Fixed-time Sliding Mode Tracking Control for Autonomous Surface Vehicles

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Abstract

In this paper, a fixed-time sliding mode trajectory tracking control method is proposed for an autonomous surface vehicles (ASV) system with perturbations and uncertainties. First, the dynamics model of ASV in three-dimensional space is simplified to the second-order dynamics with unknown model parameters and perturbations. Based on this, a sliding mode control law based on fixed time convergence is proposed to ensure that the trajectory of the ASV tracks on the reference signal in fixed time, while making the tracking error converge to 0 in fixed time. A sufficient condition for global fixed-time stability is derived using the Lyapunov function method. The proposed control method is not limited by the initial value of the system and thus outperforms the finite-time sliding mode control method. Finally, the effectiveness of the proposed method is verified by numerical simulations.

Keywords

Fixed time convergence; Sliding mode control; Autonomous surface vehicles; Lyapunov analyze.

1. INTRODUCTION

Interest in autonomous surface vehicles (ASVs) [1] has increased rapidly over the past few years. Compared to land-based vehicles, ASVs are located in relatively simpler environments with more open areas and fewer obstacles and targets for vehicle navigation. Therefore, it is used in a wide range of applications in various aspects such as courier management and transportation. Trajectory tracking control of ASVs [2] is a pressing problem in which the vehicle must precisely track a reference trajectory and thus complete the work task without relying on time constraints[3]. We know that the operation of ASVs is subject to complex external disturbances such as wind, rain, and obstacles, as well as unknown system uncertainties, both of which may negatively affect the control performance. Therefore, it is crucial to design a robust control method to overcome it.

The insensitivity of sliding mode control (SMC) to parameter variations and disturbances makes it a very effective robust control method. In recent years, some remarkable results have been achieved using SMC methods for the control of controlled systems. For example, a SMC method was proposed in the literature [4] for a class of linear systems with perturbations to achieve robust stability. In [5], a fractional-order terminal sliding-mode controller was designed to achieve trajectory tracking for a robotic manipulator with perturbations and uncertainties. The literature [6] used the SMC method for trajectory tracking control of ASVs. However, the state trajectories of the above methods all reach the sliding surface in a finite time, which makes the system reach stability in a finite time.

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Nowadays finite-time convergence has received the attention of many researchers and has led to many remarkable results [7]. For example, in the literature [8], a class of finite-time controllers was designed to ensure that the robotic manipulator tracks the upper reference trajectory in finite time. In the literature [9], an event-triggered strategy-based sliding mode controller was designed to guarantee the reachability of the sliding surface in a finite time. The literature [10] studied the finite-time control problem for unmanned vehicles. However, it still has poor convergence accuracy for systems with high convergence time requirements, such as unmanned vehicles, robotic manipulators, etc.

To achieve faster convergence accuracy, fixed-time convergence methods are proposed. Fixed-time convergence is a hot topic and a large number of researchers have devoted themselves to this research [11]. A very remarkable result was published this year in the literature [12], which applied the fixed-time convergence algorithm to different SMC methods to ensure the fixed-time stability of the system. A fixed-time attitude coordinated tracking control problem was solved for spacecraft in the literature [13]. A fixed-time convergent adaptive non-singular terminal sliding mode controller was proposed in the literature [14] for a robotic manipulator. To study the control problem of a second-order nonlinear system under the influence of nonmatching perturbations, a fixed-time sliding-mode controller was proposed in [15] to ensure the robust stability of the system with mismatch disturbance. Meanwhile, a new fixed-time observer is proposed in [16] to estimate the composite perturbations of a reusable launch vehicle, including fault information and uncertainty

Based on the above analysis, this paper proposes a fixed-time convergence SMC algorithm for ASV systems with perturbations and uncertainties to track the target trajectory and ensure the global robust stability of the closed-loop system. The controller has faster convergence accuracy than finite-time control in a complex and variable environment. Finally, it is proved by the Lyapunov function that all the state trajectories reach the sliding mode surface in fixed time and the error trajectories also achieve fixed time stability on the sliding manifold, and the simulation results verify the effectiveness of the proposed control method.

2. PROPERTIES

2.1. Problem statement and lemma

Consider a nonlinear system given by

$$\begin{aligned} \dot{x} &= f(x,t) \\ x(0) &= x_0 \end{aligned} \tag{1}$$

where $x \in R^n$ represents the state information, $f(x,t)$ is a continuous nonlinear part.

