Simple Application of AOA Model in Multi machine Cooperative Passive Location

Shiyi Liu, Zhanbo Wang, Xin Hu

School of Mathematics and Computational Science, Hunan University of Science and Technology, Xiangtan, 411100, China

Abstract

Unmanned aerial vehicle (UAV) location is an important tool to obtain information. Bearing only passive location has been paid much attention because of its advantages of small computation and high accuracy. This paper starts from the relevant theories, analyzes the geometric characteristics of the problem, establishes coordinates based on the formation of UAVs, builds a model to solve the coordinates of UAVs with Matlab, and finally solves the problem with iterative thinking. For the first question of question 1, we establish a polar coordinate system with the central UAV as the pole O and discuss the position relationship according to the characteristics of the problem. We use the sine theorem to list the trigonometric equation, specify that the deviation range of the receiving UAV is at a reasonable position, use the phase angle to roughly estimate the orientation between UAVs, and then use the algorithm to calculate the approximate positioning of the UAV, According to the requirements of angle only positioning given by the topic, AOA passive positioning algorithm is used, and Matlab software is used to assist calculation. By giving the form of polar diameter and polar angle, the passive positioning model of any UAV in the circular surface is established. For the second question of question 1, based on the first question model, another UAV is selected as the signal sending machine on the basis of the known signal sending UAV, which cannot make accurate positioning. Therefore, we assume that two new UAVs with unknown numbers can meet the requirements of the topic and establish a relationship model of UAV positions. Through the method of traversal, the unknown number UAV and the known UAV are successively brought into the first question model to solve, and different coordinate sets are obtained. The intersection of coordinate sets is the corresponding position of the unknown UAV. In addition to the known sending UAV, at least two sending UAVs can be added to effectively locate the UAV.

Keywords

AOA passive location model, Iterative thought, Traversal search algorithm, Local optimal adjustment.

1. INTRODUCTION

1.1. Background

No matter in the field of modern commerce or military technology, the development and research of UAV is gradually valued by all countries because of its sensitive, compact, easy to operate and other characteristics[1]. The bearings only passive location method has the advantages of strong concealment, strong anti-interference, strong endurance, and is widely used in navigation and wireless networks[2]. The combination of the two represents an advanced development trend. When UAV clusters fly in formation, in order to reduce

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

interference from other factors, bearings only passive positioning[3][4] is preferred to locate UAVs, and the formation is adjusted through the received direction information[5].

1.2. Problem Restatement

In a UAV cluster flying in a certain formation, each UAV has a corresponding fixed number, and the relative position of each UAV in the cluster remains unchanged.

Question 1: There are ten UAVs forming a circular formation (as shown in Figure 1), among which nine UAVs numbered FY01~FY09 are evenly distributed on a circle with the number FY00 as the center. It is known that these 10 UAVs are always at the same altitude during flight based on the altitude information they receive.

Now, 3 UAVs with known numbers in the center of the circle and formation are transmitting signals, and 7 UAVs with slightly deviated positions are receiving signals. It is necessary to establish a positioning model to solve the position of UAVs with passive receiving signals.

It has been determined that FY00 and FY01 are transmitting signals, and there are several unmanned aerial vehicles with unknown numbers transmitting signals. If the unmanned aerial vehicles transmitting signals have no deviation, how many more unmanned aerial vehicles are required to transmit signals in addition to FY00 and FY01 in order to accurately locate the position of the remaining unmanned aerial vehicles?

Keep formation, and the nine UAVs are evenly distributed on the circumference with the other one as the center and 100m as the radius. The initial position of the UAV is known, and there is a deviation in the position at this time. At present, up to 3 UAVs on FY00 and the circumference are selected to transmit signals. The remaining UAVs are adjusted to the position without deviation according to the received direction information, ignoring the adjustment time, and finally 9 UAVs are evenly distributed on a certain circumference. Please provide the adjustment scheme.

Question 2: In practice, the UAV cluster can also be in a conical formation (as shown in Figure 2), and the distance between two adjacent UAVs is the same, so the adjustment scheme of UAV is still given using bearings only passive location method.

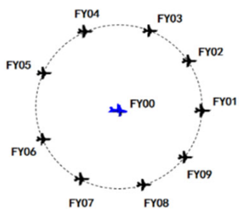

Figure 1. Schematic Diagram of Round UAV Formation

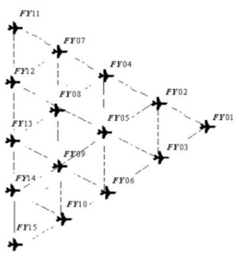
Figure 2. Schematic Diagram of Conical UAV Formation

2. PROBLEM ANALYSIS

This paper mainly determines the position of the UAV receiving the signal based on the bearings only passive location method of the UAV, and gives an adjustment scheme accordingly, so that the UAV can be adjusted to a given formation.

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

2.1. Analysis of Question 1

The first question: It is mainly required to use the UAV FY00 located at the center of the circle, and select any two UAVs FY0M and FY0N located on the circumference to send signals. The remaining seven UAVs will give positioning according to the received direction information and the selected position of the three UAVs without deviation. The polar coordinate system is established by taking FY00 as the pole and making the polar axis with the ray of the line segment where FY0M (or FY0N) is located. Once FY0M and FY0N are selected, the corresponding polar coordinates can be obtained by their numbers. The trigonometric function is used to calculate the relationship between angles, and the approximate solution of the angle is obtained by iteration in Matlab[6]. The polar coordinates of the other seven UAVs with position deviation can be further obtained for positioning.

