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Exploring the Integration of Ideological and Political Elements into Mathematical Courses

-- Take The Course of Mathematical Analysis as An Example

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Abstract

"Mathematical Analysis" is an important foundational course for mathematics majors in higher education institutions. It is widely used in many fields such as engineering, technology and scientific research. It is a powerful tool for solving practical problems and cultivating technological innovation abilities. By exploring the ideological elements in "Mathematical Analysis", students can learn the spirit of mathematicians' perseverance in exploration, cultivate students' love of life and positive spirit. The gradual infiltration of ideological and political education enhances students' sense of national pride, shapes their correct outlook and values towards life, and cultivates their resilience in the face of challenges.

Keywords

Course on Ideological and Political Education; Mathematical Analysis; Teaching Exploration.

1. INTRODUCTION

The course of ideological and political education is essentially a type of education that aims to foster moral character and cultivate talented individuals[1]. The primary task of education is to "foster virtue," focusing on the organic unity of imparting knowledge, solving doubts, and raising people and cultivating talent. This fine tradition has continued to the present day. Therefore, the concept of ideological and political education aims to strengthen the education of students' world-views, life-views, and values, inherit and innovate the excellent traditional Chinese culture, actively guide contemporary students to establish correct views of the nation, ethnicity, history, and culture, thus cultivating more comprehensive talents with moral, intellectual, physical, aesthetic, and labor development, and nurturing qualified builders and reliable successors for the cause of socialism with Chinese characteristics.

"Mathematical Analysis" is an important basic subject for various majors in higher education institutions. It has wide applications in engineering technology, scientific research, and many other fields. It is not only a powerful tool for solving practical problems but also a carrier for cultivating technological innovation ability. However, at present, teaching of mathematical analysis, so many emphasis is often placed on theoretical proof and problem-solving training, neglecting the practical significance of abstract concepts and the application of mathematical ideas, such as limits and integrals in professional fields, as well as neglecting the educational function of the course. Students may develop a fear of difficulty due to the obscure and difficult-to-understand concepts, and even after learning, they may only know the operation methods but incomprehension their essence. Therefore, we should actively explore innovative teaching models for mathematical analysis, organically combining the teaching of basic mathematics courses with the needs of majors, deeply mining ideological and political elements in various

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professional courses of colleges and universities, and integrating them into professional courses to enhance students' overall quality. This is an important path to achieving all-round education and an inevitable choice for the goal of education and teaching reform in the new era.

2. ANALYSIS OF TEACHING STATUS OF MATHEMATICAL ANALYSIS COURSE

"Mathematical analysis" is an important foundational course for undergraduate students majoring in mathematics. The course systematically introduces the basic theoretical knowledge of real analysis, limit theory, single-variable differential calculus, single-variable integral calculus, multivariable differential calculus, multivariable integral calculus, and series theory. By studying these fundamental mathematical theories and methods, students further develop their understanding of the basic concepts, theories, and skills of mathematical analysis. The course also cultivates students' abilities in rigorous logical thinking, analysis, and problem-solving, laying a solid foundation for the study of subsequent mathematics courses.

However, due to the lengthy duration, high level of difficulty, and heavy teaching workload of the current mathematical analysis course, many teachers resort to a "cramming" teaching style, which directly or indirectly affects students' interest in learning. Therefore, teaching reform is imperative. Based on the characteristics of the course, integrating ideological and political elements with professional knowledge plays an extremely important role in shaping students' outlook on life and values.

3. CURRICULUM IDEA ELEMENT DIGGING

The core goal and key content of ideological and political education in curriculum is to inject correct values into knowledge impartation and ability development, helping college students establish accurate worldviews, life philosophies, and value systems[2]. Mathematical analysis contains numerous ideological and political elements, including scientific literacy, humanistic spirit, cultural confidence, philosophical insights, ideal pursuits, and mathematical aesthetics.

3.1. Ideological and political elements in real number theory

The theory of real numbers is the topic covered in the first chapter of mathematical analysis, which serves as a bridge between high school and university mathematics. It contains rich philosophical ideas and scientific literacy. The first chapter of "Mathematical Analysis" covers irrational numbers and sets, which are related to the first and third mathematical crises. By introducing the first and third mathematical crises, it lays the foundation for learning the second mathematical crisis related to calculus with Newton and Leibniz.