Lemma 1: If exists a system

$$\begin{aligned} \dot{x}(t) &= -k\left|x\right|^{\frac{\alpha x^2}{1+\beta x^2}}\operatorname{sgn}(x(t)) + d(t) \\ x(0) &= x_0 \end{aligned} \tag{2}$$

Where $x \in R^n$, $\alpha > 0$ and $\beta > 0$ such that $\varepsilon = (\alpha/(1+\beta)) > 1$ holds. And $d(t)$ is a unknown and bounded external disturbance, *i.e.*, $\|d(t)\| < d_0$. Thus, the system is globally fixed-time stable with the settling-time T satisfying

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$$T(x_0)=\frac{1}{(k-d_0)(\varepsilon-1)}+\frac{1}{ke^{\frac{-\alpha}{2e}}-d_0} \tag{3}$$

Before designing the fixed-time sliding mode controller, the dynamic model of the 3-DOF ASV is described as

$$\begin{aligned}\dot{p}&=R(\theta)v\\ M\dot{v}&=-C(\mathrm{v})v-D(v)+\tau+\tau_d\end{aligned} \tag{4}$$

where $R(\theta)\in R^{3\times3}$ is a rotation matrix, $M\in R^{3\times3}$ is the mass matrix and includes the effects of both rigid-body and added mass, respectively.

$$R(\theta)=\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0\\ \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix} M=\begin{bmatrix}M_{11} & 0 & 0\\ 0 & M_{22} & M_{23}\\ 0 & M_{32} & M_{33}\end{bmatrix} \tag{5}$$

In relation (4), $p=\begin{bmatrix}x & y & \theta\end{bmatrix}^T$ and $v=\begin{bmatrix}u & w & r\end{bmatrix}^T$ represent the position vector in the original coordinate system and the velocity vector in the transformed coordinate system separately, in which *x*, and *y* are the coordinates of the vehicle's center of gravity, and θ is the heading, *u*, *w* are the surge velocity, the sway velocity and *r* is the yaw rate; $\tau_d\in R^3$ is the disturbance of the external environment; $\tau\in R^3$ is the control input vector of ASV system. Meanwhile, $C(v)\in R^{3\times3}$ incorporates centripetal and Coriolis effects, and $D(v)\in R^{3\times3}$ is the damping matrix.

$$C(v)=\begin{bmatrix}0 & 0 & C_{13}(v)\\ 0 & 0 & C_{23}(v)\\ -C_{13}(v) & -C_{23}(v) & 0\end{bmatrix} D(v)=\begin{bmatrix}D_{11}(v) & 0 & 0\\ 0 & D_{22}(v) & D_{23}(v)\\ 0 & D_{32}(v) & D_{33}(v)\end{bmatrix} \tag{6}$$

The model of ASV is shown in Fig. 1.

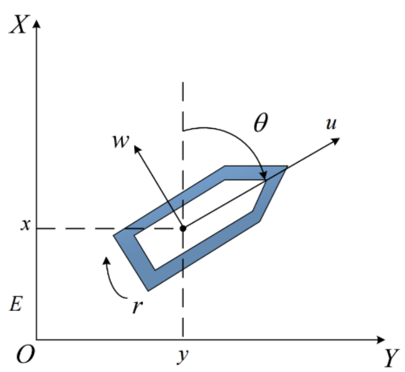

Figure 1. The description of 3-DOF ASV motion variable

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The parameters of the $C(v)$ and $D(v)$ matrices we give in the simulation section. Consider the uncertainty of the ASV system (4), *i.e.*,

$$C = C_0 + \Delta C, D = D_0 + \Delta D \tag{7}$$

Property1: The rotation matrix has the following property,

$$R^T(\theta)R(\theta) = I_3, \dot{R}(\theta) = R(\theta)W(r) \text{ and } R(\theta)W(r)R^T(\theta) = W(r) \tag{8}$$

Where $W(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Consider the uncertainty of the ASV system(6), setting $\xi_2 = \dot{p}$ from (6) and using the property of the rotation matrix $R(\theta)$, we obtain

$$\begin{aligned} \dot{\xi}_2 &= \dot{R}v + R\dot{v} \\ &= RW(r)v + RM^{-1}(-C_0(v) - D_0(v) + \tau) + d \\ &= W(r)Rv + RM^{-1}(-C_0(v) - D_0(v) + \tau) + d \end{aligned} \tag{9}$$

