beta_2\sigma_2\mathrm{t}$$

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Where $\left(\widetilde{W}_1(t), \widetilde{W}_2(t), \widetilde{W}_3(t), \widetilde{W}_4(t)\right)$is the standard Brownian motion under the equivalent probability measure Q, and the covariance matrix of $\left(\widetilde{W}_1(t), \widetilde{W}_2(t), \widetilde{W}_3(t), \widetilde{W}_4(t)\right)$ is the same as the covariance matrix of $\left(W_1(t), W_2(t), W_3(t), W_4(t)\right)$. Therefore, we can get that the driving force of various risk assets under the equivalent probability measure Q is expressed by the following formula:

$$S_1(t) = S_1(0)exp\left\{\left(r - k_1 - \frac{1}{2}\sigma_1^2 + \sigma_1\rho_{12}\beta_2\sigma_2\right)t + \sigma_1\,\widetilde{W}_1(t)\right\}$$

$$S_2(t) = S_2(0)exp\left\{\left(r - k_2 - \frac{1}{2}\sigma_2^2 + \beta_2\sigma_2\right)t + \sigma_2\,\widetilde{W}_2(t)\right\}$$

$$V(t) = V(0)exp\left\{\left(r - k_3 - \frac{1}{2}\sigma_3^2 + \sigma_3\rho_{23}\beta_2\sigma_2\right)t + \sigma_3\,\widetilde{W}_3(t)\right\}$$

$$D(t) = D(0)exp\left\{\left(r - k_4 - \frac{1}{2}\sigma_4^2 + \sigma_4\rho_{42}\beta_2\sigma_2\right)t + \sigma_4\,\widetilde{W}_4(t)\right\}$$

Now we calculate $P(0)$ under the equivalent probability measure Q. We first simplify $P(0)$ as follows:

Let $\mathrm{H}(T) = \frac{V(T)}{D(T)}$, we can get:

$$\mathrm{H}(T) = \mathrm{H}(0)exp\left\{\left(k_4 - k_3 + \frac{1}{2}\sigma_4^2 - \frac{1}{2}\sigma_3^2 + \sigma_3\rho_{23}\beta_2\sigma_2 - \sigma_4\rho_{42}\beta_2\sigma_2\right)T + \sigma_3\,\widetilde{W}_3(T) - \sigma_4\,\widetilde{W}_4(T)\right\}$$

Where $\mathrm{H}(0) = \frac{V(0)}{D(0)}$.

Now $P(0)$ under the equivalent probability measure Q can be written as

$$P(0) = e^{-rt}E_P\left[\left[S_1^{\beta_1}(T) - S_2^{\beta_2}(T)\right]^+\left\{I_{\{V(T)\geq D(T)\}} + \frac{(1-\omega)V_T}{D_T}I_{\{V(T)<D(T)\}}\right\}\right]$$

$$= e^{-rt}E_P\left[S_2^{\beta_2}(T)\right]E_P\left[\frac{S_2^{\beta_2}(T)}{E_P\left[S_2^{\beta_2}(T)\right]}\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} - 1\right)^+\left\{I_{\{V(T)\geq D(T)\}} + \frac{(1-\omega)V_T}{D_T}I_{\{V(T)<D(T)\}}\right\}\right]$$

$$= e^{-rt}E_P\left[S_2^{\beta_2}(T)\right]E_{\mathrm{Q}}\left[\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} - 1\right)^+\left\{I_{\{V(T)\geq D(T)\}} + \frac{(1-\omega)V_T}{D_T}I_{\{V(T)<D(T)\}}\right\}\right].$$

In order to calculate $P(0)$, now we first calculate $E_P\left[S_2^{\beta_2}(T)\right]$ under the risk neutral measure P,we have

$$E_P\left[S_2^{\beta_2}(T)\right] = E_P\left[S_2^{\beta_2}(0)exp\left\{\beta_2\left(r - k_2 - \frac{1}{2}\sigma_2^2\right)T + \beta_2\sigma_2\,W_2(T)\right\}\right]$$

$$= S_2^{\beta_2}(0)exp\left\{T\beta_2\left(r - k_2 - \frac{1}{2}\sigma_2^2\right) + \frac{1}{2}\sigma_2^2\beta_2^2T\right\}$$

