A Novel Fuzzy DEA/AR Model: Achieving Accuracy and Computational Improvements

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Abstract

The fuzzy data envelopment analysis model with assurance regions (fuzzy DEA/AR model) is a widely used non-parametric method for assessing the relative fuzzy efficiency values of decision making units(DMUs) with imprecise input and output variables. The conventional fuzzy DEA/AR model is α-cut based, which uses non-linear mathematical programming for calculating the upper and lower bounds of the DMUs' fuzzy efficiency values. The conventional approach has two limitations: its nonlinear characteristic always generates lower accuracy efficiency value assessments, and the α-cut based characteristic leads to a high computational load. We propose a novel fuzzy DEA/AR model which is independent of different α-cut levels and linear to address these two drawbacks. The α-cut independency feature results in a significant reduction in the computational demands, and the linear feature contributes to accuracy improvements. We also present an empirical study to demonstrate the applicability of the proposed model.

Keywords

Fuzzy DEA/AR; α-cuts ; triangular fuzzy numbers; university library.

1. INTRODUCTION

Data envelopment analysis (DEA) is a widely used non-parametric method that evaluates the relative efficiency value of decision making units (DMUs) with multiple inputs and outputs. Based on Farrell's production efficiency theory[1], DEA model can calculate the relative efficiency value of DMUs using only the input and output data; the production function is not pre-set. Since its initial introduction in 1978 by Charnes et al. [2], the DEA model has been extended resulting in many well-known derived models, such as the CCR-DEA model [2], BCC-DEA model [3], Dynamic DEA model [4], DEA/AR model [5], and so on. However, all of these DEA models have two challenges that hinder their application; these two challenges have been addressed individually in the literature, and are discussed below.

The first challenge with the DEA models is that they cannot deal with DMUs with vague or imprecise inputs and outputs. The DEA models require the accurate measurement for all of the input and output variables. However, in many cases, the obtained data for the inputs and outputs are vague or imprecise due to information deficiencies or computational errors. Recently, many researchers have made great efforts in constructing alternative fuzzy DEA models to deal with the vague variables. One way to manipulate vague data into crisp data directly is via a probability distribution, but this method needs either a priori predictable regularity or a posteriori frequency determination to construct, which is not possible in many cases. Another way is to represent the imprecise data by membership functions based on fuzzy set theory [6, 7]. This method provides an important theoretical basis for the DEA models to

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deal with imprecise data. Many researchers [8-10] have used the concept of membership functions to introduce algorithms for fuzzy DEA models. These algorithms can be classified into four categories: fuzzy ranking [11], de-fuzzification [12], tolerance [13], and α-cut based [14] methods, among which the fourth method has been widely applied in different problem domains. The α-cut based method, first proposed by Meada et al. [14], has been further improved by Satti et al. [15], Liu [16], and Zhou et al. [17]. Nowadays, fuzzy DEA models are usually transformed into a parametric programming problem with the α-cut method. DMUs' fuzzy efficiency values can be calculated by solving the parametric programming problem at different α-cut levels.

The second challenge in the DEA models is that they permit every DMU to choose the weights that are most favorable to themselves when calculating the ratio of the aggregated outputs to the aggregated inputs. Nevertheless, owing to the restrictions present in production situations in the real world, some weights must be limited to a specific region. To address this challenge, the concept of an assurance region（AR）, first proposed by Thompson et al. [18, 19], is used to restrict the ratio of any two weights to a reasonable region derived from experts' opinions [20-22]. The resulting DEA/AR model has been widely used for efficiency assessments of a commercial bank [23], hotel industry [24], and other areas.

As a countermeasure to solve the above two challenges together, Liu and Chuang [5] has merged the concept of assurance regions into fuzzy DEA models under a constant return to scale (CRS) assumption to construct the fuzzy DEA model with assurance regions (the fuzzy DEA/AR model), and Zhou et al. [17], Lai et al. [25] has further developed the fuzzy DEA/AR model based on α-cuts. These fuzzy DEA/AR models have been used for assessing the seismic efficiency of reservoir dams during earthquakes [26], the efficiency of banking industry [27], and so on. However, these fuzzy DEA/AR models still have two limitations. Firstly, they use non-linear mathematical programming for the upper and lower bounds of fuzzy efficiency values calculations, which leads to a lower accuracy of the efficiency value estimations. Some researchers (Pourmahmoud and Bafekr [28], Sanjeet [29]) have introduced a triangular fuzzy set and an intuitionistic fuzzy set to transform the fuzzy DEA/AR model (proposed by Liu and Chuang [5]) into a linear model, but they are all α-cut based, which must be solved at multiple α-cut levels. The need for multiple solutions results in the second limitation, which is high computational effort demanded due to the lack of a known rule for determining a best step size for the α-cut levels. More importantly, earlier research results on the fuzzy DEA/AR model did not consider the verification of the constraints of assurance regions in the mathematical programming proofs.

To address the limitations of accuracy and computational efficiency together, a novel fuzzy DEA/AR model is proposed in this work. The proposed model has the following three unique features: firstly, it is linear, so it can achieve more accurate efficiency values; secondly, it is independent of different α-cut levels, which minimizes the computational effort; finally, to the best of our knowledge, this is the first study to add the verification of assurance region constraints into the mathematical programming proof.

The rest of this paper is organized as follows. Section 2 reviews the fuzzy DEA/AR model proposed by Liu and Chuang [5], Lai et al. [25] and Sanjeet [29], namely the conventional fuzzy DEA/AR model, and its aggressive formulation. Section 3 presents our novel fuzzy DEA/AR model. Section 4 presents a comparative empirical analysis for the proposed model and the conventional fuzzy DEA/AR model. Section 5 gives the conclusions.

2. CONVENTIONAL FUZZY DEA/AR MODEL

This section briefly reviews the conventional fuzzy DEA/AR model proposed by Liu and Chuang [5],Lai et al. [25], Sanjeet [29], and their aggressive formulation.

