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The Damage Constitutive Model of Shale Rock Based on Weibull Distribution and Its Verification

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Abstract

With the increase of shale rock gas demand, the mining depth increases gradually and various safety accidents occur. Therefore, it is of great significance to study the constitutive behavior of shale rock under different confining pressures. Based on the existing constitutive model of rock, Weibull distribution function and confining pressure damage variable are introduced in this paper, combined with strain equivalence principle and Drucker-Prager criterion, the damage constitutive model of shale rock under different confining pressures is established and parameter expression in damage constitutive model is determined. In the verification stage, the triaxial compression test results of shale rock under different confining pressures (10, 20, 30, 40, 50 and 60 MPa) in the reference were carried out, and the results showed that: The theoretical curves of the damage constitutive model of shale rock under different confining pressures have the same trend as the test curves in the reference, indicating that the damage constitutive model constructed in this paper can better reflect the stress-strain process of shale rock under triaxial compression. In the test process, if the stress decreases, the damage constitutive model cannot accurately reflect the stage of stress rising again. The damage constitutive model established in this paper can well fit the stress-strain curves of elastic and plastic stages of shale rock under different confining pressures. The damage constitutive model constructed in this paper has certain reference value for deep shale rock excavation.

Keywords

Shale rock; Damage constitutive model; Weibull distribution; Confining pressure.

1. INTRODUCTION

As a clean energy, shale rock gas is widely used around the world. With the increase of shale rock gas exploitation depth, various safety accidents occurred in the exploitation process, resulting in many casualties and economic losses. This is due to the insufficient understanding of the strength and damage characteristics of shale rock under different stress states. Therefore, in order to improve the safety of deep shale rock gas mining and reduce safety accidents, it is necessary to establish a damage constitutive model that can reflect the strength change and damage development law of shale rock under different ground stresses.

At present, researchers have studied the influence of different confining pressures on rock physical and mechanical properties through a large number of tests and constitutive models. In test, Li, C [1] and Yang, S [2] through triaxial compression tests under different confining pressures, it is found that with the increase of confining pressure, the initiation stress and elastic modulus of shale rock increase. Hu, J [3] through the change trend of energy, the

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influence of shale rock bedding structure and confining pressure on shale rock is studied. It is found that with the increase of confining pressure, the releasable energy of shale rock increases during loading. Cheng, C [4] through triaxial cyclic compression test, it is found that the change trend of peak strength of shale rock is the same as that in ordinary triaxial compression test, with the increase of confining pressure, the peak strength of shale rock increases. In constitutive model, at present, most damage variables are represented by Weibull [5] distribution, and the damage constitutive model is constructed by introducing strain equivalent principle and continuous damage theory. Ren, C [6] proposed the dual variables of plastic strain and damage variable, and the damage constitutive model constructed has good fitting for rocks under different confining pressures. However, in the strain softening stage, the model is quite different from the test data. Liu, Y [7] carried out triaxial compression tests of layered composite rock under different confining pressures. The damage constitutive model was constructed by defining the axial strain as the random distribution variable in the composite rock. On the basis of Wang ' s model [8], Wang, J [9] optimized the constitutive model to simulate the strain softening characteristics of lime mudstone better. Zong, Y [10] defines damage by acoustic emission and rock volume change, and the damage constitutive model has a high degree of fitting to the stress-strain process of rock. Li, F [11] constructed the damage constitutive model through Weibull distribution and Drucker-Prager criterion.

On the basis of the above research, the damage constitutive model of shale rock micro-element strength obeying Weibull distribution and Drucker-Prager criterion is proposed in this paper. The triaxial compression test data of shale rock under different confining pressures in reference [12] are fitted to verify the rationality of the damage constitutive model constructed in this paper, which provides certain reference for shale rock gas mining engineering.

2. DAMAGE CONSTITUTIVE MODEL OF SHALE ROCK

2.1. Damage Variable and Damage Constitutive Model

According to the concept of confining pressure inhibiting damage [13], when the confining pressure is high, the initial cracks inside the rock are closed under the action of confining pressure. With the increase of confining pressure, the rock becomes dense, and the elastic modulus is significantly enhanced, indicating that confining pressure plays a strengthening role on the rock. The constitutive model of rock under non-destructive condition is as follows:

$$\sigma = E\varepsilon \tag{1}$$

Where, σ is effective stress, E is elastic modulus under non-destructive condition, ε is axial strain of shale rock.

