ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

Fuzzy Comprehensive Evaluation of Grouting Treatment Effect of Goaf Foundation Based on Grey Correlation Analysis

Yongchao Yang

School of Civil Engineering, Henan Polytechnic University, Jiaozuo, Henan, 454000, China

Abstract

In order to evaluate the grouting treatment effect of goaf foundation more systematically and comprehensively, taking the goaf treatment project of a school district in Xinmi as the research object, a grey relational fuzzy comprehensive evaluation method is proposed based on the principle of grey relational analysis and fuzzy mathematics. Combined with the actual working conditions and collected data, 5 first-level evaluation indexes and 18 second-level evaluation indexes were analyzed and determined. Ahp was used to determine the weight of each index, grey correlation coefficient was used as the membership degree of second-level indexes, and a grey correlation fuzzy comprehensive evaluation model was built by analyzing the engineering data. This model is used to evaluate the grouting effect of goaf treatment project in a school district of Xinmi, and the conclusion is that the grouting effect is "good", which is basically consistent with the follow-up monitoring results of surface settlement, which verifies the feasibility of this method in the evaluation of the grouting effect of goaf foundation, provides theoretical support for the safe use of the school district after completion, and also provides a reference for similar projects in the future.

Keywords

Goaf foundation; Grouting effect; Comprehensive evaluation method; Evaluation index system.

1. INTRODUCTION

With the steady development of the integration of urban and rural areas, the construction land is increasingly tight, and many buildings will inevitably be built above the goaf. When disturbed by the new load, the original stress state of the goaf and its overlying strata will change, resulting in uneven settlement of the stable surface and endangering the safety of new buildings and residents on the surface [1,2]. Grouting and filling method means to drill holes on the ground and use grouting devices such as high-pressure pumps to inject grout into rock cracks to form grout stones inside rocks and fill in the interior of rock layers, so as to improve the bearing capacity and compressive deformation of rocks and effectively control the surface residual settlement of goaf under important buildings (structures) such as houses and roads [3,4]. The grouting treatment of goaf is a hidden project, and there are many kinds of post-construction quality detection methods, which can be roughly divided into three categories: data analysis, geophysical detection and drilling detection. Xue Yiguo et al. [5] used the P-Q-t curve method to study the grouting effect of the water gusher fault in Qingdao Jiaozhou undersea Tunnel, and determined that the grade of surrounding rock after grouting was significantly improved and the grouting effect was remarkable by integrating geophysical exploration and drilling methods. Li Shuzhi [6] used borehole TV to compare and analyze the filling situation of rock cracks and voids before and after grouting, and evaluated the grouting

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

effect combined with the loss of borehole washing fluid. Zhang Xuan [7] evaluated the grouting treatment effect of goaf by integrating geophysical exploration methods such as direct current sounding, Rayleigh surface wave, electromagnetic CT and seismic CT. Although the grouting testing technology has been relatively mature, the evaluation of grouting effect in engineering practice often relies on a single test result and does not pay enough attention to the correlation between various test indexes. The evaluation focuses on post-construction quality testing, and the results are independent and incompatibility [8-10]. Therefore, the evaluation of grouting effect should be a systematic work, which should involve geological exploration before construction, data analysis during construction and quality inspection after construction. Due to the concealment and complexity of grouting engineering, the collection of engineering data can not cover all aspects, and the traditional fuzzy comprehensive evaluation can not effectively reach a conclusion. Grey correlation analysis is just suitable for this kind of situation.

In the course of the continuous development of grey system theory, grey correlation analysis has become one of the hot topics in recent years. As a new method to study uncertainty problems such as small amount of data and lack of information, grey correlation analysis mainly conducts in-depth research on known parts of data and further explores favorable information, so as to accurately describe the operation behavior and evolution law of the system and effectively monitor it [11]. Nowadays, with the rapid development of information technology, "small data" and "poor information" and other common situations in engineering are still urgent problems to be solved, and the emergence of grey theory provides corresponding solutions. Based on the geometric features of sequence curves, it is possible to determine whether the relationship between each sequence is close. Previous grey relational analysis models, whether based on point correlation coefficient or generalized grey relational analysis with overall planning, measure similarity degree from the perspective of proximity [12].