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Consider that there are n DMUs, for which each DMU_j ($j=1,2,\ldots,n$) consumes m inputs X_{ij} ($i=1,2,\ldots,m$) to produce s outputs Y_{rj} ($r=1,2,\ldots,s$). Suppose the *k*th DMU assessed is denoted as DMU_k. According to Charnes et al. [3], the conventional DEA model, assuming constant returns-to-scale, is designed to calculate the efficiency value of DMU_k for crisp data. This can be described as follows:

$$E_k = \max \frac{\sum_{r=1}^{s} u_r Y_{rk}}{\sum_{i=1}^{m} v_i X_{ik}}$$
$$s.t.\begin{cases} \dfrac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \le 1 & \forall j \\ u_r \ge \varepsilon > 0 \quad \forall r, v_i \ge \varepsilon > 0 & \forall i \end{cases} \tag{1}$$

where, v_i and u_r, treated as unknown variables, donate the weights for the ith input and the rth output, respectively, and ε is a small non-Archimedean number [3]. Model (1) can be converted to model (2) by applying the Charnes-Cooper transformation [30]:

$$E_k = \max \sum_{r=1}^{s} u_r Y_{rk}$$
$$s.t.\begin{cases} \sum_{i=1}^{m} v_i X_{ik} = 1 \\ \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0 \quad \forall j \\ u_r \ge \varepsilon > 0 \quad \forall r, v_i \ge \varepsilon > 0 \quad \forall i \end{cases} \tag{2}$$

Nevertheless, model (2) permits each DMU_j to choose favorable weights for their inputs and outputs. The issue encountered with this approach is that there are many cases for which the input and output weights must be limited to a specific region for the production mechanisms to work in real world. The AR, proposed by Thompson et al. [18, 19], is used to restrict the weights in some reasonable regions based on expert opinions. Liu and Chuang [5],Lai et al. [25]and Sanjeet [29]have all used the following mathematical inequalities to describe the domainsof the relative importance of the input and output weights:

$$\frac{L_p^I}{U_q^I} \le \frac{v_p}{v_q} \le \frac{U_p^I}{L_q^I}, 1 \le p < q = 2,3,\ldots,m, \frac{L_p^O}{U_q^O} \le \frac{u_p}{u_q} \le \frac{U_p^O}{L_q^O} 1 \le p < q = 2,3,\ldots,s. \tag{3}$$

L_p^I and U_p^I denote the relative importance of the *p*th($1 \le p < q = 2,3,\ldots,m$) input elicited from the experts. While L_p^O and U_p^O denote the relative importance of the *p*th($1 \le p < q = 2,3,\ldots,s$) output elicited from the experts. To simplify the calculations, let $C_{pq}^L = \frac{L_p^I}{U_q^I}$, $C_{pq}^U = \frac{U_p^I}{L_q^I}$, $D_{pq}^L = \frac{L_p^O}{U_q^O}$, and $D_{pq}^U = \frac{U_p^O}{L_q^O}$. These AR constraints are added into model (2), which results in model (4) as follows:

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$$E_k = \max \sum_{r=1}^{s} u_r Y_{rk}$$
$$s.t.\begin{cases} \sum_{i=1}^{m} v_i X_{ik} = 1 \\ \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, j = 1,2,\dots,n, \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\dots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\dots,s, \\ v_i \ge \varepsilon > 0, \quad i = 1,2,\dots,m, u_r \ge \varepsilon > 0, \quad r = 1,2,\dots,s, \end{cases} \tag{4}$$

Suppose the inputs and outputs of each DMU are not crisp but imprecise. Let $\tilde{X}_{ik}$ and $\tilde{Y}_{rk}$ with membership functions $\mu_{\tilde{X}_{ik}}$ and $\mu_{\tilde{Y}_{rk}}$ respectively be the *i*th and *r*th fuzzy input and output of DMU_k. This results in Model (5)as follows:

$$\tilde{E}_k = \max \sum_{r=1}^{s} u_r \tilde{Y}_{rk}$$
$$s.t.\begin{cases} \sum_{i=1}^{m} v_i \tilde{X}_{ik} = 1 \\ \sum_{r=1}^{s} u_r \tilde{Y}_{rj} - \sum_{i=1}^{m} v_i \tilde{X}_{ij} \le 0, j = 1,2,\dots,n, \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\dots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\dots,s, \\ v_i \ge \varepsilon > 0, \quad i = 1,2,\dots,m, u_r \ge \varepsilon > 0, \quad r = 1,2,\dots,s, \end{cases} \tag{5}$$

Liu and Chuang [5],Lai et al. [25]have used $(\tilde{X}_{ij})_\alpha$ and $(\tilde{Y}_{rj})_\alpha$ to represent the α-cuts of $\tilde{X}_{ij}$ and $\tilde{Y}_{rj}$ respectively. For $x_{ij} \in \tilde{X}_{ij}$, $y_{rj} \in \tilde{Y}_{rj}$, $(\tilde{X}_{ij})_\alpha$ and $(\tilde{Y}_{rj})_\alpha$ can be described as follows:

$$(\tilde{X}_{ij})_\alpha = \left[(X_{ij})_\alpha^L, (X_{ij})_\alpha^U\right] = [\min_{x_{ij}} \{x_{ij} \in \tilde{X}_{ij} \left| \mu_{\tilde{X}_{ij}}(x_{ij}) \ge \alpha\right.\}, \quad \max_{x_{ij}} \{x_{ij} \in \tilde{X}_{ij} \left| \mu_{\tilde{X}_{ij}}(x_{ij}) \ge \alpha\right.\}]$$

$$(\tilde{Y}_{rj})_\alpha = \left[(Y_{rj})_\alpha^L, (Y_{rj})_\alpha^U\right] = [\min_{y_{rj}} \{ y_{rj} \in \tilde{Y}_{rj} \left| \mu_{\tilde{Y}_{rj}}(y_{rj}) \ge \alpha\right.\}, \max_{y_{rj}} \{ y_{rj} \in \tilde{Y}_{rj} \left| \mu_{\tilde{Y}_{rj}}(y_{rj}) \ge \alpha\right.\}],$$

Based on Zadeh's extension principle [6,7], the upper and lower bounds of the fuzzy efficiency value $\tilde{E}_k$ in (5) at a specific α-cut value, namely $(E_k)_\alpha^U$ and $(E_k)_\alpha^L$, can be calculated by:

$$(E_k)_\alpha^U = \max_{\substack{x_{ij} \in \tilde{X}_{ij} \\ y_{ij} \in \tilde{Y}_{ij} \\ \forall i,r,j}} \begin{cases} \max(\sum_{r=1}^{s} u_r y_{rk}) \\ s.\ t.\ \sum_{i=1}^{m} v_i x_{ik} = 1, i = 1,2,\dots,m, \\ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, j = 1,2,\dots,n, \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\dots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\dots,s, \\ v_i \ge \varepsilon, u_r \ge \varepsilon, r = 1,\dots,s, \end{cases} \tag{6}$$