In this paper, through the strain equivalence principle [14], the damage variable of shale rock is defined by D, and the damage constitutive model of rock is established:

$$\sigma_1 = (1-D)E_0\varepsilon + 2\mu\sigma_3 \tag{2}$$

$$D = \frac{E_S}{E_0} \tag{3}$$

where, D is damage variable, E0 is initial elastic modulus of shale rock, ES is elastic modulus of shale rock under confining pressure, μ is Poisson ' s ratio of shale rock, σ3 is confining pressure.

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Since there are a large number of micro-cracks and pores in shale rock, in order to better reflect the distribution law of internal defects in shale rock, Weibull distribution is introduced to describe the strength of micro-units in shale rock [15-17]:

$$P(a)=\frac{n}{m}(\frac{a}{m})^{n-1}\exp\left[-\left(\frac{a}{m}\right)^{n}\right] \tag{4}$$

where, P(a) is probability of shale rock internal strength distribution, a is variables of random distribution, n and m are parameter of distribution.

According to Eq. (4) and Eq. (3), can be drawn:

$$D=\int_0^a P(a)\mathrm{d}\varepsilon=1-\exp\left[-\left(\frac{a}{m}\right)^{n}\right] \tag{5}$$

Combining Eq. (2) and Eq. (5), we can obtain:

$$\sigma_1=E\varepsilon\exp\left[-\left(\frac{a}{m}\right)^{n}\right]+2\mu\sigma_3 \tag{6}$$

According to Drucker-Prager failure criterion of rock, it can be found the shale rock variables of random distribution is:

$$a=bI_1+\sqrt{J_2} \tag{7}$$

where, b is shale rock parameters, I1 is the first invariant of stress tensor, J2 is the second invariant of stress deviator.

The relevant expressions of b, I and J are as follows:

$$I_1=\frac{(\sigma_1+2\sigma_3)E\varepsilon_1}{\sigma_1-2\mu\sigma_3} \tag{8}$$

$$\sqrt{J_2}=\frac{(\sigma_1-\sigma_3)E\varepsilon_1}{\sqrt{3}(\sigma_1-2\mu\sigma_3)} \tag{9}$$

$$b=\frac{\sin\phi}{\sqrt{9+3\sin^2\phi}} \tag{10}$$

where, σ_1, σ_2 and σ_3 is test results, $\sigma_2=\sigma_3$, ϕ is internal friction angle of shale rock.

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2.2. Parameter Determination of Damage Constitutive Model

In order to determine the parameters of the damage constitutive model, the boundary conditions of the stress-strain curve of shale rock are found according to the stress-strain figure of shale rock under the confining pressure of 10 MPa in the reference [11]:

$$
\begin{aligned}
&(1)\ \varepsilon = 0, \sigma_1 = 0 \\
&(2)\ \varepsilon = 0, \frac{\mathrm{d}(\sigma_1)}{\mathrm{d}\varepsilon} = E \\
&(3)\ \sigma_1 = \sigma_{1\max}, \varepsilon = \varepsilon_{\max} \\
&(4)\ \sigma_1 = \sigma_{1\max}, \frac{\mathrm{d}\sigma_{1\max}}{\mathrm{d}\varepsilon} = 0
\end{aligned}
;
$$

