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# The Application of "Disproof Method" in Mathematics

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## Abstract

**It is characterised by its simplicity and practicality, and its unique approach to problem solving and thinking is of great significance to the development of students' logical and creative thinking. However, there is a lack of comprehensive research on the "disproof method". In view of this, this article starts from the meaning and classification of the disproof method, and makes an overall analysis of its theoretical basis and application conditions.**

## Keywords

**Disproof method; Thinking skills; Applied research.**

## 1. INTRODUCTION

When solving mathematical problems, we usually start from the front and follow the conventional way of thinking, this kind of forward thinking can solve many mathematical problems, but not everything. This is because we often encounter problems that are more tedious, difficult or even impossible to solve from the front, so we can choose to start from a dialectical point of view, using the reverse way of thinking, overcoming the traditional thinking stereotypes, from the opposite direction of our customary thinking to think and analyse, which is a frequent method of problem solving in mathematics - the disproof method.

## 2. MEANING AND CLASSIFICATION OF THE DISPROOF METHOD

Some mathematical problems are difficult to solve if they are carried out in the positive direction, so you should turn to the negative direction, i.e. use the "positive difficulty is negative" strategy.

The converse method is also divided into the reductio ad absurdum (i.e. there is only one reverse case) and the exhaustive method (i.e. there is more than one reverse case), noting that all the reverse cases should be listed out and negated one by one. The general steps of the converse method are: counterfactual (foundation), reductio ad absurdum (key), and truth (purpose).

## 3. RATIONALE FOR THE DISPROOF METHOD

The basic laws of logical thinking, which must be observed in all correct thinking, include the law of identity, the law of contradiction, the law of alignment, and the law of sufficient reason. The Law of Contradiction and the Law of Neutralization are the main laws involved in the Law of Contradiction and the Law of Neutralization.

The law of contradiction, which is written in its general form as: and A and not A.

The law of the row, which must be true in one case, is either A or non-A.

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The Law of Contradiction looks at the transformation of propositions and essentially exposes the incompatibility of the new proposition with the known mathematical theory and the conditions of the proposition.

The reasoning process of $(p \frown \neg q)$ $(p \rightarrow q)$ is the reasoning process of this incompatibility from concealment to manifestation.

## 4. DISPROOF METHOD OF SOLVING PROBLEMS

The basic steps in solving a problem by the inverse method are divided into three steps.

1. Contradictory: The conclusion is negated by assuming that the conclusion of the original proposition does not hold and that its converse holds.

2. Contradictory: The "converse" is used as a condition to derive a self-contradictory conclusion through a series of reasoning arguments with known conditions such as a theorem.

3. Conclusion: If the contradiction leads to the conclusion that the converse is not true, then the original proposition is correct.

## 5. APPLICATION OF THE DISPROOF METHOD (FOUR MAIN TYPES)

### 5.1. Function types

Example 1: Let the quadratic function f(x)=x2+px+q,find the proof that at least one of |f(1)|, |f(2)|, |f(3)| is not less than 1/2.

Proof :Suppose that |f(1)|，|f(2)|，|f(3)| are less than 1/2 ,then we have

$$|f(1)|+|f(2)|+|f(3)| < 2 \quad ①$$

On the other hand, there are the properties of absolute value inequalities，then we have

$$|f(1)|+2|f(2)|+|f(3)| \geqslant |f(1)|-2|f(2)|+|f(3)|=|(1+p+q)-2(4+2p+q)+(9+3p+q)|=2 \quad ②$$

Clearly ① and ② contradict each other, so the hypothesis does not hold and the conclusion of the original proposition is correct.

### 5.2. Column type

Example 2: Let the $\{S_n\}$ series be an isometric series with common ratio q and be the sum of its first n terms.

Prove that the $\{S_n\}$ series is not an isoperimetric series.

Is the $\{S_n\}$ series is an isometric series? and justify.

Proof :(1) Suppose the $\{S_n\}$ series is isometric, then S22=S1S3 ,i.e.a12 $(1+q)^2$ =a1*a1*(1+q+q2), Since a1≠0, the $(1+q)^2$ =1+q+q2, i.e. q = 0. This contradicts the common ratio q ≠0, so the series is not isometric.

Solution:(2) When q=-1, $\{S_n\}$ is an equivariant series; when q ≠ 1, $\{S_n\}$ is not an equivariant series, otherwise 2S2=S1+S3, That is 2a1 (1+q)= a1+a1 (1+q+q2), giving q = 0, which contradicts the common ratio q ≠ 0.