After calculating $E_P\left[S_2^{\beta_2}(T)\right]$ under the risk-neutral measure P, let's calculate

$$E_{\mathrm{Q}}\left[\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} - 1\right)^+\left\{I_{\{\mathrm{H}(T)\geq 1\}} + (1-\omega)\mathrm{H}(T)I_{\{\mathrm{H}(T)<1\}}\right\}\right]$$

$$= E_{\mathrm{Q}}\left[\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} - 1\right)\left\{I_{\left\{H(T)\geq 1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}} + (1-\omega)\mathrm{H}(T)I_{\left\{H(T)<1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right\}\right]$$

$$= K_1 + K_2 + K_3 + K_4$$

Where

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202104_7(4).0034

$$K_1 = E_{\mathrm{Q}}\left[\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} I_{\left\{H(T)\geq 1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right]$$

$$K_2 = -E_{\mathrm{Q}}\left[I_{\left\{H(T)\geq 1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right]$$

$$K_3 = (1-\omega)E_{\mathrm{Q}}\left[\mathrm{H}(T)\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} I_{\left\{H(T)<1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right]$$

$$K_4 = -(1-\omega)E_{\mathrm{Q}}\left[\mathrm{H}(T) I_{\left\{H(T)<1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right]$$

We only need to derive the expressions of $K_1 - K_4$ to get the pricing formula of fragile power exchange options. We noticed that $\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}$ and $H(T)$ are random variables, and it is easy to observe that $\left(\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right), \log\big(H(T)\big)\right)$ is a bivariate normal distribution, and has

$$\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right) = \log\left(\frac{S_1^{\beta_1}(0)}{S_2^{\beta_2}(0)}\right) + \left\{\beta_1\left(r - k_1 - \frac{1}{2}\sigma_1^2 + \sigma_1\rho_{12}\beta_2\sigma_2\right)T\right\}$$
$$+\sigma_1\beta_1\,\widetilde{W}_1(T) - \beta_2\left(r - k_2 - \frac{1}{2}\sigma_2^2 + \beta_2\sigma_2\right)T + \sigma_2\,\widetilde{W}_2(T)$$

$$\log\big(H(T)\big) = \log\big(H(0)\big) + \left\{\left(k_4 - k_3 + \frac{1}{2}\sigma_4^2 - \frac{1}{2}\sigma_3^2 + \sigma_3\rho_{23}\beta_2\sigma_2 - \sigma_4\rho_{42}\beta_2\sigma_2\right)T\right.$$
$$\left. + \sigma_3\,\widetilde{W}_3(T) - \sigma_4\,\widetilde{W}_4(T)\right\}$$

We can calculate some properties of the two-dimensional normal distribution $\left(\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right), \log\big(H(T)\big)\right)$ as follows:

$$A_1 = E\left[\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right)\right] = \log\left(\frac{S_1^{\beta_1}(0)}{S_2^{\beta_2}(0)}\right)$$
$$+\left\{\beta_1\left(r - k_1 - \frac{1}{2}\sigma_1^2 + \sigma_1\rho_{12}\beta_2\sigma_2\right)T - \beta_2\left(r - k_2 - \frac{1}{2}\sigma_2^2 + \beta_2\sigma_2\right)T\right\}$$

$$A_2 = E\big[\log\big(H(T)\big)\big] = \log\big(H(0)\big) + \left(k_4 - k_3 + \frac{1}{2}\sigma_4^2 - \frac{1}{2}\sigma_3^2 + \sigma_3\rho_{23}\beta_2\sigma_2 - \sigma_4\rho_{42}\beta_2\sigma_2\right)T$$

$$B_1 = Var\left[\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right)\right] = (\beta_1^2\sigma_1^2 + \beta_2^2\sigma_2^2 - 2\rho_{12}\beta_1\beta_2\sigma_1\sigma_2)T$$

$$B_2 = Var\big[\log\big(H(T)\big)\big] = (\sigma_3^2 + \sigma_4^2 - 2\rho_{34}\sigma_3\sigma_4)T$$

$$C = Cov\left(\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right), \log\big(H(T)\big)\right) = (\beta_1\sigma_1\sigma_3\rho_{13} - \beta_1\sigma_1\sigma_4\rho_{14} - \beta_2\sigma_2\sigma_3\rho_{23} + \beta_2\sigma_2\sigma_4\rho_{24})T$$

$$\rho_{12} = Corr\left(\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right), \log\big(H(T)\big)\right) = \frac{C}{\sqrt{B_2}\sqrt{B_1}}$$