Substitute Eq. (7) to (10) into Eq. (6) and derive ε:

$$
\begin{cases}
\dfrac{\mathrm{d}\sigma_1}{\mathrm{d}\varepsilon} = E\exp\left[-\left(\dfrac{a}{m}\right)^n\right] + \left\{E\varepsilon\exp\left[-\left(\dfrac{a}{m}\right)^n\right]\right\}\left[-\left(n\dfrac{a^{n-1}}{m^n}\right)\right](\dfrac{\mathrm{d}a}{\mathrm{d}\varepsilon} + \dfrac{\mathrm{d}a}{\mathrm{d}\sigma_1}\dfrac{\mathrm{d}\sigma_1}{\mathrm{d}\varepsilon}) \\
\dfrac{\mathrm{d}a}{\mathrm{d}\varepsilon} = b\dfrac{(\sigma_1 + 2\sigma_3)E}{\sigma_1 - 2\mu\sigma_3} + \dfrac{(\sigma_1 - \sigma_3)E}{\sqrt{3}(\sigma_1 - 2\mu\sigma_3)} = \dfrac{a}{\varepsilon} \\
\dfrac{\mathrm{d}a}{\mathrm{d}\sigma_1}\dfrac{\mathrm{d}\sigma_1}{\mathrm{d}\varepsilon} = \dfrac{\mathrm{d}\sigma_1}{\mathrm{d}\varepsilon}E\varepsilon[b\dfrac{-2\mu\sigma_3 - 2\sigma_3}{(\sigma_1 - 2\mu\sigma_3)^2} + \dfrac{\sqrt{3}\sigma_3 - 2\sqrt{3}\mu\sigma_3}{3(\sigma_1 - 2\mu\sigma_3)^2}
\end{cases}
\tag{10}
$$

According to conditions (3), (4) and Eq. (11):

$$
\begin{cases}
\sigma_{1\max} = E\varepsilon_{\max}\exp\left[-\left(\dfrac{a_{\max}}{m}\right)^n\right] + 2\mu\sigma_3 \\
\dfrac{\mathrm{d}\sigma_{1\max}}{\mathrm{d}\varepsilon} = \left\{E\exp\left[-\left(\dfrac{a_{\max}}{m}\right)^n\right]\right\}\left[1 - n\left(\dfrac{a_{\max}}{m}\right)^n\right] = 0
\end{cases}
\tag{11}
$$

A can be obtained by simplifying Eq. (12):

$$
\begin{cases}
\exp\left[-\left(\dfrac{a_{\max}}{m}\right)^n\right] = \dfrac{\sigma_{1\max} - 2\mu\sigma_3}{E\varepsilon_{\max}} \\
\left\{\dfrac{\sigma_{1\max} - 2\mu\sigma_3}{\varepsilon_{\max}}\right\}\left[1 + \ln(\dfrac{\sigma_{1\max} - 2\mu\sigma_3}{E\varepsilon_{\max}})\right] = 0
\end{cases}
\tag{12}
$$

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$$\begin{cases} n = \dfrac{-1}{\ln(\dfrac{\sigma_{1\max} - 2\mu\sigma_3}{E\varepsilon_{\max}})} \\ m = a_{\max} n^{\frac{1}{n}} \end{cases} \tag{13}$$

Substituting Eq. (14) into Eq. (5) and (6) can obtain damage constitutive models of shale rock under different confining pressures:

$$\begin{cases} D = 1 - \exp[-(\dfrac{a}{a_{\max} n^{\frac{1}{n}}})^{\frac{-1}{\ln(\frac{\sigma_{1\max} - 2\mu\sigma_3}{E_0\varepsilon_{\max}})}}] \\ \sigma_1 = E\varepsilon\exp[-(\dfrac{a}{a_{\max} n^{\frac{1}{n}}})^{\frac{-1}{\ln(\frac{\sigma_{1\max} - 2\mu\sigma_3}{E_0\varepsilon_{\max}})}}] + 2\mu\sigma_3 \end{cases} \tag{14}$$