Olson[13,14] et al. proposed a grey relational degree analysis method for solving interval-number multi-attribute decision making problems, and applied it to the simulation of various fuzzy multi-attribute decision models to obtain relatively systematic probabilistic explanations related to the simulation results. Amanna[15] et al. combined GM and GRA models to conduct in-depth discussion on intelligent reasoning engine and adaptive algorithm in wireless communication. Liu Weifeng [16] gave a generalized interval grey relational degree model, and explained the calculation method and feasibility of the model with specific cases. Huangshi [17] incorporated the maximum entropy principle into the study, and took Xinghe Oilfield in Shaanxi-Gan-ning Basin as an example to explore. It is known from the research that although grey correlation analysis is rich in theoretical research, there are few researches on the application of evaluating the grouting effect of goaf. Therefore, this paper combines it with fuzzy comprehensive evaluation, comprehensively analyzes the influencing factors of grouting detection technology and grouting effect, and establishes the evaluation model of grouting treatment effect of goaf foundation. The model is used to evaluate the effect of the grouting treatment project in the goaf of Xinmi Party School.

2. FUZZY COMPREHENSIVE EVALUATION MODEL OF GROUTING EFFECT BASED ON GREY CORRELATION ANALYSIS

2.1. The establishment of evaluation index system

The Analytic Hierarchy Process (AHP)[18-20] is mainly based on the analysis of target elements to build an interconnected cascade level, which clearly reflects the relationship between various elements. Through the combination of qualitative and quantitative methods, the decision-making process is made mathematical and the decision results are objectified, so as to simplify the evaluation of the treatment effect of goaf foundation. Combined with previous literature, field construction and post-construction quality detection, the evaluation of grouting

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

effect of goaf foundation should take into account construction quality, post-construction detection, field test and other factors, and determine 5 first-level evaluation indicators and 18 second-level indicators, as shown in Figure 1.