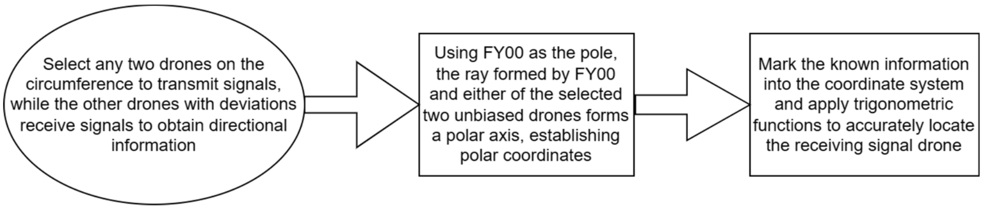

Figure 3. Flow chart of the first question and answer

The second question: FY00 and FY01 are required to transmit signals according to the known topic. In addition, there are several UAVs with unknown number and unknown number transmitting signals. First, consider that there are three UAVs sending signals. In addition to FY00 and FY01, another UAV is selected on the circumference to send signals, and another UAV is selected to receive signals. At this time, a polar coordinate point of the UAV receiving signals can be obtained by substituting the model of the first question. Assuming that FY0M is selected as the UAV sending signals, FY00, FY01 and FY0M send signals, and FY0J receives signals to obtain a polar coordinate about FY0J(ρ_{jm},θ_{jm}) , then when FY0M takes all possibilities except FY0J, a total of 7 polar coordinates are obtained, and these 7 coordinates are recorded as setsA_1;Assuming that FY0N is selected as another UAV to send signals, FY00, FY01 and FY0N send signals, and FY0J receives signals to obtain a polar coordinate about FY0J(ρ_{jn},θ_{jn}) , then when FY0N takes all possibilities except FY0J, a total of 7 polar coordinates are obtained, and these 7 coordinates are recorded as sets A_2;There must be one $\tau\epsilon A_1$ And $\tau\epsilon A_2$, τ It is the coordinate of UAV FY0J with position deviation.

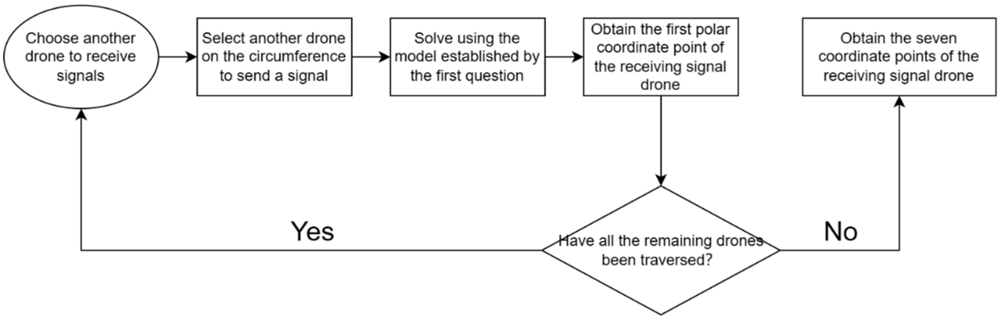

Figure 4. Flow chart of the second question and answer

The third question: This question gives the radius of the round formation that the UAV group needs to maintain, that is, the distance from all UAVs in the formation to the central UAV numbered FY00.After a preliminary analysis of the data given in the question, the given initial

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

position of the UAV is displayed on the coordinate axis, it can be clearly found that the initial formation is not a regular circle, and it does not meet the requirements of formation in the question. Through further analysis of the subject, we know that we need to adjust the distance between each UAV and FY00, and also need to keep the center angle formed by the connection between each two adjacent UAVs and the central UAV of FY00 the same. For a given bivariate problem, our most common idea is to reduce the dimension of the problem model, first solve the problem arising from a variable, and then transform the problem into a more familiar one-dimensional problem[7]. The two variables in this question make it easier to solve the distance between each UAV in the formation and the FY00 central fighter first, because each UAV is on the line from the central UAV to its original initial position, which means that if the UAV does not need to receive other information, we can first let the UAV return to the coordinates given by us along this line, Then, two error free UAVs, FY00 and FY01, start to fine tune the angle of the aircraft already in other circles, find the standard position of each UAV in the formation, give the adjustment plan, and complete the solution of the third problem.

2.2. Analysis of Question 2

In this question, the formation of UAVs becomes a conical formation, and the distance between any two UAVs is required to be equal. First, fix the number FY05 as the center of formation circle, group the remaining UAVs, draw concentric circles, and use geometric characteristics to plan, so that the ideal positions of as many UAVs as possible are located on the circumference[8]. For the remaining UAVs $FY11$、$FY15$After the other UAVs have adjusted their positions, the received signals will be used for adjustment.