In the realm of real number theory, we can begin by discussing the history and evolution of numbers with students. From the well-known natural, whole, and rational numbers, it was once believed that all numbers could be expressed as a ratio of two integers, or a rational number. It was also thought that between any two given line segments, a third line segment could always be found, dividing the two into integer segments. The Greeks referred to these two line segments as "measurable." However, the diagonal of a square and one of its sides were found to be immeasurable, or unable to be expressed as a ratio of integers, leading to the first mathematical crisis. Today, we know that $\sqrt{2}$ is an irrational number, and the set of rational numbers is comprised solely of ratios of integers and cannot include irrational numbers. How, can we come to comprehend and understand incommensurability?

The Pythagorean school proposed the concept of "monas" to try and eliminate it. A "monas" is such a small unit of measurement that it cannot be measured itself, yet must remain a unit of measurement. However, this led to a new paradox: Zeno proposed that a "monas" is either zero or not zero. If it is zero, then adding an infinite number of "monads" still would not result in a length. If it is not zero, then a finite line segment made up of an infinite number of "monads"

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should be infinitely long. In any case, it is a contradiction no matter how you look at it. About a century later, the Greek mathematician Eudoxus (circa 408 BC-355 BC) solved this crisis partially. He cleverly introduced a new concept of "the ratio of two quantities," which avoided the essence and complexities of irrational numbers $\sqrt{2}$, and used the method of proportion in geometry to deal with incommensurability. As a result, the foundation of geometry became solid, and it stood out among all branches of mathematics. Euclid also adopted this approach in his book "Elements of Geometry," which made geometry the basis of almost all rigorous mathematics for the next two thousand years. However, the complete resolution of this crisis relied on the expansion of the number system in the 19th century. It wasn't until humanity recognized the foundation of real numbers and established the real number system that this crisis was considered to be completely resolved, which happened over two thousand years later.

By introducing the history of mathematics, not only can we attract students' interest in learning, but we can also help them understand that some truths in geometry are not related to arithmetic, and geometric quantities cannot be completely represented by integers and their ratios. On the contrary, numbers can be expressed by geometric quantities. The revered status of integers was challenged, and the mathematical viewpoint of ancient Greece was greatly impacted. Therefore, geometry began to occupy a special position in Greek mathematics. This also reflects that intuition and experience are not necessarily reliable, and reasoning and proof are. From then on, the Greeks started with "self-evident" axioms, through deductive reasoning, and thus established a system of geometry. This was a mathematical revolution and a natural product of the first mathematical crisis.

Through the study of the history of number system development, students will also gain an understanding of the twists and turns in the development of mathematics and the relentless persistence of mathematicians in solving problems. From this crisis in mathematics, we can also learn several lessons: 1. From a historical perspective, Greek mathematics evolved from arithmetic to geometry. 2. From a thinking perspective, intuition and experience may not be reliable, but reasoning and proof are. 3. From a mathematical perspective, logic must not have any flaws or omissions in basic principles within a mathematical system, and any such occurrence must be rectified. 4. From a learning perspective, while studying mathematics and science, one may encounter many setbacks and pressures, but we should not be afraid of difficulties and persist in the pursuit of truth!

By incorporating the history of mathematics, students will not only learn about the development of real numbers, but also lay a solid foundation for future learning. This will prevent the study of "Mathematical Analysis" from getting lost in complex and tedious formulas and theorems right from the start, and will also ignite students' desire to learn subsequent courses.

3.2. Ideological and political elements in the limit theory

The theory of limits is one of the most fundamental and central topics in "Mathematical Analysis." It serves as a bridge between the abstract concept of finite to infinite learning for students and lays the foundation for subsequent calculus studies. For most students, it is extremely challenging to comprehend. The study of functions through the tool of limit thinking involves theory and methods related to limits almost throughout the whole process.

3.1.1 Ideological and political elements in the function limit

In ancient China, there was an early idea of limits, such as "taking half of a one foot stick every day and it will never run out" from the book of Zhuangzi and Liu Hui's method of cutting a circle, "cutting it finer loses little, cutting repeatedly until it can no longer be cut, it then becomes one with the circumference without losing anything.[3]" These concepts not only inspire students' interest in learning and cultivate innovative spirits but also strengthen their cultural confidence.

