A Hybrib Method for Solving Nonlinear Fuzzy Volterra Integro-Differential Equations

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Abstract

In this paper, a hybrid method for solving nonlinear fuzzy Volterra integro-differential equations with degenerate kernel is introduced. The proposed method combines Laplace transform with Adomian Decomposition Method, which is abbreviated as LADM. The nonlinear fuzzy Volterra integro-differential equation is converted into two crisp integral equations by using fuzzy numbers in the form of parameters. Then the Laplace transform is used to process the differential part of the equation, and the nonlinear part is dealed with the Adomian Decomposition Method. The numerical solution of the equation is obtained. Some examples are illustrated to the robustness, efficiency and the applicability of the proposed method.

Keywords

Fuzzy integro-differential equations; Volterra integro-differential equations; Laplace transform; Adomian decomposition methods.

1. INTRODUCTION

Fuzzy integro-differential equations provides the universality of uncertainty or ambiguity for mathematical modeling of real-world problems. The study of fuzzy integro-differential equations is of more interest and fast growing, especially in fuzzy control, which has been developedrecently. The concepts of fuzzy sets and arithmetic operations were proposed by Zadeh [1, 2] and further enriched by Tsumoto and Tanaka [3]. Later, Dubois and Prade [4] introduced the concept of left and right (LR) fuzzy numbers and gave formulas for calculating fuzzy number operations. Puri and Ralescu [29] proposed two definitions for fuzzy derivative of fuzzy functions. Kaleva [30] solved fuzzy differential equations based on h-difference. The concept of integration of fuzzy functions was first introduced by Dubois and Prade [5]. Wu and Ma [28] were the first proposers of fuzzy integral applications, in which they mainly studied the second type of fuzzy Fredholm integral equations. Friedman and Ma [31] proposed an embedding method to solve the fuzzy Volterra and Fredholm integral equations. Babolian and Sadeghi [32] used Adomian method to solve the second kind of Fredholm fuzzy linear integral equation. Attari and Yazdani [33] proposed a homotopy perturbation algorithm to solve the second kind of nonlinear fuzzy Volterra Fredholm integral equation. More numerical methods can be find in [6–8, 8–16].

The Volterra integro-differential equations appeared in many physical applications such as neutron diffusion and biological species coexisting together with increasing and decreasing rates of generating. The fuzzy nonlinear Volterra integro-differential equation (FVIDE) is given as follows:

$$\tilde{u}'(x) = \tilde{f}(x) + \int_0^x \widetilde{K}(x,t)F(\tilde{u}(x))dt\,, \tilde{u}(0) = (\bar{a}, \underline{a}), t \in J := (a,b], \tag{1.1}$$

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where the function $\tilde{f} : J \times \mathbb{R}_{\mathcal{F}} \to \mathbb{R}_{\mathcal{F}}$ and the crisp function $\widetilde{K}(x,t)$, are continuous and $\tilde{u}(0)$ is a fuzzy number. $F(\tilde{u}(x))$ is a nonlinear function of $\tilde{u}(x)$ such as $\tilde{u}^3(x)$,$sin(\tilde{u}(x))$, $con(\tilde{u}(x))$. In this paper, the hybrid method LADM is used to solve Eq(1.1).

This work is organized as follows: Section 2 devotes to the preliminaries mainly on the basic fuzzy definitions and fuzzy Laplace transformation method. Section 3 is the algorithm of LADM method. Numerical examples are given in Section 4 and illustrate the practicability of the method. Some of the conclusions of this paper and future work are given in Section 5.

2. PRELIMINARIES

This section devotes to some useful preliminaries. Firstly the basic definitions offuzzy number and metric are introduced. Given a nonempty set X, *a fuzzy subset* A *is characterized by a membership function* $f_A(x) : X \to [0,1]$ *which represents the "grade of membership" of* x *in* A.

Definition 2.1. [22,23] Let $\mathbb{R}$ be the set of reals and $u : \mathbb{R} \to [0,1]$. u is called a fuzzy real number if it has the following properties

(i)u is an upper semi-continuous function on $\mathbb{R}$,

(ii) u is a convex fuzzy set, i.e., $u(\lambda x_1 + (1-\lambda)x_2) \geq min\{u(x_1), u(x_2)\}$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$,

(iii) u is normal, i.e., there exists $x_0 \in \mathbb{R}$ such that $u(x_0) = 1$.

(iv) The closure of the support of u is compact in $\mathbb{R}$.