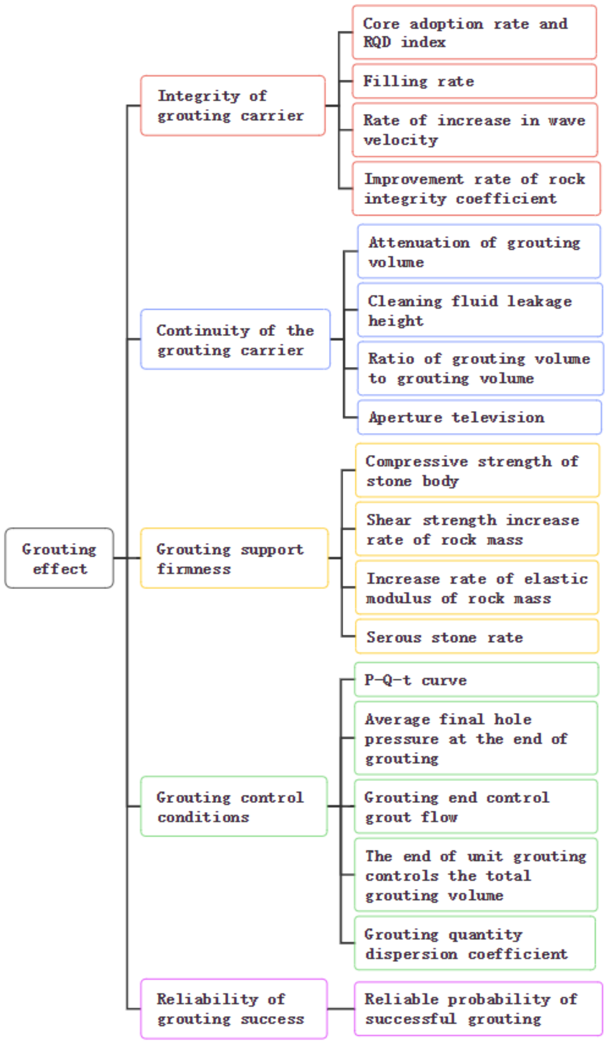

Figure 1. Evaluation model of grouting treatment effect of goaf foundation

2.2. AHP method to determine index weight

In the evaluation system, the influence degree of each factor on the evaluation object is different, which can be quantified and weighted to reflect the influence degree of each factor on the evaluation object. AHP can combine qualitative and quantitative methods and determine the weight through multi-objective decision analysis. According to the criterion layer and the index layer in the grouting evaluation index, the 1-9 scale method is used to describe the influence of each factor on the evaluation objective, and the corresponding score is given according to the advice of experts. The evaluation matrix $A=(aij)n\times n$ is constructed, and the scale is shown in the following table. Using MATLAB software to calculate the maximum eigenvalue of the matrix λmax, the corresponding eigenvector is the weight of each level of factors. The consistency test of judgment matrix uses the consistency ratio CR as the test condition. When $CR<0.1$, the matrix consistency is considered to meet the standard; if $CR\geq0.1$, the judgment matrix needs to be readjust.

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

Table 1. Scale table

	meaning
1	a and b are of equal importance
3	a is slightly more important than b
5	a is significantly more important than b
7	a is more important than b
9	a is absolutely more important than b
2, 4, 6, 8	The median of the above tests
reciprocal	If a is more important than b is m, then b is more important than a is 1/m

Table 2. Randomness Index Table (RI)

order	1	2	3	4	5	6	7	8	9
RI value	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46

Table 3. V-U judgment matrix

Evaluation and classification of treatment effect of grouting in goaf V	U_1	U_2	U_3	U_4	U_5
U_1	1	2	7	1/2	3
U_2	1/2	1	5	1/4	2
U_3	1/7	1/5	1	1/8	1/5
U_4	2	4	8	1	5
U_5	1/3	1/2	5	1/5	1

Table 4. U_1-U_{1-i} judgment matrix

Grouting carrier integrity U_1	U_{11}	U_{12}	U_{13}	U_{14}
U_{11}	1	5	1/2	1/3
U_{12}	1/5	1	1/6	1/7
U_{13}	2	6	1	1/2
U_{14}	3	7	2	1

Table 5. U2-U2-i judgment matrix

Grouting carrier continuity U_2	U_{21}	U_{22}	U_{23}	U_{24}
U_{21}	1	5	1/2	1/4
U_{22}	1/5	1	1/7	1/9
U_{23}	2	7	1	1/3
U_{24}	4	9	3	1

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

Table 6. U3-U3-i judgment matrix

Grouting carrier firmness U3	U_{31}	U_{32}	U_{33}	U_{34}
U_{31}	1	1/2	1	1/4
U_{32}	2	1	2	1/3
U_{33}	1	1/2	1	1/4
U_{34}	4	3	4	1

Table 7. U_3-$U_{3\text{-}i}$ judgment matrix

Grouting construction control condition U_4	U_{41}	U_{42}	U_{43}	U_{44}	U_{45}
U_{41}	1	1/2	1/3	1/2	6
U_{42}	2	1	1/3	1	7
U_{43}	3	1/2	1	2	9
U_{44}	2	1	1/2	1	7
U_{45}	1/6	1/7	1/9	1/7	1

Table 8. U4-U4-i judgment matrix

Grouting construction control condition U_4	U_{41}	U_{42}	U_{43}	U_{44}	U_{45}
U_{41}	1	1/2	1/3	1/2	6
U_{42}	2	1	1/3	1	7
U_{43}	3	1/2	1	2	9
U_{44}	2	1	1/2	1	7
U_{45}	1/6	1/7	1/9	1/7	1

Table 9. Consistency test

Judgment matrix	Maximum characteristic root	CI	RI value	CR value
V-U	5.1632	0.0408	1.11	0.0368
U_1	4.0766	0.0255	0.882	0.0289
U_2	4.1042	0.0347	0.882	0.0394
U_3	4.0206	0.0069	0.882	0.0078
U_4	5.102	0.0255	1.11	0.023

2.3. Determine feature sequence and character sequence

The data sequence that reflects the behavior characteristics of the system is the parent sequence, let it be $U = \{U_1, U_2, \ldots, U_n\}$; The sequence of factors that affect the behavior of the system is the characteristic sequence, let it be