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$$(E_k)_\alpha^L = \min_{\substack{x_{ij}\in\tilde{X}_{ij} \\ y_{ij}\in\tilde{Y}_{ij} \\ \forall i,r,j}} \begin{cases} \max(\sum_{r=1}^{s} u_r y_{rk}) \\ s.\ t.\ \sum_{i=1}^{m} v_i x_{ik} = 1, i = 1,2,\ldots,m, \\ \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, j = 1,2,\ldots,n, \\ \quad C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\ldots,m, \\ \quad D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\ldots,s, \\ \quad\quad v_i \ge \varepsilon, u_r \ge \varepsilon, r = 1,\ldots,s, \end{cases} \tag{7}$$

Note that the two-level optimization (6) and (7) cannot be calculated directly. From the results in Liu and Chuang [5], model (6) and (7) can be transformed into model (8) and (9), respectively.

$$(E_k)_\alpha^U = \max \sum_{r=1}^{s} u_r (Y_{rk})_\alpha^U$$
$$s.t. \begin{cases} \sum_{i=1}^{m} v_i (X_{ik})_\alpha^L = 1, i = 1,2,\ldots,m, \\ \sum_{r=1}^{s} u_r (Y_{rk})_\alpha^U - \sum_{i=1}^{m} v_i (X_{ik})_\alpha^L \le 0 \\ \sum_{r=1}^{s} u_r (Y_{rj})_\alpha^L - \sum_{i=1}^{m} v_i (X_{ij})_\alpha^U \le 0, j = 1,2,\ldots,n, j \ne k, \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\ldots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\ldots,s, \\ v_i \ge \varepsilon, u_r \ge \varepsilon, r = 1,\ldots,s. \end{cases} \tag{8}$$

$$(E_k)_\alpha^L = \max \sum_{r=1}^{s} u_r (Y_{rk})_\alpha^L$$
$$s.t. \begin{cases} \sum_{i=1}^{m} v_i (X_{ik})_\alpha^U = 1, i = 1,2,\ldots,m, \\ \sum_{r=1}^{s} u_r (Y_{rk})_\alpha^L - \sum_{i=1}^{m} v_i (X_{ik})_\alpha^U \le 0, \\ \sum_{r=1}^{s} u_r (Y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i (X_{ij})_\alpha^L \le 0, j = 1,2,\ldots,n, j \ne k, \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\ldots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\ldots,s, \\ v_i \ge \varepsilon, u_r \ge \varepsilon, r = 1,\ldots,s. \end{cases} \tag{9}$$

The analysis of model (8) and model (9) reveals two drawbacks: they are subject to various α-cut levels, which leads to high computational requirements; and they are in the non-linear mathematical programming form because of $(X_{ij})_\alpha^U$, $(X_{ij})_\alpha^L$, $(Y_{rj})_\alpha^U$ and $(Y_{rj})_\alpha^L$, so the accuracy of theefficiency values calculated is poor.

3. THE NOVEL FUZZY DEA/AR MODELFORMULATION

To solve the above drawbacks (high computation requirements and poor accuracy), a novel fuzzy DEA/AR model is proposed in this section.

For the poor accuracy performance of the fuzzy efficiency values, up to now, only Sanjeet [29] introduced intuitionistic fuzzy set theory to transform the non-linear mathematical programming form of model (8) and (9) into a linear form. However, this method needs a pre-set probability distribution, which is impossible in many cases. In this work, a triangular fuzzy number for characterizing fuzzy variables in model (8) and (9) is introduced. The reason we use a triangular fuzzy number is its simplicity in computation and its ease of extension to other fuzzy numbers. Therefore, we have Definition3.1 as follows:

Definition3.1[31]: Consider a fuzzy set $\tilde{A}$ of z, a triangular fuzzy number (TrFN) can be denoted as $\tilde{A}=\{l^1,l^2,l^3\}$, where $l^1 \le l^2 \le l^3$, l^1 and l^3 are called the lower and upper limits of $\tilde{A}$ respectively, l^2 is called the most likely value of $\tilde{A}$, When $l^1=l^2=l^3$, $\tilde{A}$ becomes a crisp variable. $\forall z \in \tilde{A}$, The membership function of $\tilde{A}$, represented by $\mu_{\tilde{A}}(z)$, can be described as follows：

$$\mu_{\tilde{A}}(z)=\begin{cases} 0 & z \le l^1 \quad or \quad z \ge l^3 \\ \dfrac{z-l^1}{l^2-l^1}, & l^1 \le z \le l^2 \\ \dfrac{l^3-z}{l^3-l^2} & l^2 \le z \le l^3 \end{cases}$$

The α-cuts of the triangular fuzzy numbers are needed, so we have Definition3.2 as follows:

Definition3.2 [32]: The α-cut of the fuzzy set $\tilde{A}$, which is referred as α-cut level (as shown in Figure1), is defined as $\tilde{A}_\alpha$: $\tilde{A}_\alpha=\{z \mid \mu_{\tilde{A}}(z) \ge \alpha, z \in \tilde{A}\}$, where $\alpha \in [0,1]$.

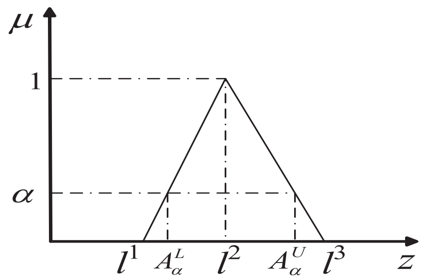

Figure 1. α-cut level of the fuzzy set $\tilde{A}$

For a TrFN $\tilde{A}=\{l^1,l^2,l^3\}$ and $\alpha \in [0,1]$, the confidence interval can be denoted as follows [32]:

$\tilde{A}_\alpha=\left[A_\alpha^L, A_\alpha^U\right]=\left[l^1+\alpha(l^2-l^1), l^3-\alpha(l^3-l^2)\right]$.

