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Summary of Elastic Foundation Model and Comparison of Calculation Theory

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Abstract

The establishment of elastic foundation model and its solution are very important engineering topics. The commonly used elastic foundation models and corresponding solution methods are summarized by literature summary method, so as to facilitate the practical application of engineering reference. The results show that the elastic foundation model mainly includes Winkler foundation model, two parameter foundation model, three parameter foundation model, elastic half space foundation model, finite compression layer foundation model and layered transverse isotropic elastic half space model. Winkler foundation model, two parameter foundation model, three parameter foundation model and elastic half space foundation model are simple, but they are not consistent with the stratification characteristics of actual soil, and it is difficult to determine the model parameters. The finite compression layer foundation model and the layered transversely isotropic elastic half space model can consider the layered characteristics of soil, and the finite compression layer foundation model is simpler than the layered transversely isotropic elastic half space model, and has a greater engineering application prospect. The solution methods of elastic foundation model mainly include link method, initial parameter method, power series approximation method, finite difference method and finite element method. Among them, link method, power series approximation method, finite difference method and finite element method can solve different foundation models and load situations, which has great engineering application prospects.

Keywords

Elastic foundation model; Solution method; Soil stratification; Engineering application.

1. INTRODUCTION

Elastic foundation beam model is a very important calculation method to calculate the interaction between foundation and foundation. Different from the beam in the building structure, the foundation beam always keeps in contact with the foundation, that is, the foundation is always under the pressure of the foundation beam, which will produce settlement deformation. Accordingly, the foundation beam has bending deformation. It can be considered that the foundation beam and foundation meet the displacement coordination conditions on the contact surface.

Scholars at home and abroad have done a lot of research work on the establishment and solution of elastic foundation model. This paper introduces the main elastic foundation models with engineering application prospects at home and abroad, and summarizes the solution methods of the main models.

2. ELASTIC FOUNDATION MODEL

2.1. Winkler Foundation Model

Winkler foundation model assumes that the foundation beam and foundation have coordinated vertical displacement, and the pressure between the foundation beam and foundation is directly proportional to the vertical displacement. Namely

$$p(x, y) = kw(x, y) \tag{1}$$

Where, $p(x, y)$ and $w(x, y)$ are foundation reaction and settlement at the position (x, y), respectively; k represents the coefficient of subgrade bed.

According to the classical elastic theory, the stress diffusion phenomenon will appear when the upper load is transmitted from the beam to the foundation, that is, the elastic foundation will not only produce settlement deformation in the action area of the force, but also produce deformation outside the action area of the force. Formula (1) corresponding to Winkler foundation model is obviously inconsistent with this. This is mainly because Winkler foundation model makes the assumption that shear deformation and settlement only occur in the base range, which are inconsistent with the reality. In addition, according to the Winkler foundation model, the foundation bed coefficient is a constant. However, for the foundation composed of layered soil, how to take a reasonable value of the foundation bed coefficient is still an open problem. Despite its shortcomings, Winkler foundation model has been widely used because of its clear physical meaning and simple calculation.

2.2. Two Parameter Foundation Model

The Winkler foundation model ignores the shear deformation. In order to consider the influence of shear deformation and try to establish the interaction between independent Winkler springs, the most intuitive method is to introduce a parameter representing the interaction between springs, so a two parameter elastic foundation model is formed. The classical two parameter elastic foundation models mainly include Filonenko-Borodich model [1], Hetenyi model [2] and Pasternak model [3].