$U_1 = \{U_{11}, U_{12}, \ldots, U_{1m}\}$

$U_2 = \{U_{21}, U_{22}, \ldots, U_{2m}\}$

$\vdots$

$U_m = \{U_{m1}, U_{m2}, \ldots, U_{nm}\}$; Construct evaluation set

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

$$V = \{V_1, V_2, V_3, V_4,\} = \{Excellent, good, qualified, unqualified\}$$

2.4. Membership degree determined by grey correlation analysis

Grey correlation analysis is a multifactor statistical method based on grey system, according to the factors of sampling data, the available methods of grey correlation degree, the strength, size, and sort of the relationship between various influencing factors of digital processing. This method is compared with the conventional analysis of multiple factors analysis is more intuitive, suitable for processing has a certain gray level of the sample data, can be used on concealment goaf grouting treatment project. Traditionally, grey correlation analysis method is the optimal value of all parameters as the target matrix, according to the literature research the reverse application [34], will be measured as the target matrix, it combined with matrix evaluation grades as matrix analysis, grey correlation coefficient of solving steps are as follows:

Establish a grey relational basic data analysis matrix Xi ', where xi0' is the measured value of evaluation indicators during grouting construction, and xi1 ', xi2', xi3 ', and xi4' are the median of the grading interval for excellent, good, qualified, and unqualified evaluation indicators.

$$X_i{}' = \begin{bmatrix} x_{i0}{}'(1) & x_{i1}{}'(1) & \cdots & x_{i4}{}'(1) \\ x_{i0}{}'(2) & x_{i1}{}'(2) & \cdots & x_{i4}{}'(2) \\ \vdots & \vdots & \ddots & \vdots \\ x_{i0}{}'(m) & x_{i1}{}'(m) & \cdots & x_{i4}{}'(m) \end{bmatrix}$$

According to the following formula, dimensionless normalization is carried out on the analysis sample data, and the normalization analysis matrix Xi is obtained.

$$x_{ij}(k) = \frac{x'_{ij}(k) - \min_k \{x'_{ij}(k)\}}{\max_k \{x'_{ij}(k)\} - \min_k \{x'_{ij}(k)\}}$$

$$(i = 1,2, \dots ,5; j = 0,1, \dots ,4; k = 1,2, \dots , m)$$

$$X_i = \begin{bmatrix} x_{i0}(1) & x_{i1}(1) & \cdots & x_{i4}(1) \\ x_{i0}(2) & x_{i1}(2) & \cdots & x_{i4}(2) \\ \vdots & \vdots & \ddots & \vdots \\ x_{i0}(m) & x_{i1}(m) & \cdots & x_{i4}(m) \end{bmatrix}$$

Calculate the correlation coefficient of each subsequence to the parent sequence

$$r_{ij}(k) = \frac{\Delta_{min} + \rho\Delta_{max}}{\Delta_{ij}(k) + \rho\Delta_{max}}$$

$$(i = 1,2, \dots ,5; j = 1, \dots ,4; k = 1,2, \dots , m)$$

Among them, $\Delta_{ij}(k) = |x_{i0}(k) - x_{ij}(k)|$; $\Delta_{min} = \min_j \min_k |x_{i0}(k) - x_{ij}(k)|$; $\Delta_{max} = \max_j \max_k |x_{i0}(k) - x_{ij}(k)|$; ρ is the resolution coefficient, in the value of [0,1], the smaller the resolution coefficient, the greater the difference between the correlation coefficient, the stronger the differentiation ability, usually take 0.5.

2.5. Grey relational fuzzy comprehensive evaluation

According to the evaluation model of grouting treatment effect of goaf foundation, the multi-stage fuzzy comprehensive evaluation factor set U, U_i and evaluation set V are constructed. The

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

membership degree of each indicator U_{ij} in the second-level evaluation indicator U_i to each evaluation level V_i is r_{ij}, then the evaluation matrix of U_i relative to V is

$$R_i = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1k} \\ r_{21} & r_{22} & \cdots & r_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mk} \end{bmatrix}$$