Based on Definition 3.1, the fuzzy inputs $\tilde{X}_{ij}$ and outputs $\tilde{Y}_{rj}$ in models (8) and (9) can be denoted by triangular fuzzy numbers as:

$\tilde{X}_{ij}=(x_{ij}^1, x_{ij}^2, x_{ij}^3)$, $\tilde{Y}_{rj}=(y_{rj}^1, y_{rj}^2, y_{rj}^3)$.

Based on Definition 3.2, Using a specific α-cut level for $\tilde{X}_{ij}$ and $\tilde{Y}_{rj}$, the lower-bounds (i.e., $(X_{ij})_\alpha^L$, $(Y_{rj})_\alpha^L$) and upper-bounds (i.e., $(X_{ij})_\alpha^U$, $(Y_{rj})_\alpha^U$) of their membership functions can be calculated as follows:

$$(X_{ij})_\alpha^L = x_{ij}^1 + \alpha_i(x_{ij}^2 - x_{ij}^1), \alpha_i \in [0,1] \tag{10}$$

$$(X_{ij})_\alpha^U = x_{ij}^3 - \alpha_i(x_{ij}^3 - x_{ij}^2), \alpha_i \in [0,1] \tag{11}$$

$$(Y_{rj})_\alpha^L = y_{rj}^1 + \alpha_r(y_{rj}^2 - y_{rj}^1), \alpha_r \in [0,1] \tag{12}$$

$$(Y_{rj})_\alpha^U = y_{rj}^3 - \alpha_r(y_{rj}^3 - y_{rj}^2), \alpha_r \in [0,1] \tag{13}$$

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3.1. Upper bounds of the DMUs' fuzzy efficiency values

Asmodel (8) is a non-linear mathematical programming model, we replace Eq. (10)-(13) in model (8) and construct model (14) as follows:

$$(E_k)_\alpha^U = \max \sum_{r=1}^{s} u_r(y_{rk}^3 - \alpha_r(y_{rk}^3 - y_{rk}^2))$$
$$s.t.\begin{cases} \sum_{i=1}^{m} v_i(x_{ik}^1 + \alpha_i(x_{ik}^2 - x_{ik}^1)) = 1, i = 1,2,\ldots,m, \\ \sum_{r=1}^{s} u_r(y_{rk}^3 - \alpha_r(y_{rk}^3 - y_{rk}^2)) - \sum_{i=1}^{m} v_i(x_{ik}^1 + \alpha_i(x_{ik}^2 - x_{ik}^1)) \le 0, \\ \sum_{r=1}^{s} u_r(y_{rj}^1 + \alpha_r(y_{rj}^2 - y_{rj}^1)) - \sum_{i=1}^{m} v_i(x_{ij}^3 - \alpha_i(x_{ij}^3 - x_{ij}^2)) \le 0, j = 1,2,\ldots,n, j \ne k, \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\ldots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\ldots,s, \\ v_i \ge \varepsilon, u_r \ge \varepsilon, r = 1,\ldots,s. \end{cases} \tag{14}$$

It is evident that model (14) is linear, which can greatly improve the accuracy performance of efficiency values. However model (14) is still subject to various α-cut levels, so the problem of the high computational demands remains. To solve this shortcoming, we replace variables $\lambda_i = \alpha_i v_i$, $i = 1,\ldots,m$, where $0 \le \lambda_i \le v_i$, and $\eta_r = \alpha_r u_r$, $r = 1,\ldots,s$, where $0 \le \eta_r \le u_r$. This results in the construction of model (15) as follows:

$$(E_k)_\alpha^U = \max \sum_{r=1}^{s} (u_r y_{rk}^3 - \eta_r(y_{rk}^3 - y_{rk}^2))$$
$$s.t.\begin{cases} \sum_{i=1}^{m} (v_i x_{ik}^1 + \lambda_i(x_{it}^2 - x_{ik}^1)) = 1, i = 1,2,\ldots,m \\ \sum_{r=1}^{s} (u_r y_{rk}^3 - \eta_r(y_{rk}^3 - y_{rk}^2)) - \sum_{i=1}^{m} (v_i x_{ik}^1 + \lambda_i(x_{ik}^2 - x_{ik}^1)) \le 0 \\ \sum_{r=1}^{s} (u_r y_{rj}^1 + \eta_r(y_{rj}^2 - y_{rj}^1)) - \sum_{i=1}^{m} (v_i x_{ij}^3 - \lambda_i(x_{ij}^3 - x_{ij}^2)) \le 0, j = 1,2,\ldots,n, j \ne k, \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\ldots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\ldots,s, \\ v_i \ge \varepsilon, u_r \ge \varepsilon, r = 1,\ldots,s, 0 \le \lambda_i \le v_i, 0 \le \eta_r \le u_r \end{cases} \tag{15}$$

Obviously, model (15) is independent of various α-cut levels and there is no need for a pre-determined step-size, so it can dramatically reduce the computation efforts. Also, model (15) is a one-stage linear optimization model. If it always has a feasible and bounded solution, we can find the upper bounds of the DMUs' efficiency values under the assurance regions. This is given as Theorem1. Former research on the conventional fuzzy DEA/AR model has not included the verification of assurance region constraints in their proofs; this work addresses the omission as follows.

Theorem 1: Model (15) is always feasible and bounded.

Proof: In this paper, we substitute the assurance region constraints in model (15) with the matrix forms: $Cv \le 0$ and $Du \le 0$ .Then, the dual form of model (15) can be written as follows:

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$$\omega=\min[\theta-\varepsilon(\sum_{i=1}^{m}s_i+\sum_{r=1}^{s}s_r)]$$

$$s.t.\begin{cases}\delta_k y_{rk}^3+\sum_{\substack{j=1\\j\neq k}}^{n}\delta_j y_{rj}^1+\varpi D_{(k)}-s_r-\rho_r\geq y_{rk}^3\\ \delta_k(y_{rk}^2-y_{rk}^3)+\sum_{\substack{j=1\\j\neq k}}^{n}\delta_j(y_{rj}^2-y_{rj}^1)+\tau D_{(j)}-\gamma_r+\rho_r\geq y_{rk}^2-y,\\ (\theta-\delta_k)x_{ik}^1-\sum_{\substack{j=1\\j\neq k}}^{n}\delta_j x_{ij}^3+\vartheta C_{(k)}-s_i-\varphi_i\geq 0,\\ \theta(x_{ik}^2-x_{ik}^1)-\delta_k(x_{ik}^2-x_{ik}^1)+\sum_{\substack{j=1\\j\neq k}}^{n}\delta_j(x_{ij}^3-x_{ij}^2)+\psi C_{(j)}-\kappa_i+\varphi_i\geq 0,\\ \delta_j\geq 0,\varpi,\tau,\vartheta,\psi\geq 0,\\ s_r,\rho_r\geq 0, r=1,2,\ldots,s;\\ s_i,\varphi_i\geq 0, i=1,2,\ldots,m,\quad \theta\quad free.\end{cases} \tag{16}$$