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The concept of function limit can be abstracted into a model where life goals are seen as functions and the processes of achieving those goals are akin to reaching its limit. Though one's set goals may differ from reality, as long as one perseveres in their pursuit of them, they can still appreciate the beauty along the journey of life. The operational rules of limits also contain rich philosophical insights for life: one should adhere to principles when dealing with others and follow rules when performing tasks.

3.1.2 Ideological and political elements in the important limit

An important irrational number e in "Mathematical Analysis" (Volume One), which appears in the second important limit. In 1683, Swiss mathematician Jacob Bernoulli studied a sequence $\left(1+\frac{1}{n}\right)^n$ and found that it eventually settles between 2 and 3. It was not until 1728 when Swiss mathematician Euler first used to represent this limit. In our daily life, we can showcase mathematical culture education by using multimedia to display geometrical concepts like spider webs and catenary curves. This can help eliminate the misconception that math is dull and lifeless for university students and allow them to appreciate the beauty of math, understand its applications, and use a mathematical perspective to discover the beauty of life and describe it in mathematical language. This can lead to an optimistic and positive outlook on life.

3.3. Ideological and political elements in derivative and its application

3.3.1 Ideological and political elements in derivative concept

In high school, students generally learn how to find derivatives, but they may not fully understand the nature of derivatives. The concept of derivatives in mathematical analysis was originally introduced by French mathematician Fermat to study extremum problems, but it was British mathematician Newton and German mathematician Leibniz who established the concept of derivatives directly in their research on mechanics and geometry, respectively. By introducing the mathematical background, students can better understand the essence of derivatives.

3.3.2 Ideological and political elements in extreme value problem

The issue of extrema holds a significant importance in the application of derivatives. This part forms a crucial component of applied mathematics and serves as a foundation for subsequent courses in operations research and calculus of variations. The problem of extrema is not only related to the maximum and minimum in everyday life but also embodies rich philosophical lessons[4]. When teaching the issue of extrema, it is essential to emphasize the definition of extrema and maxima. Extrema are local and depend only on the function values in the vicinity of a point, while maxima are global and depend on the function values at all points. Extrema are like frogs at the bottom of a well, who only see the sky above the well; whereas, maxima see the world with a broad perspective. This can motivate students to expand their learning horizons, to take a broader perspective and to have a global outlook while reflecting on their country and the world.

3.4. The ideological and political elements of differential mean value theorem and its application

3.4.1 The ideological and political elements in Rolle's theorem

According to Fermat's theorem, Rolle's theorem is given in combination with life examples, riding a roller coaster, feeling the instantaneous speed of the highest and lowest points. If a function $f(x)$ satisfies the following conditions[5]:

(1) it is continuous on a closed interval $[a,b]$;

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(2) it is differentiable on an open interval (a,b);

(3) the function values are equal at both endpoints, i.e. $f(a)=f(b)$, then there is at least one point ξ in the interior ,such that $f'(\xi)=0$.

In mathematics, the Mean Value Theorem is mainly used to study the changes in the shape and geometric properties of functions. The related concept can be traced back to the ancient Greek mathematicians' study of the tangent to the parabolic arch, and later Archimedes applied it to calculate the area of the parabolic arch. Over centuries of development, more mathematicians such as Cauchy and Lagrange gave different proofs of the Mean Value Theorem from various perspectives and further applied it to the study of functions. By narrating historical stories, we can inspire students' interest in learning and guide them to learn the persevering spirit of mathematicians.

Finally, we can encourage students to think about the conditions that satisfy Rolle's Theorem. With the help of graphs, the conclusion we can draw is that, just like our life, it has its ups and downs. No matter we are at the pinnacle or the nadir of life, we must not forget our original intentions, maintain a peaceful attitude, and keep moving forward constantly.

Ideological and political elements in convexity and inflection point of function

Through learning about the convexity and turning points of functions, we can inspire students that ups and downs are inevitable in life. We need to take the winding roads to achieve our goals. At the turning points in our lives, we must know how to seize opportunities, maintain a positive and optimistic attitude towards life, and face it with a smile.