On the basis of Winkler foundation model, Filonenko-Borodich model introduces the membrane under tension T to connect the independent springs in Winkler foundation model, so that each spring interacts, so as to achieve the effect of continuity. The specific expression is

$$p(x, y) = kw(x, y) - T\nabla^2 w(x, y) \tag{2}$$

Based on the Winkler foundation model, Hetenyi model introduces an elastic plate with flexural stiffness D to connect the independent springs in the Winkler foundation model, so that the springs interact, so as to achieve the effect of continuity. The specific expression is

$$p(x, y) + D\nabla^4 w(x, y) = kw(x, y) \tag{3}$$

Based on the Winkler foundation model, Pasternak model introduces a shear layer with shear modulus G_p to connect the independent springs in the Winkler foundation model, so that the springs interact, so as to achieve the effect of continuity. The specific expression is

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$$p(x,y)=kw(x,y)-G_p\nabla^2 w(x,y) \tag{4}$$

2.3. Three Parameter Foundation Model

On the basis of Winkler foundation model, Livkin [4] proposed a three parameter foundation model in combination with the distribution of foundation reaction force of rectangular foundation structure

$$p(x,y)=k\left[1+\beta e^{-\alpha(m-\xi)(1-\eta)}\right]w(x,y)$$
$$m=\frac{l}{b};\xi=\frac{x}{l};\eta=\frac{y}{b} \tag{6}$$

Where, α and β are soil parameters; l and b are half the length and width of the rectangular foundation respectively.

2.4. Elastic Half Space Foundation Model

The homogeneous isotropic elastic half space model is based on the Boussinesq solution. The Boussinesq solution gives the expression of surface settlement when the surface is subjected to vertical concentrated force. For the case of surface distributed load, it only needs to integrate the Boussinesq solution.

Boussinesq solution is based on the assumption of uniform isotropic elastic half space. In reality, the foundation soil is not uniformly distributed. Gibson gives a non-uniform elastic half plane and half space model. The construction foundation soil is incompressible in the process of model establishment, and the shear modulus of foundation soil changes linearly with the depth of foundation soil. These two assumptions are inconsistent with the engineering practice, which directly limits the application of the model in practical engineering.

2.5. Foundation Model Of Finite Compression Layer

Considering that the soil is layered in real engineering [5,6], referring to the idea of layered summation, scholars at home and abroad put forward the finite compression layer model [7-9]. The finite compression layer foundation model regards the soil as the accumulation of several layers, but each layer of soil is different and has its own characteristics. It is simple to calculate the total deformation of the foundation by using the method of layered summation.

2.6. Layered Transversely Isotropic Elastic Half Space Model

Foundation soil is generally distributed in layers, and the properties of soil in each layer are similar, but the mechanical properties of soil in each layer are generally quite different. Considering the historical sedimentation of soil layer, the layered transverse isotropic elastic half space model may be more in line with the actual situation of soil. The establishment process of layered transversely isotropic elastic half space model is relatively simple, but it is difficult to solve, especially to obtain the analytical solution. Generally, only the numerical solution can be obtained, which directly affects the application of layered transversely isotropic elastic half space model in practical engineering.

3. SOLUTION OF FOUNDATION MODEL

After establishing the foundation model, if you want to make the foundation model play a practical role, you must solve the foundation model. However, the complexity of different foundation models is different. For example, Winkler foundation model, two parameter

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foundation model and three parameter foundation model are relatively simple, and the corresponding solution method is also relatively simple. The non-uniform elastic half space model and the layered transversely isotropic elastic half space model are more complex. For some special cases, analytical solutions can be obtained, but generally, numerical solutions can only be used. According to whether the explicit solution expression can be obtained, the solution methods can be divided into three categories: analytical method, numerical method and semi analytical and semi numerical method. Among them, the analytical method mainly combines the differential equations and boundary conditions corresponding to the model to establish equations for solution. Numerical solution generally refers to the finite element method. Semi analytical and semi numerical method combines the respective advantages of numerical method and analytical method, and simplifies the two methods into one method. The core of this method is the assumption of analytical function. The general analytical function should not only meet the continuous differentiable conditions, but also meet its displacement boundary and stress boundary conditions. Finally, the calculation process should be as simple as possible.

For the beam model on elastic foundation, the typical solution methods include link method, initial parameter method, power series approximation method, finite difference method and finite element method.