The lumped disturbance $d(t)$ contains two portions, which are the external disturbance and the disturbance due to modeling uncertainty, and can be expressed as

$$d = RM^{-1}\left(-\Delta C(v)v - \Delta D(v)v + \tau_d\right) \tag{10}$$

where ΔC and ΔD represent the uncertainty. Next, setting $\dot{\xi}_1 = \xi_2$ and $v = R^{-1}(\xi_1)\xi_2$, then system (4) can be converted as

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= f(\xi_1, \xi_2) + R(\theta)M^{-1}\tau + d \end{aligned} \tag{11}$$

where $f(\xi_1, \xi_2) = W(v)\xi_2 - R(\xi_1)M^{-1}C(v) + D(v)v$ is a nonlinear part.

Assumption 1: The disturbance d is bounded, *i.e.*, there exists a positive constant d_0, such that $\|d(t)\| < d_0$.

2.2. Problem statement and lemma

To describe the dynamic equation of the tracking error of the ASV, the desired signals p_d and v_d are given here

$$\begin{aligned} \dot{p}_d &= R(\theta)v_d \\ M\dot{v}_d &= -C_d(\mathrm{v})v_d - D_d(v) + \varphi_d \end{aligned} \tag{12}$$

where $p_d = \begin{bmatrix} x_d & y_d & \theta_d \end{bmatrix}^T$ represents the desired position vector, $v_d = \begin{bmatrix} u_d & w_d & r_d \end{bmatrix}^T$ represents the desired velocity vector and φ_d represents the input of the desired system. By setting $\xi_{d1} = \xi_d$ and $\xi_{d2} = \dot{\xi}_d$ the above system (12) is rewritten as

$$\begin{aligned} \dot{\xi}_{d1} &= \xi_{d2} \\ \dot{\xi}_{d2} &= Q_d(\xi_d, \varphi_d) \end{aligned} \tag{13}$$

where $Q_d(\xi_d, \varphi_d) = R(\xi_d)M^{-1}(\varphi_d + (C(v_d) - D(v_d)) + MW(v_d)v_d)$. For the desired trajectories $\xi_d = \begin{bmatrix} \xi_d & \dot{\xi}_d \end{bmatrix}$, the following assumption is true.

Assumption 2: The desired trajectories is bounded, *i.e.*, there exists a positive constant φ_0 such that $\|\varphi_d\| < \varphi_0$.

2.3. The tracking error dynamics

All of the above assumptions are reasonable due to practical constraints. Thus, the dynamics equation of tracking error of the ASV can be expressed as

$$\begin{aligned} \dot{\chi}_1 &= \chi_2 \\ \dot{\chi}_2 &= f(\xi_1, \xi_2) + R(\theta)M^{-1}\tau - Q_d(\xi_d, \varphi_d) + d \end{aligned} \tag{14}$$

where $\chi_1 = \xi_1 - \xi_{d1} = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \end{bmatrix}^T$ represents position tracking error, $\chi_2 = \xi_2 - \xi_{d2} = \begin{bmatrix} \chi_{21} & \chi_{22} & \chi_{23} \end{bmatrix}^T$ is velocity tracking error. To achieve the ASV track the reference signal, it is essential to introduce tracking errors. The control of the system (4) is then converted to the control of the error system (14). Once all the tracking errors converge in a fixed time, then the ASV system reaches fixed time stability.

3. DESIGN OF FIXED TIME SLIDING MODE CONTROL LAW

SMC is a commonly used method in nonlinear systems with remarkable robustness to uncertain parameters and external disturbances. The sliding mode control law brings the state or error to the sliding mode surface and keeps it there for the subsequent time. SMC consists of two main steps, the first is to select a suitable sliding mode surface. The second step is to design the sliding mode control law to achieve robust stability. The purpose of this paper is to propose a fixed convergence nonlinear sliding mode surface and sliding mode controller that can achieve global robust fixed time stability of the system.