The weight $A_i = (a_{i1}, a_{i2}, \dots, a_{im})$, then the comprehensive evaluation matrix of the second-level index can be expressed as

$$B_i = A_i \times R_i = (b_{i1}, b_{i2}, \dots, b_{ik},)$$

The fuzzy comprehensive evaluation matrix of first-level index is

$$R = (B_1, B_2, \dots, B_n)$$

$$B = A \times R = (b_1, b_2, \dots, b_k)$$

According to the principle of maximum membership degree, the element of evaluation set corresponding to maximum membership degree is taken as the evaluation result

$$V = \{V_i | V_i \rightarrow max b_i\}$$

3. ENGINEERING APPLICATION

3.1. Project Overview

Taking the evaluation of the grouting effect of goaf foundation in a campus in Xinmi as an example, the evaluation system constructed above is adopted for verification. The school area is located at the northeast corner of the intersection of Qingping Street and Yuecui Road, Xinmi City. The overall terrain of the project area is flat, high in the north and low in the south, with a slope of about 3%. The ground elevation of the study area is 255m~268m, the lowest point is the center of the collapse pit, and the ground relative elevation difference is about 8m. There are four circular collapse pits scattered in the area. The distribution of the pits is shown in Figure 2.

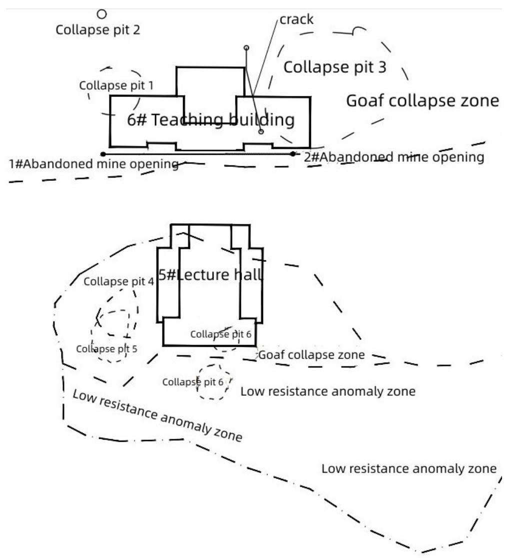

Figure 2. Distribution of caving pits in goaf

Combined with engineering practice experience and grouting operation requirements, the water-solid ratio of curtain hole grout (water: solid material) is 1:1.3, solid ratio (cement: fly ash) is 3:7, according to previous engineering experience, the curtain hole grout should be mixed with 2% of the total solid mass of accelerator (water glass). The ratio of water to solid

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

(water: solid material) in the grouting hole is 1:1.2, and the ratio of solid (cement: fly ash) is 3:7. For the foundation grouting treatment of the goaf site in this project, two circles of curtain holes are arranged around building 5 and building 6. The layout is triangular, and the grouting holes are arranged in the shape of clubs. The distance between curtain holes is 6.5m; The grouting holes are arranged in plum shape, and the spacing between the holes is 10m. There are 155 grouting holes, 17 rows of grouting holes, 1 to 7 rows with a distance of 8.66m. In this project, there are 334 grouting holes, 179 curtain holes and 155 grouting holes in the goaf site of Building 5 and 6 of the project. Borehole locations are shown in Figure 3.

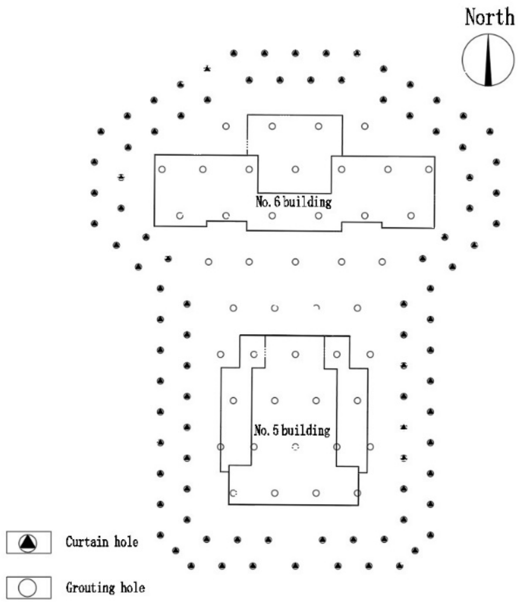

Figure 3. Borehole location distribution

3.2. Fuzzy comprehensive evaluation based on grey correlation analysis

(1) The weight of primary index and secondary index

According to the judgment matrix established in Section 2.2, the weight of each judgment matrix is calculated by using the root method through MATLAB