3.1. Chain Rod Method

According to the interaction between foundation and foundation beam, foundation beam can be regarded as an infinite statically indeterminate structure. The link method simplifies the foundation into several links, so the beam can be regarded as a finite statically indeterminate structure. As shown in Figure 1 [10], the foundation beam is divided into *n* equal parts, and then a rigid chain rod is connected with the foundation at the center of each beam section, that is, the foundation under the elastic foundation beam is transformed into a finite number of supporting chain rods, so that the elastic foundation beam is transformed into a finite statically indeterminate structure. According to the calculation process of the force method, taking the statically determined beam as the basic system, hinge constraints are imposed on the beam at joint 1,2... n in the beam, and the section bending moments X_1, X_2,..., X_n are applied at these positions. There are n in total. According to the rotation coordination conditions at each hinge, *n* equations can be obtained, and *n* section bending moments can be solved.

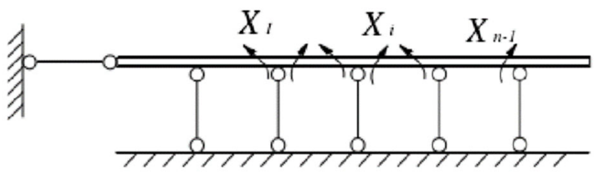

Figure 1. Calculation diagram of chain rod method

The solution of link method is not limited to Winkler foundation model, but can be solved for different foundation models and load situations. Theoretically, the more the number of links, the higher the accuracy of the solution, but the higher the corresponding calculation cost.

3.2. Initial Parameter Method

For the elastic foundation beam corresponding to Winkler foundation model, the equilibrium equation of foundation beam expressed by differential equation is

$$EI\frac{\mathrm{d}^4 w(x)}{\mathrm{d}x^4} = q(x) - p(x) = q(x) - kw(x) \tag{6}$$

Where, EI represents the bending stiffness of the beam, $q(x)$ represents the line load on the beam. In particular, when the line load is not considered and

$$s = \left(\frac{4EI}{k}\right)^{1/4} \tag{7}$$

Then equation (6) can be transformed into

$$\frac{\mathrm{d}^4 w(x)}{\mathrm{d}x^4} + \frac{4}{s^4} w(x) = 0 \tag{8}$$

The corresponding solution is

$$w(x) = C_1 \mathrm{ch}\frac{x}{s}\cos\frac{x}{s} + C_2 \mathrm{ch}\frac{x}{s}\sin\frac{x}{s} + C_3\,\mathrm{sh}\frac{x}{s}\cos\frac{x}{s} + C_4\,\mathrm{sh}\frac{x}{s}\sin\frac{x}{s} \tag{9}$$

After the deflection expression is obtained, the four coefficients in formula (9) can be obtained according to the relationship between bending moment, shear force, distributed force and deflection, combined with known physical quantities, so as to determine the deflection expression and realize the solution of Winkler foundation model. Such a solution method is called out parameter method [11]. In addition, combined with Laplace Hankel transform and matrix transfer method, the initial parameter method is also used to solve the spatial axisymmetric problem of layered foundation [12]. Considering the engineering application prospect of the solution under the general load of layered foundation, how to apply the initial parameter method to the solution under the general load of layered foundation is a direction of great research value in the future.

3.3. Power Series Approximation Method

Considering that the distribution of foundation reaction force is unknown in general, it is very difficult to solve the foundation model. In order to facilitate the solution, the foundation reaction can be approximately expressed as a finite power series polynomial [13]

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \tag{10}$$

Obviously, as long as the undetermined coefficient $a_i (i = 1, 2, 3, \ldots, n)$ is obtained, the foundation model can be solved.

The deflection curve equation of the foundation beam can be obtained by acting the foundation reaction on the foundation beam and combining with other external forces borne by the foundation beam; When the foundation reaction acts on the foundation, the foundation settlement can be obtained. According to the equality of foundation settlement and foundation beam deflection everywhere, the undetermined coefficient can be determined.

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Obviously, when the number of terms of the power series polynomial shown in equation (10) is more, the calculation result is more accurate in theory. In addition, similar to the chain link method, the solution of the power series approximation method is not limited to the Winkler foundation model. It can solve different foundation models and load situations, and has great engineering application prospects.