3.5. Ideological and political elements in calculus

3.5.1 Development history of calculus

Throughout history, mathematics and philosophy have been inseparable, and in a sense, mathematics itself is philosophy. Differential and integral calculus, as the core concepts of mathematical analysis, serve as the best interpretation of basic philosophical principles and dialectics, whether from their own perspective or in relation to each other. The history of the development of calculus is a history of human civilization exploring nature and society. The creation of calculus was the result of the unremitting efforts of scientific masters in the 17th century, such as the Bernoulli family, Newton, Leibniz, Euler, Cauchy, and other mathematical giants, whose spirit of pursuing truth deserves to be learned and emulated by future generations. When introducing basic concepts and theorems of calculus such as infinitesimal, mean value theorem, fundamental theorem of calculus, L'Hopital's rule, Taylor series, etc., it is important to explain the mathematical story behind them. From practical issues such as instantaneous rate of change, tangent lines, measurements, gravity, etc., after half a century of fermentation, to the general refinement of fluxion, differential calculus, and other concepts, the creation of calculus went through a process of repeated sublimation from concrete to abstract, knowledge and practice. Through various teaching methods such as case studies, it is important to reproduce and showcase the original path of Newton, Leibniz, and others' invention of calculus. These research experiences are invaluable raw materials for cultivating students' innovative ability.

3.5.2 Aesthetics in integral science

In the sections of multiple integrals, curve integrals, and surface integrals, various functions, fractal shapes, and spatial solids can be showcased through multimedia, allowing for the appreciation of the beauty of mathematics, the sparking of learning interests, the cultivation of innovative thinking, and the improvement of aesthetic literacy.

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3.6. Ideological and political elements in the series theory

3.6.1 Ideological and political elements in the series of several terms

When dealing with a series of numbers, does the result involve adding an infinite number of numbers together or does it approach infinity? A story that can be shared with students is that of Achilles, the hero in Homer's epic who was known for his speed but could not catch the tortoise. Achilles always had to reach the starting point of the tortoise, therefore the tortoise always remained ahead. This is an apparent paradox and was part of the second crisis in mathematics, involving the infinite divisibility of time and space, and the continuity of motion, which led to contradictions. After many people's hard work for many years, calculus emerged in the late 17th century to solve such paradoxes with infinitesimals. The study of series of numbers involves a wealth of philosophical questions in mathematics, guiding students towards adopting the rigorous scientific attitude of scientists and their persistent quest for truth.

3.6.2 Ideological and political elements in Fourier series

The images of Fourier series of general cycles can improve students' mathematical aesthetic literacy, appreciate some fractal graphs, stimulate students' interest in learning and have a certain desire to explore the follow-up courses, and thus tell the relationship between the party and the masses as concentric circles, which is also the embodiment of fractal graphs, so as to realize the ideological and political thinking of the course.

4. TEACHING STRATEGIES FOR INTEGRATING IDEOLOGICAL AND POLITICAL THINKING INTO PROFESSIONAL COURSES

4.1. Three values of ideological and political curriculum

Curriculum thinking and politics is not curriculum help thinking and politics, nor is it ideological and political help curriculum, but mutual promotion. Curriculum ideology and politics contain three values, namely: emotional value, willingness value, connection value, the three values affect each other, can not be separated.

4.1.1 Emotional value

Emotions often arise from unmet needs and gaps, acting as a gatekeeper for long-term memory and affecting learning. The study of mathematical analysis is often dry, difficult and abstract, causing students to retreat. Behind this is the pain they experience from learning, so it is important to pay attention to the emotional needs of students. Introducing political education in the curriculum is a way to meet these emotional needs and create an impact. For example, while teaching math, incorporating historical stories can impact the emotions of the students and resonate with the persistent spirit of mathematicians, making the knowledge easier to remember.

4.1.2 Willing value

Behind the concept of "volition" lies a person's understanding of their own identity and their values. Volition is often directed towards specific individuals or situations, and the more specific it is, the more easily we can be moved by it. Teachers' volition is to see their students gain knowledge and to impart correct values, life views, and world views. As for students, their volition of course is to be able to learn math knowledge with ease and happiness, to smoothly complete their studies, to learn useful knowledge for society, and ultimately to give back to society.

4.1.3 Connection value

Connect with history to exhibit cultural confidence; connect with reality to learn from the spirit of predecessors; connect with the world to bring value to society.

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In short, all values are driven by motives, guiding us to conduct ideological and political education in courses from multiple perspectives.

4.2. Three strategies for integrating curriculum ideology and politics into professional courses

The course "Mathematical Analysis" covers various abstract theoretical knowledge such as limits, real numbers, functions, calculus, and series, with a high level of difficulty. Teachers should avoid the "transmission" method during lectures and use a flexible approach to explain the knowledge alongside appropriate case analysis, naturally integrating ideological elements to inspire students to explore the essence behind abstract theoretical knowledge, stimulate their interest in exploring the unknown, and fulfill the fundamental task of cultivating students' morality.