4.3 Inequality type

Example 3: We know that a + b + c > 0, ab + bc + ca > 0 and abc > 0. Prove that a,b,c > 0.

Proof :Suppose a < 0, ∵ abc > 0, ∴ bc < 0, and a + b + c > 0, ∴ b + c = -a > 0.

∴ ab + bc + ca = a(b + c) + bc < 0, which contradicts the question.

If a = 0, then contradict abc > 0, ∴ must have a > 0, the same can be proved: b > 0, c > 0.

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### 5.3. Geometric type

Example 4: As shown in the diagram, the two chords NP and MQ of a circle O intersect at point A, and neither NP nor MQ crosses the point O. Prove that the chords NP and MQ cannot bisect each other.

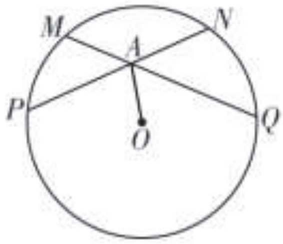


**Figure 1.** Example 4 Figure

Proof: Suppose NP and MQ are bisecting each other and the bisecting point is A. By the vertical path theorem, OA $\perp$ NP.at the same time OA $\perp$ MQ.

∴ NP // MQ, which contradicts the known intersection of NP and MQ,

∴ NP, MQ cannot be bisected by each other.

## 6. POINTS TO NOTE IN APPLYING THE DISPROOF METHOD

For the same proposition, reasoning from different perspectives can often lead to different contradictory results of different nature and thus different methods of proof, some of which are complicated and some of which are simple and quick. Therefore, in the proof by disproof method should be from the characteristics of the proposition to choose the appropriate method of reasoning.

1. The conclusion must be correctly "negated". The correct negation of the conclusion is the primary issue in the use of disproof.

2. The "characteristics of reasoning" must be clear. It is the task of the reflexive method to derive contradictions from the negative conclusion, but it is not possible to predict what kind of contradiction will arise. It is usually considered in the context of the relevant area of the proposition, and this is precisely the characteristic of reflexive reasoning. All that is required is that the conclusion is correctly negated and that the rules of reasoning are strictly adhered to in a step-by-step manner, so that the proof ends as soon as the contradiction appears.

3. Understanding the "types of contradiction". There are many different kinds of contradictions in the reasoning process. The result of the reasoning may be contradictory to the hypothesis or part of it, to a known true proposition (definition or axiom, or theorem, or property), to a provisional hypothesis or to a pair of contradictory results.

The converse method is a concise and practical way of solving mathematical problems, and is an important mathematical idea. Learning to use the inverse method will enable us to grasp the idea of logical mathematical reasoning and the mathematical method of indirect proof, to improve our powers of observation, thinking and discrimination, and to develop the habit of rigorous study. I believe that only by understanding this knowledge and building on it with continuous training and summaries can we become proficient in its use.

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# 7. THE VALUE OF THE APPLICATION OF THE DISPROOF METHOD

## 7.1. Cultivate counterfactual thinking

The practice of the disproof method can reverse our fixed thinking stereotypes, change our previous unchanging directional thinking, instead of the "back to front" inverse thinking process, which not only avoids the superficiality and staleness of our thinking, but also improves the flexibility and agility of our thinking, while the training of the inverse method can also simplify the arithmetic process The application of the inverse method not only allows us to identify and analyse this knowledge more clearly, but also gradually develops our awareness of reverse thinking and promotes innovation in thinking.

## 7.2. Promoting the formation of mathematical thinking

It is well known that China's primary mathematics education is significantly better than that of the West, but why is the independent creativity of Chinese students not as good as that of Western students when it comes to university? It is because Chinese students at university level lack creativity, they are still pulled along by their teachers and do not think ahead of themselves, so it is difficult for them to achieve anything, let alone win a Nobel Prize.

Our teaching of mathematics has been heavily weighted towards problem solving and a sea of problems, purely for the sake of doing problems, and not much in the way of stimulating and creative thinking and summarising ideas from the problems themselves. This situation is long overdue for some reflection. This is the essential requirement for mathematical learning and the basic guarantee for improving the quality of mathematics, as well as the current needs of social development and an important measure to improve the overall quality of the nation.

Euclid was fond of using the inverse method, which he considered to be a mathematician's sword, and it was more brilliant and far-sighted than the chess game of sacrificing a piece to win the whole game. Whereas the chess player merely sacrifices a pawn or a piece, the mathematician simply gives up the whole game to his opponent, making him think that he has got it all.