3. GENERAL ASSUMPTIONS

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

1. Assume that the radius of circular formation is known, which is r;

2. It is assumed that there is no influence of electromagnetic environment and other factors, that is, there is no error in the direction information received by the UAV;

3. Assume that the adjustment time is ignored when the UAV is adjusting;

4. Assume that the position of UAVs in all formations is only slightly deviated;

5. It is assumed that there are no extreme weather conditions or unmanned aerial vehicles out of control during flight.

4. VARIABLES AND ABBREVIATIONS

Symbol	Definition	Units
r	Radius of circular formation	m
d	Spacing between two adjacent UAVs in conical formation	m
α_i	Direction information received by UAV	degree
ρ	Distance between passive signal receiving UAV and circle center	m
θ	Direction angle of passive signal receiving UAV in polar coordinates	degree
β	The angle between two signal transmitting UAVs on the circumference and the pole	degree
FY0I	No. of the first aircraft	/
A_i	Get the set of UAV coordinates	/
b_i	UAV number in polar coordinates	/
($x_{acp}{}^{New}$,	New position coordinates of UAV after one iteration	/

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

5. MODELING AND SOLVING

5.1. Model I: UAV positioning and adjustment

According to the information in the question stem, it can be seen that the relative position relationship between UAVs in formation remains unchanged, and 10 UAVs are always at the same altitude during flight, so it can be converted into a model on a two-dimensional plane.

5.1.1 Model I-1: Using planar geometry to locate UAVs

Optional UAV FY0M and FY0N on the circumference,$M, N \in \{1,2,3,4,5,6,7,8,9\}$, as two other UAVs transmitting signals on the circumference;

On the two-dimensional plane, point M represents UAV FY0M, and point N represents UAV FY0N;Take FY00 as pole O and ray OM as polar axis to establish polar coordinate system, then $M(r,0)$, $N(r, \frac{2}{9}\pi|M-N|)$ .Assume that the radius r of circular UAV formation is known. Select any one UAV FY0D with slight position deviation,$D \in \{1,2,3,4,5,6,7,8,9\}$, mark as point $D(\rho,\ \theta)$.

After selecting M point, set the number of FY0M tob_1, number increases tob_9, FY0N number is b_n, FY0D Nob_d.Specify current number$b_i, i > 9$When, represents the number b_{i-9}UAV.

The figure is as follows:

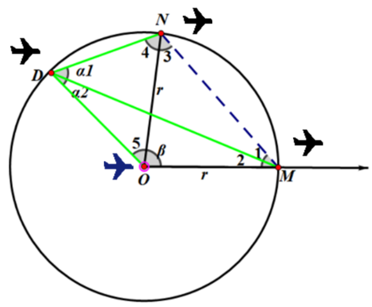

Figure 5. Polar coordinate model diagram

According to the question, click D to get three angles of information. We define:

$$\angle NDM = \alpha_1,\ \angle MDO = \alpha_2\ \ \angle NDO = \alpha_3,\ \ \angle NOM = \beta = \frac{2}{9}\pi|M-N|$$
$$\angle NMD = \angle 1,\ \angle DMO = \angle 2,\ \angle DNO = \angle 4,\ \angle MDO = \angle 5. \quad (1)$$

1) Scenario 1: Number of signal receiving UAV FY0D b_d Number of the third signal transmitting UAV FY0Nb_nSatisfaction formula: $b_n < b_d < b_{n+3}$.

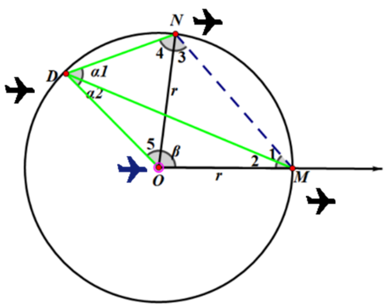

Figure 6. Polar model diagram of case 1

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

stayΔDONUsing the sine theorem

$$\frac{DO}{\sin\angle 4} = \frac{DN}{\sin\angle 5} = \frac{r}{\sin\alpha_3} , \tag{2}$$

令$\frac{r}{\sin\alpha_3} = k$, from which we can get

$$\begin{cases} DO = k\cdot\sin\angle 4 \\ DN = k\cdot\sin\angle 5 \end{cases} , \tag{3}$$

Applying the law of sines in ΔDONThe sum of known triangle internal angles is 180 °, that is

$$\angle 1+\angle 2 + \alpha_3 = 180° , \tag{4}$$

Applying the law of sines in ΔMNDUsing the sine theorem

$$\frac{MN}{\sin\ \alpha_1} = \frac{DN}{\sin\ \angle 1} \tag{5}$$

Applying the law of sines in ΔMODUsing the sine theorem

$$\frac{MO}{\sin\ \alpha_2} = \frac{DO}{\sin\ \angle 2} \tag{6}$$

$$\angle 1 +\angle 2 = \frac{\pi-\beta}{2} \tag{7}$$

$$\alpha_1+\alpha_2=\alpha_3 \tag{8}$$

Combined with Formula (4), (5), (6), (7), (8), we can get$\angle 1$， $\angle 2$， $\angle 4$， $\angle 5$， And write an appendix program in Matlab. Given an initial value ∠ 1, the values of ∠ 4 and ∠ 5 can be calculated through iteration,

$$\begin{cases} \angle 4=\eta_1 \\ \angle 5=\eta_2 \end{cases} , \tag{9}$$

Applying the law of sines in $\triangle DON$ using the sine theorem again in

$$\frac{\rho}{\sin\eta_1} = \frac{r}{\sin\alpha_3} , \tag{10}$$

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

Solved $\rho = r \cdot \frac{\sin\ \eta_1}{\sin\ \alpha_3}$,

And

$$\theta = \beta + \eta_2 , \tag{11}$$

2) Scenario 2: Number of signal receiving UAV FY0D b_d Number of the third signal transmitting UAV FY0Nb_nSatisfaction formula: $b_{n+2} < b_d < b_{n+5}$.