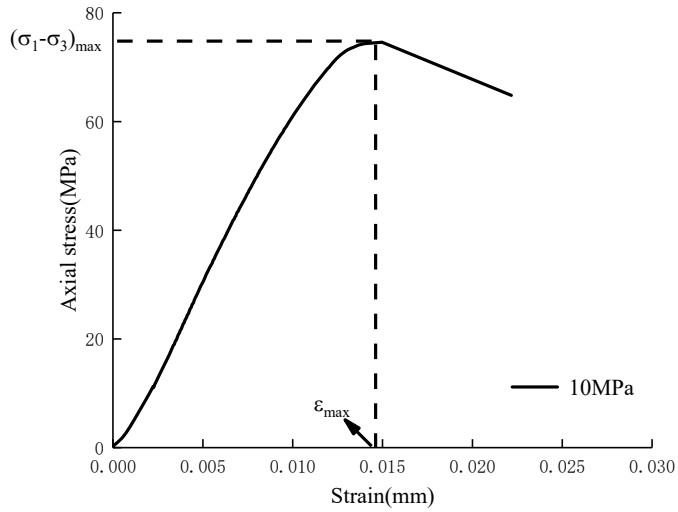

Figure 1. Boundary conditions of stress-strain curve of shale rock

3. VERIFICATION AND ANALYSIS OF DAMAGE CONSTITUTIVE MODEL

3.1. Reference Verification

At present, Zhao, B described shale rock strength under different confining pressures by six strength criteria. In this paper, shale rock tests under different confining pressures are selected to verify the rationality of the damage constitutive model proposed in this paper. The relevant data obtained from triaxial compression tests of shale rock under different confining pressures are as follows Table 1, and the elastic modulus and Poisson ' s ratio of shale rock under uniaxial compression is 5272.8 MPa and 0.24, respectively.

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Table 1. Test data under different confining pressures

Specimen number	σ_1 (MPa)	σ_3 (MPa)
C1	74.59	10
C2	87.33	20
C3	105.25	30
C4	118.17	40
C5	129.24	50
C6	141.35	60

Figure 2 is the elastic modulus of shale rock under different confining pressures. It can be seen from the Figure 2 that with the increase of confining pressure, the elastic modulus of shale rock increases, and the greater the confining pressure, the greater the amplitude of the increase, which shows that under the condition of high confining pressure, the internal fracture of shale rock is closed more closely under the action of confining pressure. Under the action of load, high confining pressure can effectively prevent the development of internal fracture of shale rock.

According to Figure 2, the relationship between confining pressure and elastic modulus E is:

$$E=897.14*\exp(-\sigma_3/-27.76)+4270 \tag{16}$$

The relationship between elastic modulus and confining pressure can be set as:

$$E=E_0*f*\exp(\sigma_3/g)+h \tag{15}$$

where, f, g and h are fitting parameters.

The damage constitutive model of shale under different confining pressures can be obtained by substituting Eq. (17) into Eq. (15):

$$\sigma_1=(E_0*f*\exp(\sigma_3/g)+h)\varepsilon\exp[-(\frac{a}{a_{\max}n^{\frac{1}{n}}})^{\frac{-1}{\ln(\frac{\sigma_{1\max}-2\mu\sigma_3}{E_0\varepsilon_{\max}})}}]+2\mu\sigma_3 \tag{16}$$