To achieve this goal, we must first understand the three values of ideological education in the curriculum. We need to unconsciously integrate ideological education into professional knowledge by starting from specific events or cases, relying on stories as carriers, such as the stories of mathematicians. Stories are often emotionally rich and can move people's hearts. Coupled with authentic and credible case analysis or knowledge exercises, students can deeply understand the ocean of mathematics and achieve learning goals while also embodying their values[6].

4.2.1 Cognitive strategy

Some students may encounter cognitive obstacles when studying mathematical analysis. Initially, their perception may be similar to that of secondary school mathematics, with clearly expressed concepts and principles. However, the fact is that mathematical analysis requires strong theoretical foundations and may be quite abstract, which can pose certain difficulties in understanding. Therefore, as educators, we should provide adequate knowledge bridging between high school and university levels, and supplement instruction with relevant materials to overcome the cognitive barriers. When cognition changes, students' attitudes towards learning will naturally change, and the fear of mathematical analysis will diminish, leading to significant improvement in learning outcomes.

4.2.2 Affective strategy

Emotions arise from disparities. Teachers can make use of mathematical stories to convey the perseverance and hard work of mathematicians, inspiring students to adopt the same spirit. This can cultivate a fearless attitude towards difficulties and embrace challenges. By introducing the historical achievements of ancient mathematicians, cultural confidence can be enhanced and students can be encouraged to take responsibility as the masters of their own fate, striving to contribute to their country. Additionally, designing relevant cases for different knowledge areas, especially those related to real-life, can allow students to truly appreciate the charm of mathematics. Once emotions are involved, the learning attitude naturally changes and the learning outcomes will improve accordingly.

4.2.3 Behavioral strategy

Behavior is based on facts and beliefs, and after cognitive and emotional enhancement, teachers need to play a role of exemplifying guidance. Guiding students through questioning and setting up real-life examples are effective ways to integrate ideological and political elements into teaching. Teachers can also demonstrate how to solve challenging problems, allowing students to acquire knowledge through imitation and highlighting the values of the curriculum. Additionally, various teaching methods can be used to increase the appeal and interest of the course, reflecting mathematical aesthetics, philosophy, perseverance, cultural confidence, as well as patriotism.

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In short, by integrating ideological and political elements into mathematical analysis courses at the cognitive, emotional, and behavioral levels, we can ensure that the course has a foundation, direction, and results in ideological and political education. Ultimately, the goal of ideological and political education can be achieved.

5. PERORATION

"Mathematical analysis" is an important basic course in universities, which contains rich ideological and political elements. How to effectively integrate the content and spirit of ideological and political education with the mathematical analysis course is an important research topic. In the teaching process, we can further explore the ideological and political elements of the course, appropriately introduce ideological and political case studies, increase students' interest in learning, cultivate their rigorous academic attitude and perseverance in overcoming difficulties, and cultivate their awareness and ability to raise questions, explore problems, and solve problems, truly achieving "organic unity of value guidance, knowledge education, and ability cultivation". Let mathematics analysis teaching play its due role in implementing the goal of cultivating students with high moral character.

REFERENCES

[1] JIN Liuhe, GAO Zhe. Some reflections on "curriculum thinking and politics" [J]. Modern vocational education, 2017(18):1.

[2] ZHANG Fengqin, YANG Jianya. Study on the classroom teaching mode of Mathematical Analysis[J]. Educational theory and practice,2010(36):54-55.

[3] YAN Wenyong, WU Yamei et al. Study on the Ideological and Political Reform of Higher Mathematics Course[J]. Journal of Chengdu Institute of Technology, 2020,23(2):97-99.

[4] GAO Hongya. Some Cases of Ideological and Political Education Through all Courses in the Course of “Mathematical Analysis”[J]. Journal of Bao Ding University, 2020, 33(5):112-115.

[5] School of Mathematical Sciences, East China Normal University. Mathematical Analysis: Volume one[J]. 5th edition. Beijing: Higher Education Press,2019.

[6] WU Huizhuo. Exploration and Reflaction on Infiltrating Ideological and Political Education Based on Curriculum in Higher Mathematics Teaching [J]. College mathematics, 2019,35(5):40-43.