Denote the set of all fuzzy real number u with $\mathbb{R}_{\mathcal{F}}$. For any $x_0 \in \mathbb{R}$, denote χ_{x0} be the characteristic function at x_0, it is obvious that $\chi_{x0} \in \mathbb{R}_{\mathcal{F}}$ [23].

Definition 2.2. [24,25] Given $0 \leq r \leq 1$, a fuzzy number u in parametric form is represented by an ordered function pairs $\left(\underline{u}(r), \overline{u}(r)\right)$ satisfying

(i) $\underline{u}(r)$ is a bounded left continuous non decreasing function,

(ii) $\overline{u}(r)$ is a bounded left continuous non increasing function,

(iii) $\underline{u}(r) \leq \overline{u}(r)$.

For $u = (\underline{u}, \overline{u}), v = (\underline{v}, \overline{v}) \in E$ and $\lambda \in R$, the sum of $u + v$ and the scalar multiplication λu can be defined by

$$\left(\underline{u+v}\right)(r) = \underline{u}(r) + \underline{v}(r), \quad \left(\overline{u+v}\right)(r) = \overline{u}(r) + \overline{v}(r), \quad \forall r \in [0,1],$$

and

$$\lambda u = \begin{cases} (\lambda\underline{u}, \lambda\overline{u}), \lambda \geq 0, \\ (\lambda\overline{u}, \lambda\underline{u}), \lambda \leq 0. \end{cases}$$

Definition 2.3.[22,23] For $0 \leq r \leq 1$, the r-level set of a fuzzy number u is defined by

$$[u]^r := \{x \in \mathbb{R} \mid u(x) \geq r\}, 0 < r \leq 1, [u]^0 := \overline{\{x \in \mathbb{R} \mid u(x) \geq r\}}.$$

It is well known that for each $r \in [0,1]$, $[u]^r$ *is a bounded closed interval of* $\mathbb{R}$, *i.e.,*$[u]^r = [u_-^{(r)}, u_+^{(r)}]$ *with* $u_-^{(r)} \leq u_+^{(r)}, u_-^{(r)}, u_+^{(r)} \in \mathbb{R}$.

Definition 2.4.[19]For any two fuzzy numbers u and v, define $D: \mathbb{R}_{\mathcal{F}} \times \mathbb{R}_{\mathcal{F}} \to \mathbb{R}^+ \cup \{0\}$ *by*

$$D(u,v) := \sup_{r\in[0,1]} D_H([u]^r, [v]^r) = \sup_{r\in[0,1]} max\{|\,\underline{u}(r) - \underline{v}(r)\,|, |\,\overline{u}(r) - \overline{v}(r)\,|\},$$

It can be obtained that D *is a metric on* $\mathbb{R}_{\mathcal{F}}$. $(\mathbb{R}_{\mathcal{F}}, D)$ *is a complete metric space with the properties.*

Definition 2.5.[17] Let $I = [0,a] \subset \mathbb{R}$ be a closed and bounded interval. A mapping $x: I \to \mathbb{R}_{\mathcal{F}}$ is bounded, if there exists r > *0* such that

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$$D\big(x(t),\tilde{0}\big) < r\,, \forall t \in I,$$

For any u, v, w, z $\in \mathbb{R}_{\mathcal{F}}$ and $\lambda \in \mathbb{R}$, some useful properties of D are given by

(i) $D(u+w, v+w) = D(u,v)$,

(ii) $D(u+v, w+z) \le D(u,w) + D(v,z)$,

(iii) $D(u,v) \le D(u,w) + D(w,v)$,

(iv) $D(\lambda u, \lambda v) = |\lambda| D(u,v)$,

(v) $D\big(u \tilde{*} v, \tilde{0}\big) = D\big(u,\tilde{0}\big) D\big(v,\tilde{0}\big)$ with the fuzzy multiplication $\tilde{*}$ is based on the extension principle that can be proved by α-cuts of fuzzy numbers $u, v \in \mathbb{R}_{\mathcal{F}}$.