Suppose a specific solution for model (16) is described as formula (17):

$$\begin{aligned}&\delta_j=0, j=1,2,\ldots,n; j\neq k;\\ &\delta_k=1;\varpi=\tau=\vartheta=\psi=0;\\ &s_r=\rho_r=0, r=1,2,\ldots s;\\ &s_i=\varphi_i=0, i=1,2,\ldots,m;\theta=1.\end{aligned} \tag{17}$$

Firstly, it is evident that this solution satisfies all of the constraints of model (16);consequently, model (16) is always feasible. Secondly, it is obviously independent of different α-cut levels. Finally, in the feasible solution shown in formula (17), $\theta=1$ and the slack variables ($s_r=s_i=0$) are equal to zero. Hence, the optimum value of model (16) is $\omega^*\leq 1$, so model (16) is bounded.

Furthermore, based on the duality theorem in linear programming, the optimal values of model (16) and (15) are equal (i.e. $\omega^*=(E_k)_\alpha^{U^*}\leq 1$), and the solution above is also a feasible solution to model (15).Hence, we have that model (15) is also feasible and bounded. This completes the proof.

3.1. Lower bounds of the DMUs' fuzzy efficiency values

As model (9) is also a non-linear mathematical programming model, we replace Eq. (10)-(13) in model (9) and construct model (18) as follows:

$$(E_k)_\alpha^L=\max\sum_{r=1}^{s}u_r(y_{rk}^1+\alpha_r(y_{rk}^2-y_{rk}^1))$$

$$s.t.\begin{cases}\sum_{i=1}^{m}v_i(x_{ik}^3-\alpha_i(x_{ik}^3-x_{ik}^2))=1,\quad i=1,2,\ldots,m,\\ \sum_{r=1}^{s}u_r(y_{rk}^1+\alpha_r(y_{rk}^2-y_{rk}^1))-\sum_{i=1}^{m}v_i(x_{ik}^3-\alpha_i(x_{ik}^3-x_{ik}^2))\leq 0,\\ \sum_{r=1}^{s}u_r(y_{rj}^3-\alpha_r(y_{rj}^3-y_{rj}^2))-\sum_{i=1}^{m}v_i(x_{ij}^1+\alpha_i(x_{ij}^2-x_{ij}^1))\leq 0, j=1,2,\ldots,n, j\neq k,\\ C_{pq}^L v_q\leq v_p\leq C_{pq}^U v_q,\forall 1\leq p<q=2,3,\ldots,m,\\ D_{pq}^L u_q\leq u_p\leq D_{pq}^U u_q,\forall 1\leq p<q=2,3,\ldots,s,\\ v_i\geq\varepsilon,u_r\geq\varepsilon, r=1,\ldots,s.\end{cases} \tag{18}$$

Although model (18) is linear, it is still subject to various α-cut levels. Similar to the procedure in Section 3.1, a variable exchange is conducted to solve this issue. We replace $\lambda_i=\alpha_i v_i$ where

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$0 \le \lambda_i \le v_i$ and $\eta_r = \alpha_r u_r$ where $0 \le \eta_r \le u_r$ in model (18). Then, model (18) can be written as model (19):

$$(E_k)_\alpha^L = \max \sum_{r=1}^{s} (u_r y_{rk}^1 + \eta_r (y_{rk}^2 - y_{rk}^1))$$
$$s.t. \begin{cases} \sum_{i=1}^{m} (v_i x_{ik}^3 - \lambda_i (x_{ik}^3 - x_{ik}^2)) = 1, \quad i = 1,2,\ldots,m, \\ \sum_{r=1}^{s} (u_r y_{rk}^1 + \eta_r (y_{rk}^2 - y_{rk}^1)) - \sum_{i=1}^{m} (v_i x_{ik}^3 - \lambda_i (x_{ik}^3 - x_{ik}^2)) \le 0; \\ \sum_{r=1}^{s} (u_r y_{rj}^3 - \eta_r (y_{rj}^3 - y_{rj}^2)) - \sum_{i=1}^{m} (v_i x_{ij}^1 + \lambda_i (x_{ij}^2 - x_{ij}^1)) \le 0, j = 1,2,\ldots,n; j \ne k; \\ C_{pq}^L v_q \le v_p \le C_{pq}^U v_q, \forall 1 \le p < q = 2,3,\ldots,m, \\ D_{pq}^L u_q \le u_p \le D_{pq}^U u_q, \forall 1 \le p < q = 2,3,\ldots,s, \\ v_i \ge \varepsilon, u_r \ge \varepsilon, r = 1,\ldots,s, \quad 0 \le \lambda_i \le v_i, 0 \le \eta_r \le u_r \end{cases} \tag{19}$$

It is evident that model (19) is a linear programming optimization model, and more importantly, it is independent of various α-cut levels. If it always has feasible and bounded solutions, then we can find more accurate solutions for the lower-bounds of the DMUs' efficiency values under the assurance regions. This is given as Theorem 2.

Theorem 2: Model (19) is always feasible and bounded.