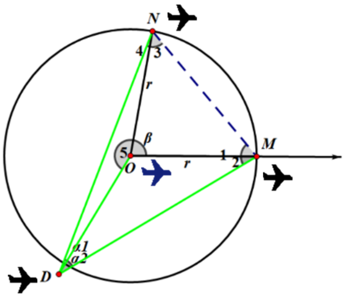

Figure 7. Polar model of case 2

At this time, the expressions of (4), (5) and (6) remain unchanged, but

$$\alpha_1 = \alpha_2 + \alpha_3, \tag{12}$$

$$\angle 1 - \angle 2 = \frac{\pi - \beta}{2} , \tag{13}$$

Causativeη_1， η_2Solution transformation, but the coordinates of FY0D$(\rho, \ \theta)$The expression is still the same as case 1

3) Scenario 3: Number of received signal UAV FY0Db_dNumber of the third signal transmitting UAV FY0Nb_nSatisfaction formula: $b_{n+4} < b_d < b_{n+7}$.

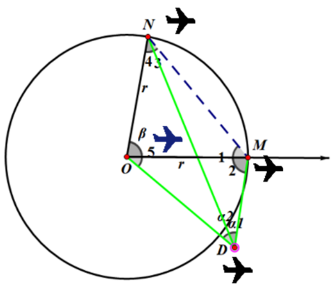

Figure 8. Polar model of case 3

At this time, the expressions of (4), (5) and (6) remain unchanged, but

$$\alpha_2 = \alpha_1 + \alpha_3, \tag{14}$$

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

$$\angle 1 - \angle 2 = \frac{\pi - \beta}{2} , \tag{15}$$

Causative η_1， η_2 Solution transformation, D point polar coordinates (ρ, θ) by

$$\begin{cases} \rho = r \cdot \dfrac{\sin \eta_1}{\sin \alpha_3} \\ \theta = \beta - \eta_2 \end{cases} , \tag{16}$$

4) Scenario 4: Number of received signal UAV FY0D b_d Number of the third signal transmitting UAV FY0N b_n Satisfaction formula: $b_{n+6} < b_d < b_{n+9}$.

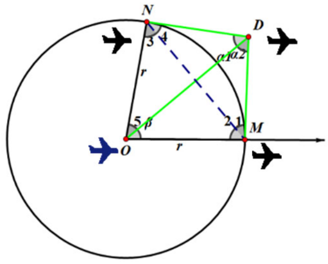

Figure 9. Polar Model of Case 4

At this time, the expressions of (4), (5) and (6) remain unchanged, but

$$\alpha_1 = \alpha_2 + \alpha_3, \tag{17}$$

$$\angle 2 - \angle 1 = \frac{\pi - \beta}{2} , \tag{18}$$

Causative η_1， η_2 Solution transformation, but the coordinates of FY0D (ρ, θ) The expression is still the same as case 3

In summary:

When point D is numbered $b_d \in (b_n, b_{n+5})$ Its polar coordinates (ρ, θ) by

$$\begin{cases} \rho = r \cdot \dfrac{\sin \eta_1}{\sin \alpha_3} \\ \theta = \beta + \eta_2 \end{cases} , \tag{19}$$

When point D is numbered $b_d \in (b_{n+4}, b_{n+9})$ Its polar coordinates (ρ, θ) by

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

$$\begin{cases} \rho = r \cdot \dfrac{\sin\ \eta_1}{\sin\ \alpha_3} \\ \theta = \beta - \eta_2 \end{cases}, \tag{20}$$

Where, angle α_3 =Angle NDO , Angle η_1 =Angle DNO , Angle η_2 =Angle DON .UAV Nob_i, $i > 9$When, represents the numberb_{i-9}UAV.

5.1.2 Model I-2: Positioning comparison method

According to the known topic, FY00 and FY01 are required to transmit signals. In addition, there are several UAVs with unknown number and unknown number transmitting signals. In the case of only two launching aircraft, it is impossible to achieve effective positioning of UAVs. Now a UAV FY0J is selected（$J \in \{2,3,4,5,6,7,8,9\}$Receive signal.

First, consider to select another UAV FY0M at random except FY00 and FY01（$M \in \{2,3,4,5,6,7,8,9\}$）transmit signals. The situation of selected UAVs is discussed by using the ergodic search method. For each selected UAV to transmit signals, the model of the first question can be used to get the information about FY0J(ρ_{jm}, θ_{jm}）When all UAVs except FM0J are taken, 7 different coordinates are obtained and recorded asA_1,

$$A_1 = \{(\rho_{jm}, \theta_{jm}), m = 2, \ldots, j-1, j+1, \ldots, 9\} \tag{21}$$

At this time, it is impossible to determine which coordinate is the true FY0J coordinate.

Now, 1 UAV FY0N except FY00 and FY01 is selected（$N \in 2,3,4,5,6,7,8,9$，$N \neq M$）Transmit signals. After each UAV other than FM0J is selected, 7 different coordinates about FY0J are obtained again and recorded asA_2,

$$A_2 = \{(\rho_{jn}, \theta_{jn}), n = 2, \ldots, j-1, j+1, \ldots, 9\} \tag{22}$$

There must be a coordinate $\tau \epsilon A_1 \cap A_2$Is the position information of UAV with slight deviation.