Definition 2.6.[35] Fuzzy Laplace Transformation: The fuzzy Laplace transform of a fuzzy real valued function f(t) is defined as follows:

$$F(s) = L\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt = \lim_{T\to\infty} \int_0^{T} e^{-st} f(t)dt,$$

whenever the limit exist. The symbol L is fuzzy Laplace transformation, which acts on fuzzy real valued function f = f(t) and generates F(s) = L {f(t)}. The lower and upper Laplace transform of a fuzzy real valued function f(t) are given as follows:

$$F(s,\alpha) = L\{f(t,\alpha)\} = [L\{\underline{f}(t,\alpha)\}, L\{\overline{f}(t,\alpha)\}].$$

Definition 2.6.[34] Fuzzy Convolution Theorem: The convolution of two fuzzy real valued functions f, g defined for $t \ge 0$ by

$$(f * g)(t) = \int_0^{t} f(t)\, g(t-T)dT.$$

Theorem 2.1[34] If f and g are piecewise continuous fuzzy real valued function on [0,∞) with exponential order p , then

$$L\{(f * g)(t)\} = L\{f(t)\} \cdot L\{g(t)\} = F(s) \cdot G(s),$$

where L represents the Laplace transform.

3. LADM ALGORITHM

Consider the following FVIDE:

$$\tilde{u}'(x) = \tilde{f}(x) + \int_0^{x} \tilde{K}(x,t) F(\tilde{u}(x))dt\,, \tilde{u}(0) = \tilde{a} = (\bar{a}, \underline{a}), t \in J := (a,b], 0 \le \alpha \le 1. \quad (3.1)$$

The parametric forms of Eq(3.1) are written as

$$\begin{cases} \underline{u'}(x,\alpha) = \underline{f}(x,\alpha) + \int_0^{x} K(x,t) F(\underline{u}(t,\alpha))dt, \\ \overline{u'}(x,\alpha) = \bar{f}(x,\alpha) + \int_0^{x} K(x,t) F(\bar{u}(t,\alpha))dt. \end{cases} \quad (3.2)$$

The Adomian decomposition method identifies the nonlinear terms F_1 and F_2 by the decomposing series

$$F_1\big(t, x(t)\big) = \sum_{n=0}^{\infty} A_n(t), \quad F_2\big(t, x(t)\big) = \sum_{n=0}^{\infty} B_n(t), \quad (3.2)$$

with the Adomian polynomials A_n and B_n are given by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[F_1\left(\sum_{i=0}^{n} \lambda^i x_i\right)\right]_{\lambda=0}, n = 0,1,2,\cdots.$$

$$B_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[F_2\left(\sum_{i=0}^{n} \lambda^i x_i\right)\right]_{\lambda=0}, n = 0,1,2,\cdots$$

Substitute Eqs.(3.1)-(3.2) into Eq.(1.5) to obtain

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$$\sum_{i=0}^{\infty} x_i(t) = f(t) + \int_0^t K_1(t,s) \sum_{n=0}^{\infty} A_n(s)ds + \int_0^1 K_2(t,s) \sum_{n=0}^{\infty} B_n(s)ds. \tag{3.3}$$

The iteration of SADM [30] is as follows

$$x_0(t) = f(t),$$
$$x_{i+1}(t) = \int_0^t K_1(t,s)A_i ds + \int_0^1 K_2(t,s)B_i ds, i = 0,1,\cdots$$

In 1999, Wazwaz [30] proposed a MADM on the SADM, which is based on the function $f(t)$ can be decomposed into two parts $f_1(t)$ and $f_2(t)$, That is

$$f(t) = f_1(t) + f_2(t) \tag{3.4}$$

Combining Eq.(3.3) and Eq.(3.4) yields the following iteration

$$x_0(t) = f_1(t),$$
$$x_1(t) = f_2(t) + \int_0^t K_1(t,s)A_0 ds + \int_0^1 K_2(t,s)B_0 ds,$$
$$x_{i+1}(t) = \int_0^t K_1(t,s)A_i ds + \int_0^1 K_2(t,s)B_i ds, i = 2,3\cdots$$

In 2001, Wazwaz and El-Sayed [31] proposed a MADM based on the previous research. This method represents the function $f(t)$ by Taylor series as

$$f(t) = \sum_{i=0}^{\infty} f_i(t) \tag{3.5}$$

The following new iteration can be obtained from Eq.(3.3) and Eq.(3.5)

$$x_0(t) = f_1(t),$$
$$x_{i+1}(t) = f_i(t) + \int_0^t K_1(t,s)A_i ds + \int_0^1 K_2(t,s)B_i ds, i = 1,2\cdots.$$

4. UNIQUENESS OF THE SOLUTION

Based on the following conditions, the existence and uniqueness of the solution of Eq.(1.5) is proved by the Banach fixed point theorem.