In order to simplify the calculation, we can rewrite $\left(\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right), \log\big(H(T)\big)\right)$ as

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202104_7(4).0034

$$\log\left(\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\right) = A_1 + \sqrt{B_1}\varepsilon_1$$
$$\log\big(H(T)\big) = A_2 + \sqrt{B_2}\varepsilon_2$$

Where $(\varepsilon_1, \varepsilon_2)$ is a two-dimensional standard normal distribution, and the correlation coefficient is ρ_{12}. Next we have

$$K_1 = E_{\mathrm{Q}}\left[\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} I_{\left\{H(T)\geq 1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right] = E_{\mathrm{Q}}\left[e^{A_1+\sqrt{B_1}\varepsilon_1} I_{\left\{e^{A_2+\sqrt{B_2}\varepsilon_2}\geq 1, e^{A_1+\sqrt{B_1}\varepsilon_1}\geq 1\right\}}\right]$$
$$= E_{\mathrm{Q}}\left[e^{A_1-\sqrt{B_1}(-\varepsilon_1)} I_{\left\{-\varepsilon_1\leq\frac{A_1}{\sqrt{B_1}}, -\varepsilon_2\leq\frac{A_2}{\sqrt{B_2}}\right\}}\right]$$
$$= e^{A_1+\frac{1}{2}\sqrt{B_1}} N_2\left(\frac{A_1+B_1}{\sqrt{B_1}}, \frac{A_2+\sqrt{B_1}\sqrt{B_2}\rho_{12}}{\sqrt{B_2}}, \rho_{12}\right)$$

$$K_2 = -E_{\mathrm{Q}}\left[I_{\left\{H(T)\geq 1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right] = -E_{\mathrm{Q}}\left[I_{\left\{e^{A_2+\sqrt{B_2}\varepsilon_2}\geq 1, e^{A_1+\sqrt{B_1}\varepsilon_1}\geq 1\right\}}\right]$$
$$= -E_{\mathrm{Q}}\left[I_{\left\{-\varepsilon_1\leq\frac{A_1}{\sqrt{B_1}}, -\varepsilon_2\leq\frac{A_2}{\sqrt{B_2}}\right\}}\right] = -N_2\left(\frac{A_1}{\sqrt{B_1}}, \frac{A_2}{\sqrt{B_2}}, \rho_{12}\right)$$

$$K_3 = (1-\omega)E_{\mathrm{Q}}\left[\mathrm{H}(T)\frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)} I_{\left\{H(T)<1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right]$$
$$= (1-\omega)E_{\mathrm{Q}}\left[e^{A_2+\sqrt{B_2}\varepsilon_2} e^{A_1+\sqrt{B_1}\varepsilon_1} I_{\left\{e^{A_2+\sqrt{B_2}\varepsilon_2}<1, e^{A_1+\sqrt{B_1}\varepsilon_1}\geq 1\right\}}\right]$$
$$= (1-\omega)E_{\mathrm{Q}}\left[e^{A_1-\sqrt{B_1}(-\varepsilon_1)} e^{A_2+\sqrt{B_2}\varepsilon_2} I_{\left\{-\varepsilon_1\leq\frac{A_1}{\sqrt{B_1}}, \varepsilon_2<-\frac{A_2}{\sqrt{B_2}}\right\}}\right]$$
$$= (1-\omega)e^{A_1+\frac{1}{2}\sqrt{B_1}+A_2+\frac{1}{2}\sqrt{B_2}+\sqrt{B_1}\sqrt{B_2}\rho_{12}}$$
$$\times N_2\left(\frac{A_1+B_1+\sqrt{B_1}\sqrt{B_2}\rho_{12}}{\sqrt{B_1}}, -\frac{A_2+B_2+\sqrt{B_1}\sqrt{B_2}\rho_{12}}{\sqrt{B_2}}, -\rho_{12}\right)$$