Proof: As we mentioned above, former research on the fuzzy DEA/AR model always omitted the verification of assurance region constraints in their proof. Here, we substitute the assurance region constraints in model (19) with $Cv \le 0$, $Du \le 0$, then we define the dual form of model (19) as follows:

$$T = \min[\theta^+ - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)]$$
$$s.t. \begin{cases} \phi_k y_{rk}^1 + \sum_{\substack{j=1 \\ j \ne k}}^{n} \phi_j y_{rj}^2 + \varpi D_{(k)} - s_r^- - \rho_r \ge y_{rk}^1 \\ \phi_k (y_{rk}^2 - y_{rk}^1) + \sum_{\substack{j=1 \\ j \ne k}}^{n} \phi_j (y_{rj}^2 - y_{rj}^3) + \tau D_{(j)} - \gamma_r + \rho_r \ge y_{rk}^2 - y_{rk}^1 \\ (\theta - \phi_k) x_{ik}^3 - \sum_{\substack{j=1 \\ j \ne k}}^{n} \phi_j x_{ij}^1 + \vartheta C_{(k)} - s_i^- - \varphi_i \ge 0, \\ \theta (x_{ik}^2 - x_{ik}^3) + \phi_k (x_{ik}^3 - x_{ik}^2) - \sum_{\substack{j=1 \\ j \ne k}}^{n} \phi_j (x_{ij}^2 - x_{ij}^1) + \psi C_{(j)} - \kappa_i + \varphi_i \ge 0, \\ \phi_j \ge 0, \varpi, j = 1,2,\ldots,n, j \ne k, \\ s_r^+, \rho_r \ge 0, r = 1,2,\ldots,s, \\ \tau, \vartheta, \psi \ge 0 \\ s_i^-, \varphi_i \ge 0, i = 1,2,\ldots,m; \theta^+ \ free. \end{cases} \tag{20}$$

Suppose a specific solution for model (20) is described as formula (21):

$$\begin{aligned} &\phi_j = 0, j = 1,2,\ldots,n; j \ne k; \\ &\phi_k = 1; \varpi = \tau = \vartheta = \psi = 0; \\ &s_r^+ = \rho_r = 0, r = 1,2,\ldots s; \\ &s_i^- = \varphi_i = 0, i = 1,2,\ldots,m; \theta^+ = 1. \end{aligned} \tag{21}$$

Firstly, the solution shown in formula (21) is feasible as it meets every constraint of model (20), so model (20) has feasible solution. Secondly, it is obviously independent of different

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α-cut levels. Finally, in the feasible solution shown in formula (21), $\theta^{+}=1$ and the slack variables are equal to zero ($s_i^{-}=s_r^{+}=0$). Consequently, the optimum value of the objective function of model (20) is $T^{*}\leq 1$, so model (20) is bounded.

Similar to the procedure in Section 3.1, based on the duality theorem in linear programming, the optimal values of model (20) and (19) are the same (i.e., $T^{*}=(E_k)_{\alpha}^{L^{*}}\leq 1$). Also the solution above is a feasible solution to model (19). So we have that model (19) is also feasible and bounded. This completes the proof.

From Theorem 1 and Theorem 2, model (15) and model (19) are both linear and independent of different α-cut levels. In addition, they both have bounded and feasible solutions. Hence, by solving model (15) and model (19), we can obtain more accurate solutions, which provide reduced distances between the upper and lower bounds of the DMUs' fuzzy efficiency values, with less computation work.

4. AN EMPIRICAL STUDY

In this section, we provide a comparative empirical study to verify and demonstrate the practical application of the novel fuzzy DEA/AR model. The data used in the study is from University library collections; such collections have been used in related work. The empirical analysis for this research consists of four main parts. Firstly, variables for the novel model are selected, along with the university library samples. Secondly, the novel fuzzy DEA/AR model is applied to the university libraries' data collections. In the third and fourth parts, comparisons of the novel model and the conventional model are made to demonstrate the computational and accuracy improvements.

4.1. Variables and sample selection

In determining the indicator variables to be used, approaches used in the literature to analyze the inputs and outputs for efficiency value estimations in university library systems are considered (Liu and Chuang [5], Simon et al. [33],Guccio et al. [34]). The previous studies highlighted a set of input and output variables related to funds, university library collections, building areas, and services. In choosing which of these variables to adopt, consideration is given to the availability of data, financial support for the university libraries, plus the input-output logic relationships in library efficiency estimations. Therefore, a set of one input and four output indicator variables are chosen based on these criteria, the only input in the variable set is "Funds":

Funds _ funds allocated by the university in thousands of pounds.

The outputs in the variable set are:

- *Number of Collections* _ the numbers of books, serials, and database in the university library. Size of Building
- *Space*_ the building space in square meters of the university library.
- *Number of Personnel* _ the number of staff and student volunteers in the university library.
- *Readers' Satisfaction* _ readers' satisfaction for the university library.

The second stage is to choose a sample of university libraries to analyze. Firstly, we determine the number of DMUs to use, namely the limit sample size. For a DEA model to be effective, Ali et al. [35] emphasized that the total number of DMUs must exceed twice the sum of the input and output variables. So, the limit set in this study is 10 DMUs. Secondly, the sample university libraries must be similar in nature and operation [33]. In determining the sample, we focus on the China's comprehensive list of the "985" universities project, in which the university libraries have similar nature and operation. For this study, we select 12 DMUs based on these criteria.

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The third stage accomplishes the data collection. “Funds”, “Number of Collections”, “Siz e of Building Space”, and “Number of Personnel” are crisp values, which are acquired from the university libraries’ 2018 annual audit reports. The data of “Readers’ Satisfac tion” are vague and described by triangular fuzzy numbers. Their values are obtained from the investigation of the university teachers and students using fuzzy linguistic ter ms, such as “Very Good”, “Good”, “Average”, “Poor”, and “Very Poor”; their membership functions are described in Figure2. The crisp variables “Funds”, “Number of Collection s”, “Size of Building Space”

and “Number of Personnel” can be regarded as degenerated triangular fuzzy numbers. In the process of acquiring these data collection, we met with the management team of each university library separately. After discussions, the management teams provide d the data. After a series of modification, all of the management team members of th e twelve university libraries approved the data as show in Table 1.