3.1. Design of the sliding mode surface

In this subsection, we first construct a nonlinear sliding surface $s(\chi)$ as

$$s(\chi) = \chi_2 + \lambda \|\chi_1\|^{\frac{\alpha_1 \chi_1^2}{1+\beta_1 \chi_1^2}} \operatorname{sgn}(\chi_1) \tag{15}$$

where $s(\chi) = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T$. The derivative of the sliding mode variable with respect to time is

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$$\dot{s}=f(\xi_1,\xi_2)+R(\theta)M^{-1}\tau-Q_d(\xi_d,\varphi_d)+\frac{\lambda\alpha_1\|\chi_1\|\chi_2}{1+\beta_1\chi_1^2}(\frac{2\ln\|\chi_1\|}{1+\beta_1\chi_1^2}+1)\|\chi_1\|^{\frac{\alpha_1\chi_1^2}{1+\beta_1\chi_1^2}}+d \tag{16}$$

Considering Lemma 1 and sliding variables, the sliding mode tracking control law for fixed time convergence of the error trajectory is

$$\tau_1=-MR^{-1}[f(\xi_1,\xi_2)+Q_d(\xi_d,\varphi_d)+\frac{\lambda\alpha_1\|\chi_1\|\chi_2}{1+\beta_1\chi_1^2}(\frac{2\ln\|\chi_1\|}{1+\beta_1\chi_1^2}+1)\|\chi_1\|^{\frac{\alpha_1\chi_1^2}{1+\beta\chi_1^2}}] \tag{17}$$

$$\tau_2=-MR^{-1}[k\|s\|^{\frac{\alpha_2 s^2}{1+\beta_2 s^2}}\operatorname{sgn}(s)] \tag{18}$$

$$\tau=\tau_1+\tau_2 \tag{19}$$

where $\varepsilon_2=\alpha_2/(1+\beta_2)$, and $k>d_0e^{\frac{\alpha_2}{2e}}$ as control parameter. In practical applications of ASV, the dynamics always have nonlinearities, including reference trajectories and rotation angles, which may seriously degrade the performance of the closed-loop system and even lead to instability. Considering these nonlinear parts, it is essential to construct a controller to achieve the desired performance. In this section, a sliding-mode controller (19) is designed based on fixed-time convergence to enable the tracking error trajectory (14) to be forced to drive onto a predefined sliding surface at a fixed time and achieve fixed-time stability of the state trajectory at all subsequent times. In contrast to the linear sliding surface with fixed convergence, since the sliding surface designed in this paper is a nonlinear sliding surface, all system trajectories will also converge fixedly on the sliding surface after all the sliding trajectories arrive at the sliding surface in fixed time. Therefore, the fixed time stability of the error dynamics is ensured.

3.2. Main results and stability analysis

Proposition 1: The closed-loop system (14) is globally fixed-time stable and fixed-time satisfying

$$T(\chi_0)\le\frac{1}{(k-d_0)(\varepsilon_2-1)}+\frac{1}{ke^{\frac{-\alpha_2}{2e}}-d_0}+\frac{1}{\lambda(\varepsilon_1-1)}+\frac{1}{\lambda e^{\frac{-\alpha_1}{2e}}} \tag{20}$$

where $\varepsilon_1=\dfrac{\alpha_1}{1+\beta_1}>1$, $\varepsilon_2=\dfrac{\alpha_2}{1+\beta_2}>1$.

Proof: Given the nonlinear sliding mode surface (15), then construct a suitable Lyapunov function $V_s=0.5s^Ts$. The derivative of thefunction with respect to time is given by

$$\begin{aligned}\dot{V}_s&=s^Ts\\&=s^T(f(\xi_1,\xi_2)+RM^{-1}\tau(t)-Q_d(\xi_d,\varphi_d)+\frac{\lambda\alpha_1\|\chi_1\|\chi_2}{1+\beta_1\chi_1^2}(\frac{2\ln\|\chi_1\|}{1+\beta_1\chi_1^2}+1)\|\chi_1\|^{\frac{\alpha_1\chi_1^2}{1+\beta_1\chi_1^2}}+d(t))\\&=s^T(-k\|s\|^{\frac{\alpha_2 s^2}{1+\beta_2 s^2}}\operatorname{sgn}(s)+d(t))\end{aligned} \tag{21}$$

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According to Lemma 1, it is deduced that the system (14) starts from $s(0)=s_0$ and reaches the sliding surface in a fixed time $T(s_0)$. The reachability of the sliding surface in a fixed time has been proved by the Lyapunov analysis described above. The accessibility of the sliding surface ensures that any primitive state in space can reach the sliding surface in a fixed time without restriction in the approach process $s \to 0$. In addition, $T(s_0)$ is given as follows

$$T(s_0) \le \frac{1}{(k-d_0)(\varepsilon_2-1)} + \frac{1}{ke^{\frac{-\alpha_2}{2e}} - d_0} \tag{22}$$