Model I-3: Local optimal adjustment model

i. First of all, according to the polar axis and polar diameter of the UAV in the polar coordinate system given in the title, combined with the coordinate system established with the center UAV FY00 as the origin in the first two questions, we draw the initial position distribution of the formation UAV, as shown in the following figure,

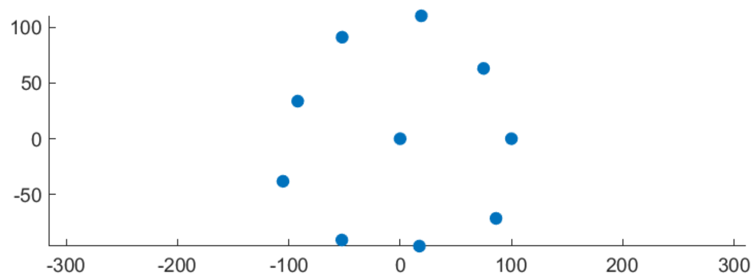

Figure 10. Initial position of UAV

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

After visualizing the initial position of UAV formation, it can be clearly observed that nine UAVs form an irregular circle around FY00 as the center.

ii. After the distance between the initial position of each UAV and the center of the circle is controlled to be 100 meters, we can obtain the ideal position coordinates of the No. FY02 UAV through the initial positioning of the two UAVs FY00 and FY01 without deviation and the ideal state of uniform distribution. The topic requires us to select the No. FY00 UAV and at most three UAVs on the circumference to transmit signals each time, The remaining UAVs are adjusted to the ideal position according to the received direction information (the time of each adjustment is ignored), so that the nine UAVs are finally evenly distributed on a certain circle, which means that the angular information formed between each UAV receiving information and the UAV is consistent. Based on this, we can further calculate and discuss.

iii. In order to facilitate the calculation, and in combination with the three-point positioning method on satellite positioning[9][10], based on this, we know that three UAVs can determine the position of a UAV at the signal receiving end. Then for each UAV, we assume that except for FY00, we can use the two UAVs farthest away from it as the signal sending end, and the coordinates of the UAV farthest away from it,We can get it from FY00 and FY01 with no deviation in initial position, as shown in the following figure.

iv. Because nine UAVs are finally evenly distributed on a certain circumference, the included angle between any UAV receiving signals and the two farthest UAVs at the signal sending end should be the same and fixed. We call this set included angle target as the target value temporarily.

v. That is, the two inclinations are identical, so the overall algorithm flow is as follows:

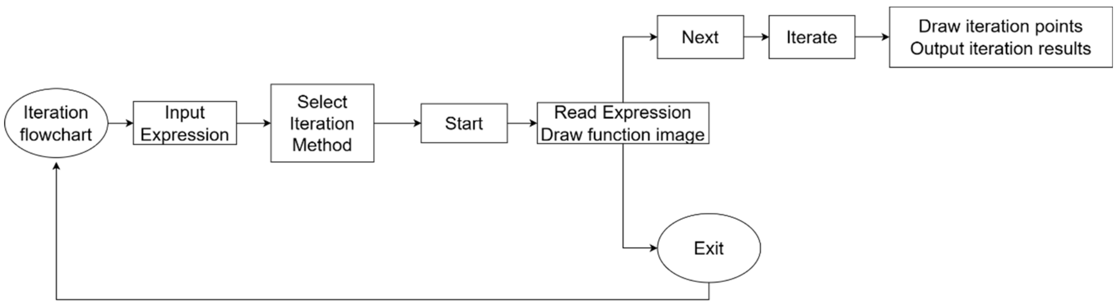

Figure 11. Iteration Flow Chart

Step 1: Calculate the included angle between each UAV and the two aircraft with the farthest linear distance on the circle except FY00 central UAV, namely （$\theta^t = \theta^1 + \theta^2 + \theta^3$）， the number 't' represents the t-th cycle, forming an array; According to the three-point positioning method, the UAV tilt angle calculation[1][2]for the i-th signal receiving end[11][12] can be expressed as

$$\begin{cases} \theta_1 = arc\cos \dfrac{(f_{x1} - x_{acp}, f_{y1} - y_{acp}) \cdot (f_{x1} - x_{acp}, f_{y1} - y_{acp})}{|(f_{x1} - x_{acp}, f_{y1} - y_{acp})| \cdot |(f_{x1} - x_{acp}, f_{y1} - y_{acp})|} \\ \theta_2 = arc\cos \dfrac{(f_{x2} - x_{acp}, f_{y2} - y_{acp}) \cdot (f_{x2} - x_{acp}, f_{y2} - y_{acp})}{|(f_{x2} - x_{acp}, f_{y2} - y_{acp})| \cdot |(f_{x2} - x_{acp}, f_{y2} - y_{acp})|} \\ \theta_3 = arc\cos \dfrac{(f_{x3} - x_{acp}, f_{y3} - y_{acp}) \cdot (f_{x3} - x_{acp}, f_{y3} - y_{acp})}{|(f_{x3} - x_{acp}, f_{y3} - y_{acp})| \cdot |(f_{x3} - x_{acp}, f_{y3} - y_{acp})|} \end{cases} \tag{23}$$