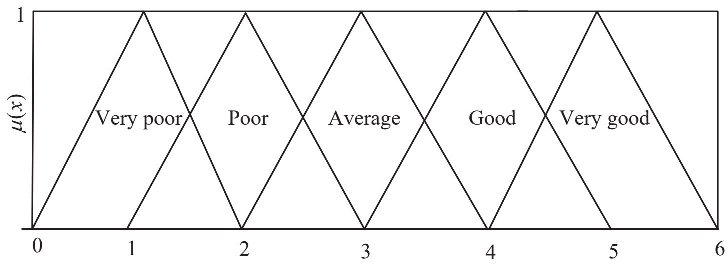

Figure 2. Fuzzy membership functions for Readers’ Satisfaction

Table 1. Data for the twelve university libraries

University library	Input	Outputs			
	Funds (thousands of pounds)	Number of Collections	Size of Building Space (m2)	Number of Personnel	Readers' Satisfaction
1	78	850000	9012	357	(3,4,5)
2	88	754000	8557	284	(2,3,4)
3	64	582460	5783	335	(3,4,5)
4	82	762800	8200	348	(3,4,5)
5	48	413500	6746	273	(2,3,4)
6	83	658000	6520	350	(2,3,4)
7	62	475000	7426	315	(2,3,4)
8	55	558000	8985	246	(3,4,5)
9	41	365000	8650	305	(3,4,5)
10	38	354300	4025	322	(3,4,5)
11	52	312700	5463	276	(3,4,5)
12	70	543600	6240	288	(2,3,4)

Finally, we determine the relative importance of inputs and outputs. After several brainstorming sessions with the curators of the twelve university libraries, the relative importance of “Number of Collections”, “Size of Building Space”, “Number of Personnel” and “Readers’ Satisfaction” are evaluated as: $u_1 \in [0.12, 0.43]$, $u_2 \in [0.19, 0.38]$, $u_3 \in [0.15, 0.31]$, $u_4 \in [0.25, 0.41]$, so the assurance regions generated from the relative importance of the outputs can be expressed as:

$$\frac{0.12}{0.38} \le \frac{u_1}{u_2} \le \frac{0.43}{0.19}, \frac{0.12}{0.31} \le \frac{u_1}{u_3} \le \frac{0.43}{0.15}, \frac{0.12}{0.41} \le \frac{u_1}{u_4} \le \frac{0.43}{0.25}, \frac{0.19}{0.31} \le \frac{u_2}{u_3} \le \frac{0.38}{0.15}, \frac{0.19}{0.41} \le \frac{u_2}{u_4} \le \frac{0.38}{0.25}, \frac{0.15}{0.41} \le \frac{u_3}{u_4} \le \frac{0.31}{0.25}.$$

Notice that there is only one input variable, so no assurance region is requested for the input variable.

4.2. Results of the empirical analysis

We use model (15) and (19), which are coded by LINGO11.0, to evaluate the upper and lower bounds of the twelve university libraries' fuzzy efficiency values (i.e.,U and L), and the results are summarized in the last column of Table 2. For a comparison of the novel fuzzy DEA/AR model proposed in this paper and the conventional fuzzy DEA/AR model proposed by [5],[25] and [29], we employ the conventional fuzzy DEA/AR model to calculate the upper and lower bounds of the twelve university libraries' efficiency values (i.e.,U and L) under different α-cut levels. These results are listed in columns 3-13 in Table 2.

According to Liu and Chuang [5], column 3 (when α =0) shows the range, determined by the upper and lower bound of the fuzzy efficiency value, in which the efficiency value is definitely going to appear. As the eleven distinct α-cut levels increase step-by-step from 0.0 to 1.0, the distances between the upper-bounds and the lower-bounds of the efficiency value decrease. When $\alpha = 1$, the upper-bound efficiency values are equal to the lower-bound efficiency values, the fuzzy efficiency becomes a crisp value. This is called the DMUs' most likely efficiency value by Liu and Chuang [5]. There is no doubt that this crisp value is an ideal value. The closer the upper and lower bounds of the fuzzy efficiency value are to this ideal value, the higher the accuracy of the efficiency value. The results reveal exceptions, such as DMU5 and DMU9, which reach the most likely efficiency values in advance at $\alpha = 0.7$ and $\alpha = 0.5$, respectively. So, when a pre-determined best step size for the α-cut levels is absent, the efficiency values at $\alpha = 0.7$ are called the relatively effective fuzzy efficiency values in [5]. Obviously, compared to the most likely efficiency values, the relatively effective efficiency values are more practical, so these are regarded as the final efficiency values by former research.

As an example, the efficiency value for DMU6 cannot exceed 0.9629 or fall below 0.4351 (when α =0). The distances between the upper-bound and the lower-bound decrease as the α-cut level increases from 0.0 to 1.0. When $\alpha = 1$, the upper-bound and the lower-bound are all 0.7238, which is called the most likely efficiency value.

Table 2. Comparison of the results between the conventional and novel fuzzy DEA/ARmodels

university library (DMU)		The conventional fuzzy DEA/AR model											The novel fuzzy DEA/AR model
		Different α-cut levels											
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
1	L	1	1	1	1	1	1	1	1	1	1	1	1
	U	1	1	1	1	1	1	1	1	1	1	1	1
2	L	0.4874	0.5168	0.5347	0.5693	0.5816	0.6053	0.6482	0.6746	0.7038	0.7319	0.7517	0.7409
	U	0.9836	0.9632	0.9415	0.9227	0.8934	0.8729	0.8482	0.8162	0.7928	0.7758	0.7517	0.7628
3	L	0.7543	0.7937	0.8124	0.8338	0.8524	0.8797	0.8905	0.9157	0.9341	0.9341	0.9341	0.9288
	U	1	1	1	1	1	0.9913	0.9716	0.9562	0.9341	0.9341	0.9341	0.9411
4	L	1	1	1	1	1	1	1	1	1	1	1	1
	U	1	1	1	1	1	1	1	1	1	1	1	1
5	L	0.7986	0.8234	0.8532	0.8816	0.9184	0.9536	0.9824	1	1	1	1	1
	U	1	1	1	1	1	1	1	1	1	1	1	1
6	L	0.4351	0.4634	0.4927	0.5272	0.5561	0.5822	0.6105	0.6428	0.6754	0.6903	0.7238	0.7106
	U	0.9629	0.9427	0.9251	0.9014	0.8891	0.8533	0.8364	0.7981	0.7732	0.7524	0.7238	0.7432
7	L	0.5287	0.5564	0.5831	0.6122	0.6421	0.6708	0.6944	0.7218	0.7532	0.7804	0.8213	0.8029
	U	0.9944	0.9956	0.9987	0.9821	0.9634	0.9425	0.9218	0.8907	0.8733	0.8529	0.8213	0.8435
8	L	1	1	1	1	1	1	1	1	1	1	1	1
	U	1	1	1	1	1	1	1	1	1	1	1	1
9	L	0.8734	0.9022	0.9233	0.9512	0.9855	1	1	1	1	1	1	1
	U	1	1	1	1	1	1	1	1	1	1	1	1
10	L	0.5037	0.5319	0.5537	0.5804	0.6028	0.6351	0.6684	0.6915	0.7268	0.7528	0.7832	0.7621
	U	0.9952	0.9968	0.9946	0.9687	0.9358	0.9139	0.8974	0.8733	0.8545	0.8264	0.7832	0.8026
11	L	0.6273	0.6418	0.6624	0.6806	0.6943	0.7094	0.7287	0.7564	0.7728	0.7964	0.8168	0.8075
	U	0.9967	0.9862	0.9618	0.9487	0.9322	0.9145	0.8901	0.8732	0.8564	0.8387	0.8168	0.8433
12	L	0.5316	0.5674	0.5967	0.6288	0.6543	0.6824	0.7108	0.7431	0.7655	0.7933	0.8364	0.8152
	U	0.9961	0.9959	0.9954	0.9971	0.9982	0.9744	0.9536	0.9264	0.8952	0.8731	0.8364	0.8529