3.4. Finite Difference Method

The core idea of finite difference method is to replace differential equations and boundary conditions with difference equations, so as to transform the solution of differential equations into the solution of linear equations, which greatly reduces the difficulty of solution. The core equations of using difference instead of differential in finite difference method include

$$\begin{aligned} \left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)_{x=x_i} &= \frac{w_{i+1}-w_{i-1}}{2\Delta x} \\ \left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right)_{x=x_i} &= \frac{w_{i+1}-2w_i+w_{i-1}}{(\Delta x)^2} \\ \left(\frac{\mathrm{d}^3 w}{\mathrm{d}x^3}\right)_{x=x_i} &= \frac{w_{i+2}-2w_{i+1}+2w_{i-1}-w_{i-2}}{2(\Delta x)^3} \end{aligned} \tag{11}$$

Where Δx is the difference segment length. For the beam, the relationship between deflection and internal force is

$$\begin{aligned} \frac{\mathrm{d}^2 w}{\mathrm{d}x^2} &= -\frac{M}{EI} \\ \frac{\mathrm{d}^3 w}{\mathrm{d}x^3} &= -\frac{Q}{EI} \end{aligned} \tag{12}$$

Where, M and Q represent bending moment and shear force respectively, the following formula can be easily obtained by substituting equation (11) into equation (12)

$$\begin{aligned} \frac{M_i}{EI_{i+1}} &= -\frac{w_{i+1}-2w_i+w_{i-1}}{(\Delta x)^2} \\ \frac{Q_i}{EI_i} &= -\frac{w_{i+2}-2w_{i+1}+2w_{i-1}-w_{i-2}}{2(\Delta x)^3} \end{aligned} \tag{13}$$

Through equation (13), the bending moment and shear force can be expressed by deflection. Then, combined with the balance of external force and internal force, the coordination conditions of beam deflection and foundation settlement, the combined force boundary and displacement boundary can form a nonlinear equation group about the deflection of discrete points. By solving the equation group, the deflection value of each discrete point can be obtained, so as to realize the solution of foundation beam.

Obviously, when the finite difference method is used to solve the foundation beam, it is not limited to the Winkler foundation model, and other foundation models can also be applied. The popularization prospect of this method is greater.

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3.5. Finite Element Method

Finite element method is a universal method for solving initial boundary value problems of differential equations. In this method, the area to be analyzed is meshed into interconnected and non overlapping elements, and then the element analysis is carried out. The overall finite element equations of all elements in the area to be analyzed are formed according to the element analysis equations, and then the overall finite element equations are solved in combination with the boundary conditions.

Finite element method can solve complex boundary problems and complex geometric problems, and the solution process is standardized. More mature commercial finite element software can be used, which can save a lot of program development time for engineers. However, the use of commercial finite element software has high requirements for users, otherwise unreasonable results may be obtained due to improper use methods.

4. CONCLUSION

The commonly used elastic foundation models and corresponding solution methods are summarized by literature summary method, so as to facilitate the practical application of engineering reference. The main conclusions are as follows:

(1) The elastic foundation model mainly includes Winkler foundation model, two parameter foundation model, three parameter foundation model, elastic half space foundation model, finite compression layer foundation model and layered transversely isotropic elastic half space model. Winkler foundation model, two parameter foundation model, three parameter foundation model and elastic half space foundation model are simple, but they are not consistent with the stratification characteristics of actual soil, and it is difficult to determine the model parameters. The finite compression layer foundation model and the layered transversely isotropic elastic half space model can consider the layered characteristics of soil, and the finite compression layer foundation model is simpler than the layered transversely isotropic elastic half space model, and has a greater engineering application prospect.

(2) The solution methods of elastic foundation model mainly include link method, initial parameter method, power series approximation method, finite difference method and finite element method. Among them, link method, power series approximation method, finite difference method and finite element method can solve different foundation models and load situations, which has great engineering application prospects.

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