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

among $(fx_i,\ fy_i)$, $i \in [1,2,3]$ It is the spatial coordinate position of three signal sending UAVs, $(fx_1,\ fy_1)$ Is the space coordinate position of FY00 UAV, and according to the subject description, we need to keep the relative position of FY00 UAV fixed, that is, always $(fx_1,\ fy_1) = (0,0)$.The spatial coordinates of the UAV receiving signals at the same time are $({x_{acp}}^{New}, {y_{acp}}^{New})$

Step 2: Set the iterative update formula of coordinates of each UAV. Here, our constraint is that "the other nine UAVs are evenly distributed on the circumference with a radius of 100m" given by the title as the constraint. Under the large condition that the height of each UAV remains unchanged, the UAV moves in a two-dimensional space coordinate system. Then, the coordinates of each receiving end UAV can be iteratively updated as

$$({x_{acp}}^{New}, {y_{acp}}^{New}) = (x_{acp} + \Delta x,\ y_{acp} + \Delta y) \tag{24}$$

At the same time, there are restrictions, namely

$$\begin{cases} \sqrt{(fx_1 - {x_{acp}}^{New})^2 - (fy_1 - {y_{acp}}^{New})^2} = 100 \\ \theta_2 = \theta_3 \end{cases} \tag{25}$$

The specific update process is as follows:

Step 2-1: Adjust the distance between all UAVs and the central UAV FY00 to ensure

$$\sqrt{(fx_1 - {x_{acp}}^{New})^2 - (fy_1 - {y_{acp}}^{New})^2} = 100 \tag{26}$$

Step 2-2: According to simple plane geometry knowledge, it is easy to know that if we can move the aircraft to be positioned and adjusted (recorded as FY0N) to the middle vertical line of the connection between the two UAVs furthest away from it, and ensure that the distance between FY0N and FY00 is always 100, then we can get an ideal figure for FY0N.

Step 2-3: Repeat this iteration for each point, and calculate the $\theta^t = \theta^1 + \theta^2 + \theta^3$

Step 2-4: Judge the objective function $\theta_1^t = \theta_2^t = \cdots = \theta_7^t = \theta_8^t = \theta_9^t$ If it is true or the precision reaches a certain value, it will exit the loop. If it is not true, it will return to Step2-1 to continue the iterative update.

(The Matlab code in the above iterative solution process is stored in the directory. After 12 iterations, we calculated the standard coordinates that the UAV should reach and stored in the variable adjustCarcoor)

After the precision reaches our target value, we stop iteration to get the coordinates we need, and then we can give the adjustment scheme of UAV from the initial position to the standard position according to the coordinates, as shown in the following table,

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

Table 1. Adjustment Scheme of UAV

UAV Number	direction	Forward length/M	Signal sending UAV
1	No adjustment required	\	\
2	Adjust 0.1 ° to east by north	-2	FY05、FY06
3	Adjust 0.21 ° to the west by east	-12	FY07、FY08
4	Adjust 0.25 ° to north by west	-5	FY08、FY09
5	Adjust 0.14 ° to north by west	2	FY09、FY01
6	Adjust 0.04 ° to the west by south	-12	FY01、FY02
7	Adjust 0.07 ° to south by west	-5	FY02、FY03
8	Adjust 0.17 ° to east by south	2	FY03、FY04
9	Adjust 0.28 ° to east by south	-12	FY04、FY05

Explanation to the above table: we need to adjust the UAVs one by one. Every time we adjust a UAV, we will query the UAV number needed to determine its position according to the above table. After the two UAVs are selected, we can send adjustment instructions to the UAVs to be adjusted through their direction information.

The following figure shows the new UAV formation after adjustment. It can be seen that the formation is evenly distributed on a certain circumference

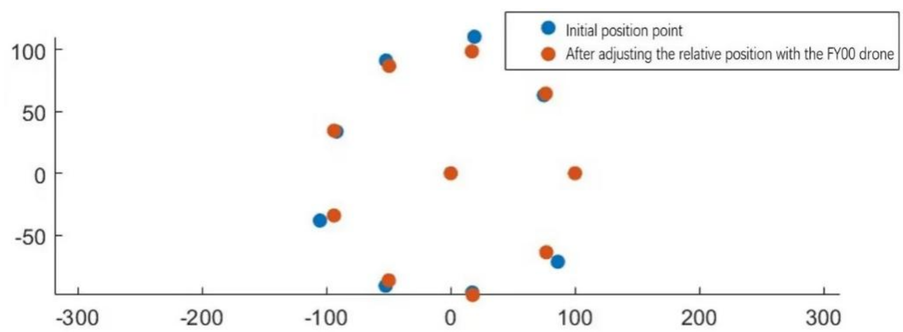

Figure 12. Polar coordinate distribution of UAV after and before adjustment

5.2. Model II: Planar circle based queue reordering

The goal of this problem is to make the whole cluster of UAVs become a standard conical formation through an adjustment scheme, that is, any two adjacent UAVs have the same distance. It is assumed that the UAV position of the cluster is only slightly biased.

Step 1: Convert to round formation

First, the target conical formation is transformed into circular formation. With $FY05$ as the center of the circle, divide $FY05$、$FY11$、$FY15$The other 12 UAVs are divided into three groups. The radius considered is$r$Circumference of O_1 And divide them into 12 equal parts. The first group is UAVs $FY02, FY03, FY04, FY06, FY08$,

$FY09$, the ideal location is evenly distributed in O_1 upper; The second group is UAVs $FY01, FY07, FY10, FY13$ the ideal location is at a radius of $\sqrt{3}r$ on the circumference of; The third group is UAVs $FY12, FY14$, the ideal location is at a radius of $2r$ on the circumference of.