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L: lower bound of the fuzzy efficiency value; U: upper bound of the fuzzy efficiency value.

4.3. Computational improvements

Based on the results of Table 2 and the above explanation, we compare the computational demands of the two models.

For the conventional fuzzy DEA/AR model, in order to determine the relatively effective fuzzy efficiency value, eleven computations are needed at the distinct α-cut levels (0.0, 0.1, 0.2, ..., 1) for every DMU. In our empirical example, there are twelve DMUs, so we have to compute 132 times, and then compare the different fuzzy efficiency values to find the relatively effective fuzzy efficiency value, which leads to substantial computational demands.

For the novel fuzzy DEA/AR model, the upper and lower bounds of the fuzzy efficiency values, shown in the last column of Table 2, are independent of different α-cut values. The proposed model doesn't need to compute the DMUs' fuzzy efficiency values at different α-cut levels and endeavor to find the relatively effective fuzzy efficiency value. This model only needs 12 computations in our empirical example. This is a 90.9% reduction of the calculation work in comparison with the 132 computations needed in the conventional model. This justifies that our novel fuzzy DEA/AR model reduces the computational demands.

4.4. Accuracy improvements

Based on the results of Table 2 and the explanation in Section 4.2, we demonstrate the accuracy improvement of our novel model.

According to Liu and Chuang [5], the closer the upper-bound and lower-bound of the fuzzy efficiency values are to the ideal value (when $\alpha = 1$), the higher the accuracy of the efficiency value calculated. In order to compare the accuracy, we compare the distances the upper and lower bounds of the fuzzy efficiency values calculated by the two models to the ideal values. These results are illustrated in Figure3, based on the data in Table2.

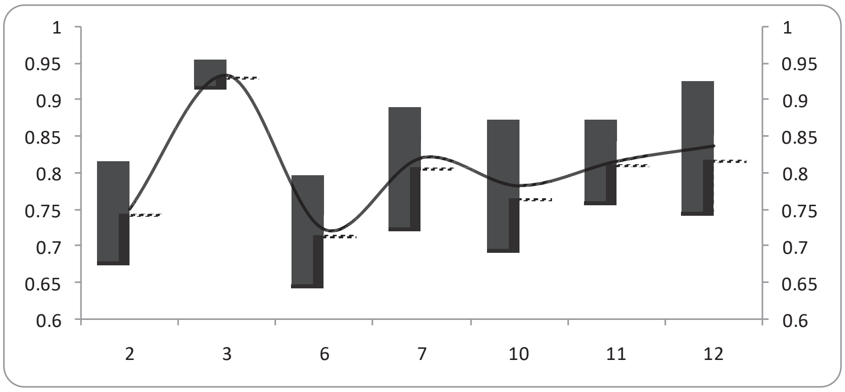

Figure 3. The comparison between the distances determined by the upper and lower bounds of the DMUs' fuzzy efficiency values calculated by the two models

The lower-bounds (i.e., L) and upper-bounds (i.e., U) of every DMU's relatively effective fuzzy efficiency value in Column 10 (when $\alpha = 0.7$) of Table 2 are represented graphically the bottom and the top of the black cylinder in Figure 3. The lower-bounds (i.e., L) and the upper-bounds (i.e., U) of every DMU's fuzzy efficiency value in Column 14 (novel DEA/AR model) of Table 2 are represented graphically the bottom and the top of the cylinder with the slanted line fill pattern in Figure 3.Column 13 reports the most likely efficiency value, which is represented graphically as the black curve in Figure 3.

From Figure 3, we find that the cylinder with the slanted line fill pattern is obviously shorter than the black cylinder. The top and bottom of these cylinders, determined by the upper and

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lower bounds of the fuzzy efficiency values in the last column of Table 2 (calculated by the novel fuzzy DEA/AR model), are obviously closer to the most likely efficiency value shown as the black curve in Figure 3, than the top and bottom of the black cylinder determined by the upper and lower bounds of the relatively effective fuzzy efficiency values (calculated by the conventional model in column 10). This means the bounds of the fuzzy efficiency values calculated by the novel DEA/AR model are closer to the ideal efficiency value (when α =1), and it justifies that the novel DEA/AR model can get accurate improvements.

5. CONCLUSIONS

The conventional fuzzy DEA/AR model has been widely applied in calculating the fuzzy efficiency values of DMUs with vague inputs and outputs. However, the conventional fuzzy DEA/AR model has two limitations: it depends on different α-cut variables, which leads to high computational demands; and it generates lower accuracy efficiency values owing to its nonlinear characteristic. To address these limitations, a novel fuzzy DEA/AR model is introduced in this work to assess the efficiency values of DMUs with fuzzy inputs and outputs. This improved model has three unique features:(1) it is independent of different α-cut variables and doesn't need a pre-set step size, which reduces the computation efforts; (2) it is linear and demonstrates an accuracy improvement; (3) We add the verification of assurance region constraints in the proof of Theorem 1 and Theorem 2, which have not been considered in the literature. The empirical study of twelve university libraries is employed to demonstrate the features and the application of this novel fuzzy DEA/AR model. Based on this novel model, we plan to investigate the sorting of fuzzy efficiency values in the future.

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