$$A=\begin{bmatrix}0.2631\\0.1496\\0.0336\\0.4536\\0.1\end{bmatrix}^T,\ A_1=\begin{bmatrix}0.1793\\0.0493\\0.2937\\0.4777\end{bmatrix}^T,$$

$$A_2=\begin{bmatrix}0.1528\\0.0408\\0.2525\\0.5539\end{bmatrix}^T,\ A_3=\begin{bmatrix}0.1214\\0.2195\\0.1214\\0.5376\end{bmatrix}^T,$$

$$A_4=\begin{bmatrix}0.1333\\0.2982\\0.3401\\0.1967\\0.0317\end{bmatrix}^T$$

Since there is only one sub-index of grouting reliability, there is no need to establish a discrimination matrix. Its value is 87.21% calculated by MATLAB programming, and the weight A5 is regarded as 1.

(2) grey relational membership degree

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

According to the principles in Section 2.3, SPSS data analysis software is used to calculate the grey correlation degree

$$R_1 = \begin{bmatrix} 0.7399 & 0.9488 & 0.8584 & 0.7415 \\ 0.7129 & 0.9048 & 0.7923 & 0.5292 \\ 1.0000 & 0.8623 & 0.7334 & 0.6118 \\ 0.9552 & 0.9640 & 0.9408 & 0.9020 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.6757 & 0.5779 & 0.4558 & 0.3371 \\ 0.6867 & 0.8622 & 0.5062 & 0.3689 \\ 0.7505 & 0.9963 & 0.6868 & 0.4588 \\ 0.7399 & 0.9488 & 0.8584 & 0.7415 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 0.9680 & 0.9390 & 0.9167 & 0.9028 \\ 0.5897 & 0.7498 & 0.8570 & 0.9712 \\ 0.5476 & 0.7422 & 0.8051 & 0.4867 \\ 0.8624 & 0.8803 & 0.9103 & 0.9817 \end{bmatrix},$$

$$R_4 = \begin{bmatrix} 0.7749 & 0.9789 & 0.7641 & 0.5510 \\ 0.8259 & 0.9457 & 0.7857 & 0.5621 \\ 0.9460 & 0.9312 & 0.7010 & 0.5174 \\ 0.9394 & 0.9140 & 0.9079 & 0.9322 \\ 0.4770 & 0.5132 & 0.6819 & 0.6708 \end{bmatrix},$$

$$R_5 = [0.8458 \;\; 0.9397 \;\; 0.7058 \;\; 0.5200]$$

(3) first-level fuzzy comprehensive evaluation

$$B_1 = A_1 \times R_1$$
$$= [0.9178 \quad 0.9285 \quad 0.8578 \quad 0.7696]$$
$$B_2 = [0.7306 \quad 0.9006 \quad 0.7392 \quad 0.5932]$$
$$B_3 = [0.7771 \quad 0.8419 \quad 0.8865 \quad 0.9096]$$
$$B_4 = [0.8712 \quad 0.9253 \quad 0.7748 \quad 0.6217]$$
$$B_5 = [0.8458 \quad 0.9397 \quad 0.7058 \quad 0.5200]$$

(4) two-level fuzzy comprehensive evaluation

$$B = A \times R = A \times \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 5.9864 \\ 6.4359 \\ 5.5070 \\ 4.5826 \end{bmatrix}$$

According to the principle of maximum membership degree, it can be determined that the grouting treatment effect of goaf foundation is good. The evaluation results are consistent with the field geophysical detection results and the late settlement observation results, which provides scientific and reasonable support for the safety of production and teaching activities in Xinmi Campus.

4. CONCLUSION

1. Based on the actual working conditions of Xinmi Campus and the limited data in the construction process, this paper constructs an evaluation system for the grouting treatment effect of goaf foundation from the characteristics of grouting carrier, grouting control conditions and reliability probability, determines 5 first-level indexes and 18 second-level indexes for evaluating the grouting effect, and determines the main control factors through the sub-analysis method. The grey correlation fuzzy comprehensive evaluation method can be used to evaluate the grouting effect of goaf foundation.

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

2. Using grey correlation coefficient instead of membership degree improves the calculation efficiency. It can not only effectively deal with the difficulties caused by incomplete data record, low precision, small amount of data and hidden grouting engineering in the construction process, but also effectively avoid the subjective influence of using traditional methods such as membership degree function to determine membership degree and ensure the objectivity of evaluation results.