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

And $FY11, FY15$ unable to O_1 the corresponding position is found on the 12 bisector of the, so wait for the other 13 UAVs to adjust before adjusting.

UAV grouping is shown in the figure below.

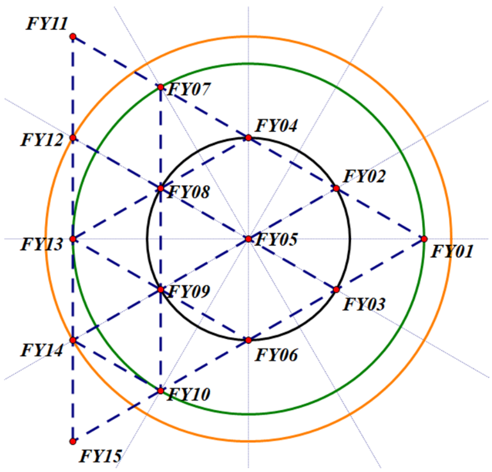

Figure 13. Ideal Position Distribution of UAV

Step 2: Adjust formation

In the third question of question 1, we have given a specific scheme to adjust UAVs to be evenly distributed on a certain circle. Here we can directly use this scheme to iterate three groups of UAVs distributed on different circles, so that they are in an ideal position.

For $FY11$、$FY15$At this time, the other UAVs are in the ideal position, so only two sets of random signal information received by them can be used to determine their angle information.

The distance can be determined by sine theorem and angle information.

6. MODEL ANALYSIS

AOA positioning accuracy analysis:

When positioning the UAV receiving radiation, due to various measurement errors, electromagnetic environment and other factors, there will be a certain error between the estimated result of the UAV to be located and its true value, which is the positioning error. Generally, the geometric precision factor (GDOP) is used as the index to measure the positioning accuracy, The size of GDOP value indicates the positioning accuracy. The larger the GDOP value is, the lower the positioning accuracy is. On the contrary, the higher the positioning accuracy is.

The position coordinates of M UAVs in two-dimensional space are$u_i = [x_i, y_i]^T, i = 1,2, \dots, M$, unknown received signal UAV source location coordinate is $s=[x,y]^T$ According to the geometric relationship, the azimuth measured by each UAV is:

$$\theta_i = \tan^{-1}\left(\frac{y-y_i}{x-x_i}\right), \quad i=1, 2, \cdots M \tag{27}$$

Differentiate both sides of the above equation at the same time:

$$d\theta_i = g_{ix}dx + g_{iy}dy + k_i, \quad i=1, 2, \cdots M \tag{28}$$

Where,$g_{ix} = -(y-y_i)/r_i^2$, $g_{iy} = (x-x_i)/r_i^2$, $k_i = -g_{ix}dx_i - g_{iy}dy_i$

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

Sort the upper differential into matrix form,

$$dz = C \cdot ds + dX_s \tag{29}$$

Where,$dz = [d\theta_1, d\theta_2, \cdots, d\theta_M]^T$ Is the error caused by AOA measurement,$ds = [dx, dy]^T$ Is the positioning error of the radiation source,$dX_s = [k_1, k_2, \dots, \ k_M]^T$ It refers to the error caused by the UAV itself, C is the coefficient matrix, and its value is:

$$C = \begin{vmatrix} g_{1x} & g_{1y} \\ g_{2x} & g_{2y} \\ \vdots & \vdots \\ g_{Mx} & g_{My} \end{vmatrix} \tag{30}$$

Use the least square method to solve the differential equation in the form of the upper matrix, and the location error of the radiation source is:

$$ds = (C^T C)^{-1} C^T (dz - dX_s) \tag{31}$$

It can be seen from the above formula that the location error of the radiation source is related to the AOA measurement error and the UAV site error. Order $F = (C^T C)^{-1} C^T$ The covariance matrix of radiation source location error can be obtained as follows:

$$P_d = E\{ds \cdot ds^T\} = F\{E\{dz \cdot dz^T\} + \{dX_s + dX_s^T\}\} F^T \tag{32}$$

Where,

$$E\{dz \cdot dz^T\} = \begin{bmatrix} \sigma_{\theta_1}^2 & & \\ & \ddots & \\ & & \sigma_{\theta_M}^2 \end{bmatrix} \tag{33}$$

$$E\{dX_s \cdot dX_s^T\} = \begin{pmatrix} g_{1x}^2 \sigma_{x1}^2 + g_{1y}^2 \sigma_{y1}^2 & & \\ & \ddots & \\ & & g_{Mx}^2 \sigma_{xM}^2 + g_{My}^2 \sigma_{yM}^2 \end{pmatrix} \tag{34}$$

Among them,σ_{θ_i} is the standard deviation of the standard error of AOA measurement of the i-th UAV. For the convenience of calculation, it is assumed that the site errors of each UAV are equal, i.e.$\sigma_{xi}^2 = \sigma_{yi}^2 = \sigma_s^2, i = 1, 2, \cdots M$.