$$K_4 = -(1-\omega)E_{\mathrm{Q}}\left[\mathrm{H}(T) I_{\left\{H(T)<1, \frac{S_1^{\beta_1}(T)}{S_2^{\beta_2}(T)}\geq 1\right\}}\right]$$
$$= -(1-\omega)E_{\mathrm{Q}}\left[e^{A_2+\sqrt{B_2}\varepsilon_2} I_{\left\{e^{A_2+\sqrt{B_2}\varepsilon_2}<1, e^{A_1+\sqrt{B_1}\varepsilon_1}\geq 1\right\}}\right]$$
$$= -(1-\omega)E_{\mathrm{Q}}\left[e^{A_2+\sqrt{B_2}\varepsilon_2} I_{\left\{-\varepsilon_1\leq\frac{A_1}{\sqrt{B_1}}, \varepsilon_2<-\frac{A_2}{\sqrt{B_2}}\right\}}\right]$$
$$= -(1-\omega)e^{A_2+\sqrt{B_2}\varepsilon_2} N_2\left(\frac{A_1+\sqrt{B_1}\sqrt{B_2}\rho_{12}}{\sqrt{B_1}}, -\frac{A_2+B_2}{\sqrt{B_2}}, -\rho_{12}\right)$$

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202104_7(4).0034

4. CONCLUSION AND DISCUSSION

This article mainly discusses the pricing formulas of vulnerable power exchange options with random default barriers. Since our proposed model involves the relevant four-dimensional Brownian motion, we propose to use the method of measurement transformation to derive the pricing formulas of vulnerable power exchange options with random default barriers. Since our proposed model assumes that the risk-free interest rate and the volatility of the underlying asset are fixed, assuming that the risk-free interest rate and the volatility of the underlying asset are random is a further research direction.

REFERENCES

[1] Herb Johnson and René Stulz:The Pricing of Options with Default Risk, Journal of Finance, vol.42 (1978) No.2, p.267-280.

[2] Turnbull J S M : Pricing Derivatives on Financial Securities Subject to Credit Risk, The Journal of Finance, vol.50 (1995) No.1, p.53-85.

[3] Klein P: Pricing Black-Scholes options with correlated credit risk, Journal of Banking & Finance, vol.20 (1996) No.7, p.1211-1229.

[4] Klein P., Inglis M : Valuation of European Options Subject to Financial Distress and Interest Rate Risk, Journal of Derivatives, vol.6 (2009) No.3, p.284-287.

[5] Chang L F , Hung M W : Valuation of vulnerable American options with correlated credit risk, Review of Derivatives Research, vol.9 (2006) No.2, p.137-165.

[6] ShuIng Liu, YuChung Liu: Pricing Vulnerable Options under Stochastic Asset and Liability, Progress in Applied Mathematics, vol.1 (2011) No.2, p.659-664.

[7] Yang S J , Lee M K , Kim J H : Pricing vulnerable options under a stochastic volatility model, Applied Mathematics Letters, vol.34 (2014) No.1, p.7-12.

[8] Jeon J , Kim G : Pricing of vulnerable options with early counterparty credit risk, The North American Journal of Economics and Finance, vol.47 (2019) No.JAN p.645-656.

[9] Tompkins R G: Power options: Hedging nonlinear risks, Journal of Risk, vol.2 (2000) No.2, p.29-45.

[10] Fischer, S: Call option pricing when the exercise price is uncertain, and the valuation of index bonds, Journal of Finance, vol.33 (1978) No.1, p.169-176.

[11] Blenman L P , Clark S P : Power exchange options, Finance Research Letters, vol.2 (2005) No.2, p.97-106.

[12] Wang, Xingchun.: Pricing power exchange options with correlated jump risk, Finance Research Letters, (2016), p.90-97.

[13] Wang X , Song S , Wang Y : The Valuation of Power Exchange Options with Counterparty Risk and Jump Risk, Journal of Futures Markets, vol.37 (2017) No.5, p.499-521.