3. Taking the goaf foundation treatment project of Xinmi Party School as an example, through the comprehensive evaluation of the grouting effect after construction, it is concluded that the treatment effect is "good". The analysis results show that the grey correlation fuzzy comprehensive evaluation method can be used well in the evaluation of grouting treatment effect of goaf foundation.

REFERENCES

[1] State Administration of Safety Regulation. Regula tions for Coal pillar setting and coal pressing in Buildings, water bodies, railways and main shafts [M]. Beijing: China Coal Industry Press,2017.

[2] Teng Yonghai, Zhang Junying. Foundation stability evaluation of old goaf [J]. Journal of China Coal Society,1997(05):58-62.

[3] Zhang Hongzhen, Deng Kazhong, Gu Wei. Study on the distribution of residual voids in old goaf [J]. Journal of Mining & Safety Engineering, 2016, 33(5): 893-897.

[4] Zhang Gangyan. Research on the Treatment Effect Detection and Comprehensive Evaluation Method of Goaf in Coal Mine [C]//. Proceedings of 2011 National Academic Conference on New Mining Surveying Technology.2011 :123-131.

[5] Xue Yiguo, Li Shucai, Su Maoxin, Zhang Xiao, Zhao Yan, Xu Zhenhao, Li Zhipeng, Zhang Gongshi. Research on comprehensive test method for grouting effect of underwater tunnel gusher in Qingdao Jiaozhou Bay [J]. Chinese Journal of Rock Mechanics and Engineering,2011,30(07):1382-1388.

[6] Li Shuzhi, Li Xueliang. Application of borehole TV detection technology in detection of grouting effect in Goaf [J]. Safety in Coal Mine,2013,44(03):147-149+152.

[7] ZHANG Xuan. Study on comprehensive geophysical detection method of grouting treatment effect in Nanqin Line Goaf [D]. Southwest Jiaotong University,2014.

[8] Hermann Haken. Grey Information: Theory and Practical Applications[J]. Grey systems: theory and application, 2011, 1(1) : 105-106.

[9] Keith William Hipel. Grey Systems: Theory and Applications[J]. Grey Systems: Theory and Application, 2011, 1(3) : 274-275.

[10] Robert Vallée. Grey Information: Theory and Practical Applications[J]. Kybernetes, 2008, 37(1) : 189-189.

[11] Liu Sifeng, Yang Yingjie, Wu Lifeng. Grey System Theory and Its Application [M]. Beijing: Science Press,2014.

[12] Sifeng Liu et al. A summary on the research of GRA models[J]. Grey Systems: Theory and Application, 2013, 3(1) : 7-15.

[13] Jijun Zhang. The method of grey related analysis to multiple attribute decision making problems with interval numbers[J]. Mathematical and Computer Modelling, 2005, 42(9) : 991-998.

[14] David L. Olson and Desheng Wu. Simulation of fuzzy multiattribute models for grey relationships.[J]. European Journal of Operational Research, 2006, 175(1) : 111-120.

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202305_9(5).0015

[15] Ashwin Amanna and Matthew J. Price and Ratchaneekorn Thamvichai. Grey systems theory applications to wireless communications[J]. Analog Integrated Circuits and Signal Processing, 2011, 69(2-3) : 259-269.

[16] Liu Weifeng. Generalized interval gray relational degree model [J]. Journal of Zhengzhou University (Science Edition),2013,45(02):41-44+89.

[17] Huang Shiyan. Application of grey evaluation entropy model to oil-bearing area evaluation [J]. Chinese Journal of Geomechanics,2006(01):77-83.

[18] WANG Y M, ELAHG T M S. An approach to avoiding rank reversal in AHP[J]. Decision Support System, 2006, 42(3):1474–1480.

[19] Stam A, Duarte Silva A P. On multiplicative priority rating methods for the AHP[J]. European Journal of Operational Research, 2003, 145(1): 92–108.

[20] Scholl A, Manthey L, Helm R, et al. Solving multiattribute design problems with analytic hierarchy process and conjoint analysis: An empirical comparison[J]. European Journal of Operational Research, 2005, 164(3): 760–777.