## 7.3. Develop rigorous thinking

Rigour of thought means that the object of the mode of thought is comprehensive, profound and complete. In the use of the inverse method of proof, to ensure strict and accurate arithmetic, reasoning to achieve precision, in the process of proof of each link must be comprehensive, can not be missed, should do "a hundred secret and no oversight". For example, in the negation of the conclusion of the original proposition, if the inverse has a variety of cases, attention must be paid to clarify the opposite of the original conclusion, the classification of the discussion, a rigorous list of all the different cases contradictory to the conclusion of the original proposition, and one by one to negate. In fact, the inverse method is inseparable from the direct method of proof, and the combination of the two often complements each other. The whole picture of the proof of a proposition is seen in terms of the converse method, but locally, the whole process of reasoning after the converse has been made is the direct method.

Sometimes we intersperse the reasoning of a direct proof with some counterfactuals to establish the correctness of certain required arguments The process of using counterfactuals Therefore, in the process of applying the method of refutation, it is important to review, practise and reinforce a correct understanding and a firm grasp of The questions that we are asked are deeper and broader in scope, and through By solving the problems, we can not only consolidate and deepen our knowledge of the inverse method and master the relevant skills, but also help to motivate our learning, develop our spirit of research and our ability to analyse and solve problems, as well as improve the rigour of our thinking.

### 7.4. Penetrating the history of mathematics

The history of mathematics is often permeated in the study of mathematics, which is a need for the study of mathematics itself and a requirement to improve our quality. Learning the inverse method is a cognitive process, and the production and development of any mathematical knowledge is not easily acquired; it is tortuous in nature and has undergone a long process of research and hard work. Traditional mathematics education only places emphasis on the development of formal logical deductive argumentation skills, while neglecting to understand, feel, and even experience and experience in learning, which are often more important and meaningful intrinsically than the mastery of mere mathematical conclusions.

When we learn mathematics, we are leaving behind the journey and experience of human exploration of mathematics, and directly applying the conclusions after the exploration of our predecessors. Such learning can easily make us feel the dullness of the learning content and methods, and the appropriate introduction of the components of the history of mathematics in the process of learning the inverse method, making full use of the materials of the history of mathematics related to teaching, reproducing the exploration journey of concepts, principles and methods, and recreating the development process of the inverse method in the learning discovering and experiencing the process of forming mathematical knowledge and the cognitive process of thinking on our own, and guiding us to learn the outstanding qualities of mathematicians, which is conducive to inspiring our love for mathematics and further grasp of mathematical thinking and methods, and can stimulate our interest and potential learning ability, while helping us to learn mathematical knowledge spontaneously, not only educating us in humanism, but also demonstrating the history of mathematics' humanist connotations.

Learning how the inverse method goes from vivid intuition to abstract thinking while also understanding mathematical knowledge has a positive impact on our learning of the inverse method, giving us a deeper view of the development of the inverse method and even of mathematics, while enabling us to understand the basic laws and fundamental ideas of the development of the inverse method, to feel the twists and turns of the development of the inverse method, to mobilise our enthusiasm and creativity in learning the inverse method, and to enable us to gain true knowledge at the This will enable us to acquire the courage to learn with tenacity and to build a perfect personality. The reflexive method is therefore of great significance in training our discursive thinking and improving our philosophical training, as well as providing theoretical guidance for mathematics education in view of the nature of mathematical development.

## 8. CONCLUSION

Mathematics is a very logical subject and the study of mathematics not only enables us to solve many practical problems, but also develops one's thinking skills in the process of learning. As a very good method of mathematical proof, its unique proof process and way of thinking can open up our minds and enhance the rigour of our reasoning, which is of great help in developing our intelligence, our logical thinking ability and our creative thinking ability.

## REFERENCES

[1] Liu Zongbao. The value and suggestions of teaching the inverse method [J]. China Journal of Science and Education Innovation,2012,(35):105-106.

[2] Wang Ling. Implementation of reverse thinking training in mathematics teaching [J]. Mathematics and Chemistry for Secondary School Students (Teaching and Learning),2014(10):53.

[3] Li Xinxing. The application of reverse thinking training in mathematics teaching [J]. Journal of Jiangsu Education College (Natural Science Edition),2011,27(01):86-88.

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[4] Zhu Bing. Talking about the application of the disproof method in high school mathematics [J]. Mathematical and rational problem solving research,2021(01):56-57.

[5] A.G.Hamilton.Logic for Mathematicians[M].Tsinghua University Press, 2013: 32-34.

[6] H.Hopf,of reduction ad absurdum conference papers,the innovation education forum,2007.