From equation (15), it can be seen that once all the state trajectories reach the sliding mode surface, the system state can be written as

$$\dot{\chi}_1 = \chi_2 = \lambda \|\chi_1\|^{\frac{\alpha_1 \chi_1^2}{1+\beta_1 \chi_1^2}} \operatorname{sgn}(\chi_1) \tag{23}$$

Invoking Lemma 1 once more, it is concluded that $\chi(t)$ starting at $\chi_1(0)=\chi_{10}$ reaches the origin in a fixed time satisfying

$$T(\chi_0) \le \frac{1}{\lambda(\varepsilon_1-1)} + \frac{1}{\lambda e^{\frac{-\alpha_1}{2e}}} \tag{24}$$

Finally, the closed-loop system (14), (15), and (19) reach the origin at a fixed time, which is bounded by (20). We can conclude that a sliding mode surface (15) ensures that the sliding trajectory arrives at the sliding manifold at a fixed time $T(s_0)$, and also ensures that the system trajectory is stable at a fixed time $T(\chi_0)$ within the sliding manifold. This completes the proof.

Remark 1: In this paper, the global robust fixed-time stabilization of the system states (4) and (14) is achieved by using a nonlinear sliding variable (15) with exponential coefficients of the state-dependent variables and the designed sliding-mode controller (19). Therefore, the closed-loop system (14) can reach the origin in a fixed time and is not limited by the initial value of the system state.

4. SIMULATION

Consider an autonomous surface vehicle systems with uncertainty and disturbance given by [9]. and $\tau_d = [2\sin(0.2t)+2 \quad 1.5\cos(t)-2 \quad \cos(2t)-1]$ as unknown disturbance. The uncertainty of the system is assumed to be $\Delta C = 0.1C_0, \Delta D = 0.1D_0$. The initial states of ASVs are given as $p=[2 \quad -3 \quad 1]$ and $v=[0 \quad 0 \quad 0]$. Some important control parameters are selected as $\alpha_1 = 1.4, \beta_1 = 0.01$, $\alpha_2 = 2.4, \beta_2 = 0.3$. The desired position signal is chosen $[\sin(t) \quad \cos(2t) \quad 2\sin(2t)]$. Meanwhile, the control gain is chosen as $k=20$. The simulation lasts for 20 seconds and the results are shown below.

Fig 2 gives a comparison of the actual running trajectory and the reference trajectory, where (a) is a comparison of the position signal and (b) is a comparison of the velocity signal. It can be seen that both position trajectory and velocity information are tracked on the reference signal. The evolution of the trajectory tracking error with time is given in Fig 3, where (a) is the position

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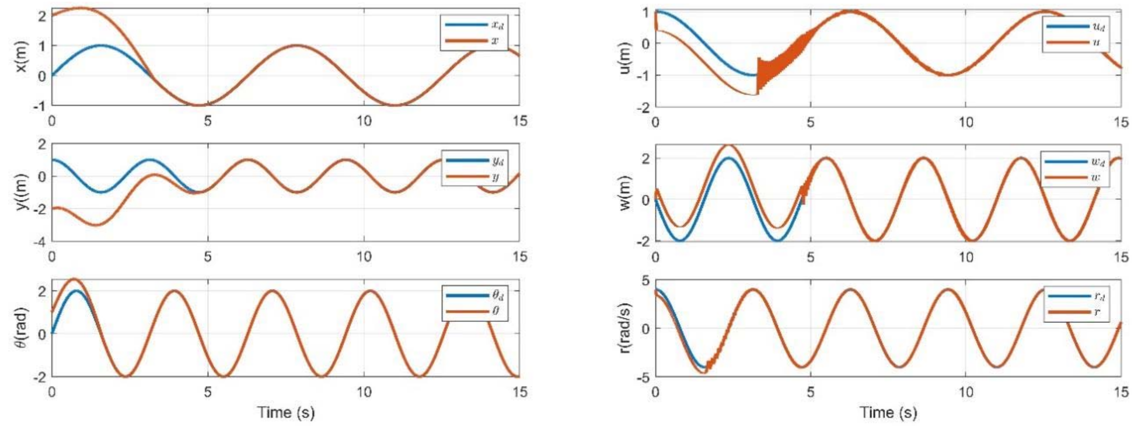
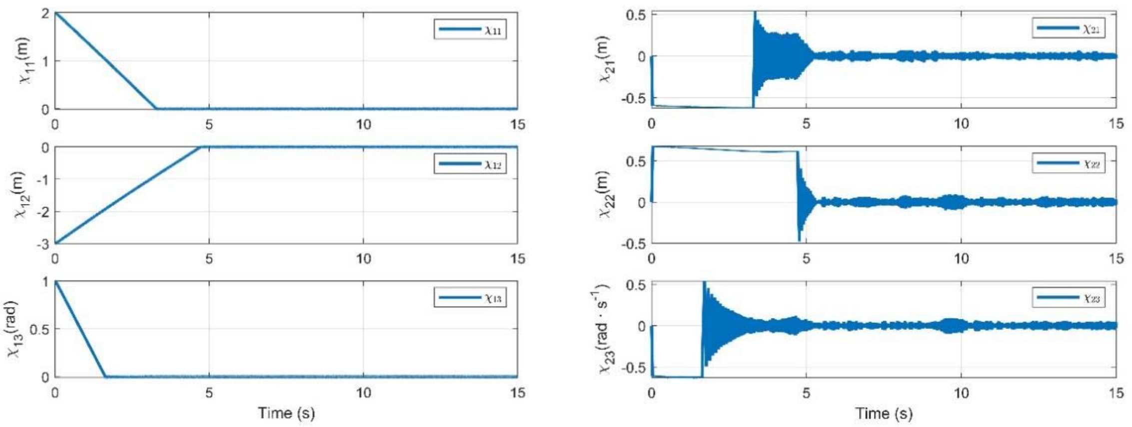
tracking error and (b) is the velocity tracking error. It can be seen that all the errors converge to 0 in a fixed time, which is sufficient evidence that the fixed-time sliding mode control law achieves robust stability of the system. A comparison of the phase diagrams of the actual and reference trajectories is given in Fig 4, and it can be seen that the tracking effect is satisfactory. The sliding mode trajectories are given in Fig 5, and it can be seen that all the sliding trajectories converge to 0.