(A1) $f: I \to E$ is a continuous fuzzy valued function.

(A2) $F_1: I \times E \to E, F_2: I \times E \to E$ are continuous functions and satisfy the Lipschitz condition, i.e., there exist $L_1 > 0$ and $L_2 > 0$ such that

$$D\left(F_1(s, x_1(t)), F_1(s, x_2(t))\right) \le L_1 D(x_1(t), x_2(t)),$$

and

$$D\left(F_2(s, x_1(t)), F_2(s, x_2(t))\right) \le L_2 D(x_1(t), x_2(t)).$$

(A3) K_1 and K_2 are continuous function and there exist $M_1 > 0, M_2 > 0$ satisfying

$$\int_0^t D\left(K_1(t,s), \tilde{0}\right) \le M_1, \quad \int_0^1 D\left(K_2(t,s), \tilde{0}\right) \le M_2,$$

with $0 < \delta := M_1 L_1 + M_2 L_2 < 1$.

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Theorem 4.1. Assume that the hypotheses $(A1),(A2)$ and $(A3)$ hold, then Eq.(1.5) has a unique solution $x(t)$.

Proof. Let $T: C([0,1],E) \to C([0,1],E)$ be an operator defined as

$$(Tx)(t) = f(t) + \int_0^t K_1(t,s)F_1\big(s,x(s)\big)ds + \int_0^1 K_2(t,s)F_2\big(s,x(s)\big)ds.$$

Then for $\forall y_1, y_2 \in E$ and $\forall t \in I$, there are

$$(Ty_i)(t) = f(t) + \int_0^t K_1(t,s)F_1(s,y_i)ds + \int_0^1 K_2(t,s)F_2(s,y_i)ds, (i=1,2).$$

From the Definition 3, there is

$$\begin{aligned}
&D\big(Ty_1(t), Ty_2(t)\big)\\
\le\ & D\left(\int_0^t K_1(t,s)F_1(s,y_1)ds, \int_0^t K_1(t,s)F_1(s,y_2)ds\right)\\
&+D\left(\int_0^1 K_2(t,s)F_2(s,y_1)ds, \int_0^1 K_2(t,s)F_2(s,y_2)ds\right)\\
\le\ & \int_0^t D\left(K_1(t,s),\tilde{0}\right)D\left(F_1\big(s,y_1(s)\big),F_1\big(s,y_2(s)\big)\right)ds\\
&+\int_0^1 D\left(K_2(t,s),\tilde{0}\right)D\left(F_2\big(s,y_1(s)\big),F_2\big(s,y_2(s)\big)\right)ds\\
\le\ & \sup_{0\le s\le 1} D\left(F_1\big(s,y_1(s)\big),F_1\big(s,y_2(s)\big)\right)\int_0^t D\left(K_1(t,s),\tilde{0}\right)ds\\
&+\sup_{0\le s\le 1} D\left(F_2\big(s,y_1(s)\big),F_2\big(s,y_2(s)\big)\right)\int_0^1 D\left(K_2(t,s),\tilde{0}\right)ds\\
\le\ & (M_1L_1+M_2L_2)D\big(y_1(s),y_2(s)\big).
\end{aligned}$$

Eq.(4.2) means that T is a contraction map. It is concluded that T has a unique fixed point $x(t)$ from the Banach fixed point theorem.

5. NUMERICAL EXAMPLES

In this section, the Adomian polynomial corresponding to the nonlinear term is shown in, and the iterative sequence corresponding to the approximate solution of the equation is referred to in section 3.

Example1 Consider the fuzzy nonlinear Volterra-Fredholm integral equations as

$$\begin{cases} \underline{x}(t) = -\frac{r^2}{300}t^4 - \frac{r^2}{400}t + \frac{r}{10}t + \int_0^t t\,\underline{x}(s)^2ds + \int_0^1 t\,s\underline{x}(s)^2ds,\ 0\le s\le t\le 1,\\ \overline{x}(t) = -\frac{(r-2)^2}{300}t^4 - \left(\frac{r}{10}-\frac{1}{5}\right)t - \frac{(r-2)^2}{400}t + \int_0^t t\,\overline{x}(s)^2ds + \int_0^1 t\,s\overline{x}(s)^2ds \end{cases} \tag{5.1}$$

The exact solution is $x(t) = [\underline{x}(t,r), \overline{x}(t,r)] = \left[\frac{r}{10}t, \frac{2-r}{10}t\right]$, $0\le r\le 1$.