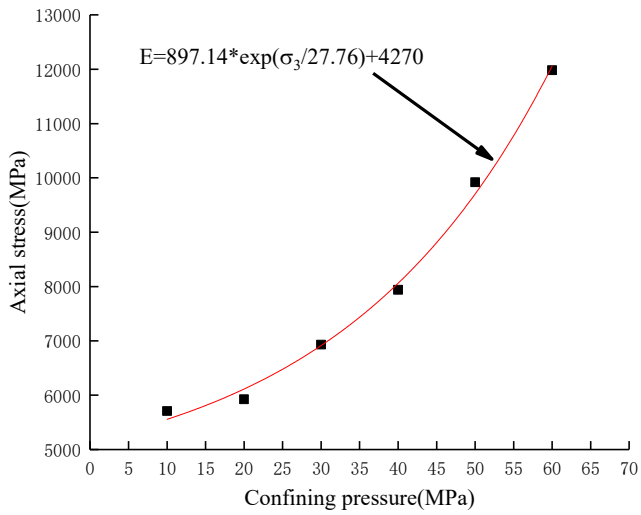

Figure 2. The elastic modulus of shale rock under different confining pressures

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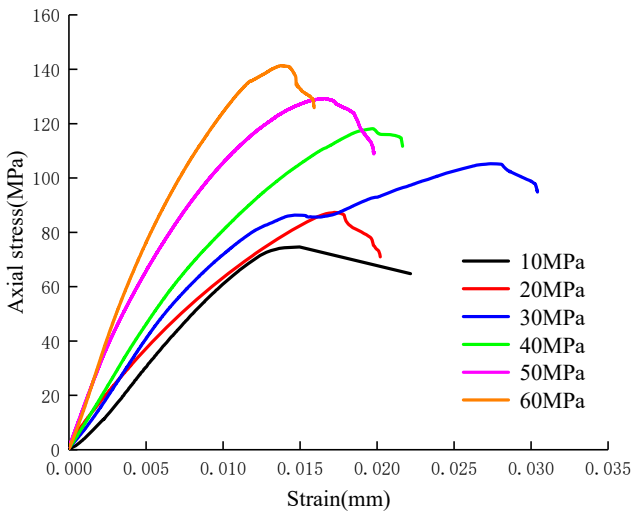

Figure 3. The axial stress-strain figure of shale rock under different confining pressures

3.2. Analysis of Effect

According to the elastic modulus and Poisson's ratio of shale rock under uniaxial compression, the damage constitutive model established in this paper is substituted. The parameter fitting is carried out by fitting software, and the development trend of shale rock damage variable under different confining pressures is obtained. At the same time, the calculation results are compared with the test results, and the results are shown in Figure 4.

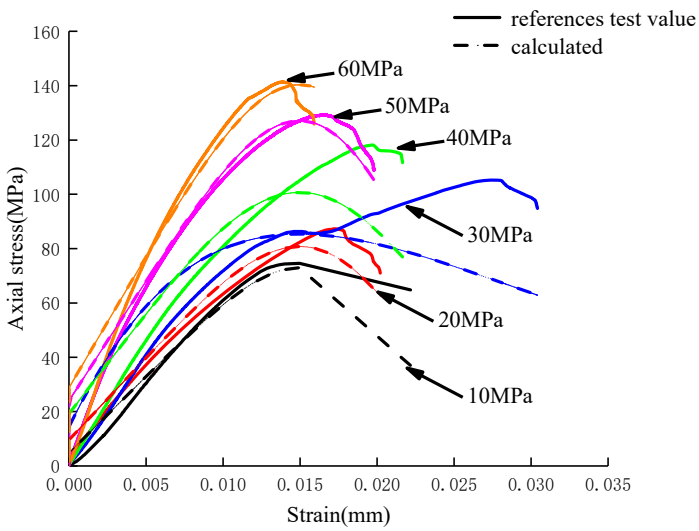

Figure 4. Teoretical and test curves under different confining pressure

It can be seen from Figure 4 that the damage constitutive models of shale rock under different confining pressures constructed in this paper can better express the variation trend of stress-strain curves of shale rock under different confining pressures. When the confining pressure is 30 MPa, the fitting degree between the calculated value and the test value is not high, which is due to the stress decrease in the loading process of shale rock, with the continuous application of axial load, the stress increases again. The damage constitutive model constructed in this paper is defined as: when the stress decreases, the rock damage. Therefore, the theoretical value

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cannot be calculated when the triaxial stress of shale rock rises again. In the elastic stage and plastic stage, the calculated value of the damage constitutive model established in this paper is basically the same as the test value in the reference, which shows that the damage constitutive model constructed in this paper can better express the development trend of shale rock in the elastic stage and plastic stage, so the constitutive model constructed in this paper has certain reference.

4. CONCLUSION

In this paper, the damage constitutive model of shale rock under different confining pressures is constructed according to the reference, and the following conclusions are obtained:

(1) In this paper, assuming that the internal micro-element of shale rock under confining pressure obeys Weibull distribution, the damage variable D is constructed according to the strain equivalent principle, and the damage constitutive model of shale rock under different confining pressures is established.

(2) The parameters of the damage constitutive model constructed in this paper are clear and can be obtained from the test, and the derivation process is clear.

(3) By fitting the test curves in the reference, it can be found that the damage constitutive model constructed in this paper can better reflect the stress-strain variation trend of shale rock under different confining pressures. However, if the stress of shale rock decreases during the test, the model cannot reflect the subsequent stress rise. The damage constitutive model constructed in this paper can better reflect the stress-strain variation trend of shale rock in elastic stage and plastic stage. The model has certain reference value.

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