(a) (b)

Figure 2. Diagram of the comparison chart of ASV's trajectory and reference trajectory.



(a) (b)

Figure 3. Diagram of the tracking errors of ASV.

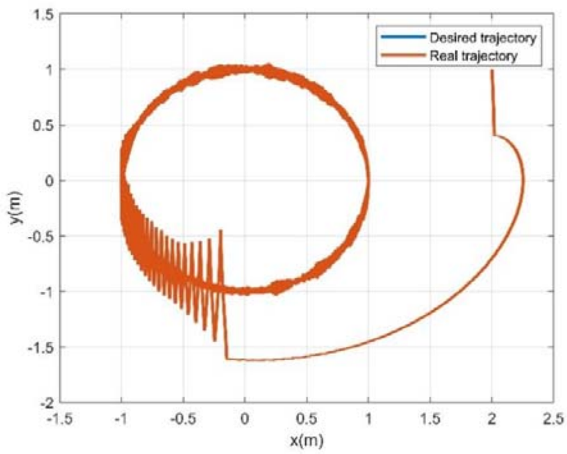


Figure 4. Diagram of reference and actual trajectories.

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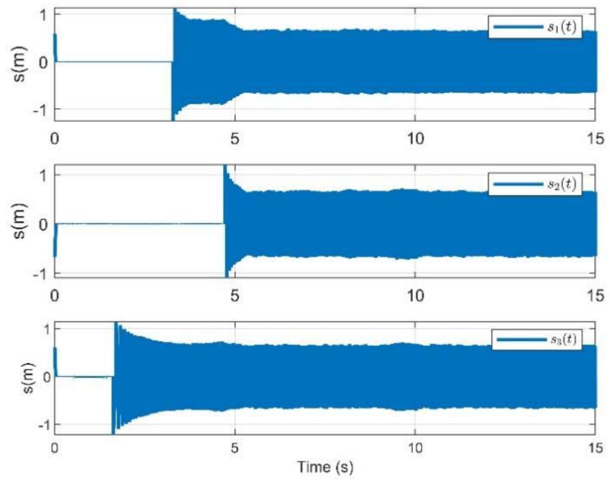

Figure 5. Diagram of sliding mold trajectory of ASV.

5. CONCLUSION

In this paper, a fixed-time SMC method is proposed to achieve tracking control and robust stability of the system for the trajectory tracking problem of the ASV system. A nonlinear sliding mode surface is designed so that not only all state trajectories reach the sliding manifold in fixed time, but also ensures that the system can converge to the origin in fixed time on the sliding mode surface as well, and the convergence time is independent of the original conditions of the system.

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