The left bound errors and the right bound errors are list in Table 1 and Table 2 for $n = 10$, respectively.

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Table 1. Left bound of error (when $r = 0.5, n = 4$)

t	E_{SADM}	$E_{MADM(1999)}$	$E_{MADM(2001)}$
0	0	0	0
0.2	8.3093e-16	1.8908e-16	4.7137e-09
0.4	1.7278e-15	3.9205e-16	9.6844e-09
0.6	2.8796e-15	6.5919e-16	1.5782e-08
0.8	4.8225e-15	1.1172e-15	2.5780e-08
1.0	9.2426e-15	2.1025e-15	5.1689e-08

Table 2. Right bound of error (when $r = 0.5, n = 4$)

t	E_{SADM}	$E_{MADM(1999)}$	$E_{MADM(2001)}$
0	0	0	0
0.2	1.6102e-11	3.9477e-12	4.2709e-07
0.4	3.3484e-11	8.2333e-12	8.8134e-07
0.6	5.6007e-11	1.3864e-11	1.4506e-06
0.8	9.3720e-11	2.3354e-11	2.3933e-06
1.0	1.7930e-10	4.4138e-11	4.7110e-06

Example2 Solve the following fuzzy nonlinear Volterra-Fredholm integral equation

$$\begin{cases} \underline{x}(t) = -\frac{r}{6}t - \frac{r^2}{144}t^5 + \frac{r^3}{864}\left(t^2-1\right) + \int_0^t t\, s\underline{x}(s)^2 ds + \int_0^1 \left(1-t^2\right)\underline{x}(s)^3 ds,\ 0 \le s \le t \le 1, \\ \overline{x}(t) = -\frac{r-2}{6}t - \frac{(r-2)^2}{144}t^5 - \frac{(r-2)^3}{864}\left(t^2-1\right) + \int_0^t t\, s\overline{x}(s)^2 ds + \int_0^1 \left(1-t^2\right)\overline{x}(s)^3 ds. \end{cases} \quad (5\text{-}2)$$

The exact solution is $x(t) = \left[\underline{x}(t,r), \overline{x}(t,r)\right] = \left[\frac{r}{6}t, \frac{2-r}{6}t\right],\ 0 \le r \le 1.$

Choose $r = 0.5$ in Eq.(5.2). The left bound errors and the right bound errors are list in Table 3 and Table 4 for $n = 4$.

Table 3. Left bound of error (when $r = 0.5, n = 4$)

t	E_{SADM}	$E_{MADM(1999)}$	$E_{MADM(2001)}$
0	2.9172e-11	9.9033e-13	1.0045e-06
0.2	2.8128e-11	9.5739e-13	4.2128e-07
0.4	2.6050e-11	9.1664e-13	1.6748e-05
0.6	2.5781e-11	1.0367e-12	1.3354e-04
0.8	3.5918e-11	1.9583e-12	5.6646e-04
1.0	1.7825e-10	9.3628e-12	1.7326e-03

Table 4. Right bound of error (when $r = 0.5, n = 4$)

t	E_{SADM}	$E_{MADM(1999)}$	$E_{MADM(2001)}$
0	1.4430e-06	2.4656e-07	2.4318e-04
0.2	1.3913e-06	2.3799e-07	2.2940e-04
0.4	1.2778e-06	2.2128e-07	5.8890e-05
0.6	1.1958e-06	2.1721e-07	9.9350e-04
0.8	1.3305e-06	2.6890e-07	4.8597e-03
1.0	2.6568e-06	5.4505e-07	1.5303e-02

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From the results of Table 1-Table 4, it is easy to obtain that the MADM which decomposes the source function into two parts improves the accuracy compared with SADM, and reduces the amount of calculation. Compared with SADM, MADM, which decomposes the source function into series, minimizes the amount of calculation, but also reduces the accuracy.

6. CONCLUSIONS

In this paper, the one-dimensional fuzzy number set is regarded as a closed convex dimension of Banach space, and the existence and uniqueness of solutions of nonlinear fuzzy Volterra-Fredholm equations are analyzed by using abstract theory. The nonlinear part of the equation is approximated by Adomian polynomials. By analyzing the results of several numerical examples, it is found that under the premise that the source term can be decomposed, the MADM obtained by decomposing the source term function into two terms has higher accuracy and better effect than SADM. Although the MADM obtained by series expansion of the source function greatly reduces the amount of calculation, it also reduces the accuracy compared with SADM.

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