Because $g_{ix}^2 + g_{iy}^2 = 1/r_i^2$, the above formula can be simplified as:

$$GDOP = \sqrt{trace(P_d)} \tag{35}$$

Based on the AOA positioning method, multiple UAVs can estimate the position of the UAV only by knowing the azimuth of the UAV to be positioned relative to other UAVs. The principle is relatively simple, but in practical engineering applications, its positioning accuracy is not high and its real-time performance is poor, because the AOA positioning accuracy is greatly affected by the multipath effect and has high requirements for the antenna, Therefore, the combination of AOA and TDOA[13] can achieve higher positioning accuracy.[14]

7. MODEL EVALUATION

7.1. The Advantages of Model

1. The first question provides a model for deriving the coordinates of drones receiving signals in general, making it easier to solve the positioning problem in circular formations.

2. When iteratively solving, we set the step size to 0.1° , with smaller values, which can improve the accuracy of the results.

3. This article has a high level of hierarchy. The positioning model and adjustment plan model in this article are simplified to complex, and gradually progress according to the requirements of the topic.

7.2. The Disadvantages of Model

1. When establishing the model, we did not consider electromagnetic interference and other situations, that is, we did not consider the error of the direction information received by the drone, which may cause certain deviations in the model solution results and have poor adaptability to more extreme situations.

2. Due to the small step size setting and long program running time, the solution cannot be completed quickly, which makes it difficult for the model to be adopted in practice.

3. The objective function of minimizing the control of drone target position adjustment using computers is an iterative process at the initial point, and the computational complexity is closely related to the setting of step size, making it impossible to adapt to general situations.

7.3. Extension of Model

This article abstracts drones as geometric points, but in reality, it does not use any of the characteristics of drones. Therefore, the model in this article can be used for spacecraft positioning, ship positioning, etc.

REFERENCES

[1] Liu Peixia. Application and trend Prospect of UAV Detection and reaction equipment Technology[J]. China Security & Protection,2023,No.202(Z1):41-46.

[2] Fang Jian. Research on Localization Methods under Multi-Scale Coverage in Heterogeneous Wireless Networks[D].Dalian University of Technology, 2022. DOI:10.26991/d.cnki.gdllu.2022.003697.

[3] ZHANG Hui，CUI Yongqiang. Research on mobile multi-station passive localization technology based on UAV platform[J/OL].Journal of Hubei University (Natural Science):1-5[2023-04-18]. http://kns.cnki.net/kcms/detail/42.1212.N.20230302.1647.002.html.

[4] ZHANG Min, ZHANG Wenjun, LI Xi, et al. Passive localization by multiple observers based on the phase difference of the arrival of a long baseline interferometer[J/OL].Journal of Electronics & Information Technology:1-9[2023-04-18].http://kns.cnki.net/kcms/detail/11.4494.tn.20230315.0918.004.html.

ISSN: 2472-3703 DOI: 10.6911/WSRJ.202308_9(8).0013

[5] Zhang Pei, You Xiaoming, Liu Sheng .Ant colony algorithm based on dynamic hierarchical clustering and neighborhood recombination[J/OL].Application Research of Computers:1-10[2023-04-18]. https://doi.org/10.19734/j.issn.1001-3695.2022.11.0556.

[6] HU Jiawei, JIA Zequn, SUN Yantao, et al. Survey of Analysis and Solutions for Multi-UAV Cooperative Mission Planning Problem Under Multi-constraint Conditions[J/OL]. Computer Science:1-27[2023-04-18]. http://kns.cnki.net/kcms/detail/50.1075.TP.20230407.1855.026.html.

[7] WANGH X, FANGL Y, BU S J, et al. Dimension reduction for longitudinal databased on martingale difference divergence [J. Chinese J Appl Probab Statist, 2023,39(1): 132-158.(in Chinese)

[8] Shen Peiping, Wang Yafei, Wu Dianxiao. lterative a geometric programming method for solving a class of minimax fractional optimization problems[J/OL]. Journal of Henan Normal University(Natural Science Edition),2023(02):56-62[2023-04-18].https://doi.org/10.16366/j.cnki.1000-2367.2023.02.006.

[9] LIU Yunbing , ZHAO Yuan , LI Zhe , et al. Research on Metering Terminal Positioning Algorithm Based on Base Station[J]. Electric Power Information and Communication Technology, 2021, 19(05): 53-58.DOI:10.16543/j.2095-641x.electric.power.ict.2021.05.008.

[10] Zhou Shuhan, Chen Mingjian, Jin Xin, et al. Doppler Positioning Performance Analysis of LEO Communication Satellites[J]. Acta Astronomica Sinica.

[11] MA Fangli, XU Yang, XU Peng. Comparison of AOA localization in ultrashort wave under various azimuth calculation methods[J]. Journal of Southwest Jiaotong University,2021,56(4):713-719.

[12] XU Guo-xun, LIANG Xiao-long, WANG Wei-jia, ZHANG Jia-qiang. Analysis on the optimal formation configuration of passive localization using AOA[J]. Transducer and Microsystem Technologies, 2017,36(09):57-60. DOI: 10.13873/j.1000-9787(2017)09-0057-04.

[13] Song Weiwei, Lin Wei, Lou Yidong, et al. Research on TDOA Positioning with BDS+5G Space-Time Datum, 2022, DOI: 10.13203/j. whugis20220348

[14] Zhu Shaoguang. Research on Multi-UAV Cooperative Path Planning for Target Positioning and Tracking[D]. Xia n University, 2021.DOI: 10.27389/d.cnki.gxadu.2021.002037.